

Towards Quantum Superpositions of a Mirror: an Exact Open Systems Analysis*

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We analyze the recently proposed mirror superposition experiment of Marshall, Simon, Penrose, and Bouwmeester, assuming that the mirror's dynamics contains a non-unitary term of the Lindblad type proportional to $-[q, [q, \rho]]$, with q the position operator for the center of mass of the mirror, and ρ the statistical operator. We derive an exact formula for the fringe visibility for this system. We discuss the consequences of our result for tests of environmental decoherence and of collapse models. In particular, we find that with the conventional parameters for the CSL model of state vector collapse, maintenance of coherence is expected to within an accuracy of at least 1 part in 10^8 . Increasing the apparatus coupling to environmental decoherence may lead to observable modifications of the fringe visibility, with time dependence given by our exact result.

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There is currently much interest in experiments to create quantum superposition states involving large numbers of particles, with the ultimate aim of testing whether quantum superpositions of macroscopic systems can be observed. Experiments performed to date involve the superposition of oppositely circulating currents in superconducting rings [1], and the diffraction of large molecules [2], which give results in accord with quantum mechanics.

Recently, Marshall *et al* [3] proposed a novel experiment which appears to be very promising, as the system to be superposed can contain up to 10^{14} atoms, within present-day technology. The experimental setup consists of an interferometer for a single photon: in each arm of the interferometer there is a cavity, and one of them (let us say, cavity A) has a movable mirror mounted on a cantilever. Under suitable assumptions, the Hamiltonian describing the composite system is [4]:

$$H = \hbar\omega_c(a_A^\dagger a_A + a_B^\dagger a_B) + \hbar\omega_m b^\dagger b - \hbar G a_A^\dagger a_A (b + b^\dagger); \quad (1)$$

here ω_c is the frequency of the photon, $a_A^\dagger$ and $a_B^\dagger$ are the creation operators for the photon in the interferometer arms A and B, respectively, while ω_m and $b^\dagger$ are the frequency and the phonon creation operator associated with the motion of the center of mass of the mirror. The coupling constant is $G = \omega_c \sigma / L$, where L is the length

of the cavity, with $\sigma = (\hbar/2M\omega_m)^{1/2}$ the width of the mirror wave packet and M the mass of the mirror.

A beam splitter places the photon in a state that is an equal superposition of being in arm A or B, so that the initial state of the composite system is:

$$|\psi_0\rangle = \frac{1}{\sqrt{2}} [|0\rangle_A |1\rangle_B + |1\rangle_A |0\rangle_B] |0\rangle_m, \quad (2)$$

with the mirror being at rest in its ground state.

Standard quantum mechanics predicts that at time t the state vector will be

$$|\psi_t\rangle = e^{-\frac{i}{\hbar} H t} |\psi_0\rangle = \frac{1}{\sqrt{2}} e^{-i\omega_c t} \left[|0\rangle_A |1\rangle_B |0\rangle_m + e^{i\kappa^2(\omega_m t - \sin \omega_m t)} |1\rangle_A |0\rangle_B |\alpha_t\rangle_m \right]; \quad (3)$$

here we have written $\kappa = G/\omega_m$ and $|\alpha_t\rangle_m$ denotes a unit normalized mirror coherent state with complex amplitude $\alpha_t = \kappa(1 - e^{-i\omega_m t})$. According to (3), the state of the mirror changes into the superposition of being at rest (when the photon travels through arm B) and oscillating between 0 and $4\kappa\sigma$ (when the photon travels in arm A and hits the mirror); thus, in order to create a spatially separated superposition of the two states $|0\rangle_m$ and $|\alpha_t\rangle_m$, one has to require $\kappa \geq 1/4$.

The physically interesting quantity of the experiment is the visibility of the photon; this quantity is related to the off-diagonal element of the reduced density matrix $\rho_p = \text{Tr}_m[\rho]$ of the photon, where ρ is the full density matrix of the photon+mirror system. In our case, $\rho = |\psi_t\rangle\langle\psi_t|$ so, after tracing over the mirror states, and using the relation ${}_m\langle 0|\alpha_t\rangle_m = e^{-\frac{1}{2}|\alpha_t|^2}$, the reduced density matrix ρ_p has as the coefficient of the off-diagonal term

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$|1\rangle_A \langle 0| \otimes |0\rangle_B \langle 1|$ the factor $\frac{1}{2}f$, with

$$f = e^{i\kappa^2(\omega_m t - \sin \omega_m t)} e^{-\kappa^2(1 - \cos \omega_m t)}. \quad (4)$$

As expected, quantum mechanics predicts full coherence when the two states $|0\rangle_m$ and $|\alpha_t\rangle_m$ of the mirror have the same spatial position (this happens when $t = 2\pi n/\omega_m$, $n \in \mathbb{N}$), while coherence is destroyed when $|\alpha_t\rangle_m$ moves away from the rest position.

Our aim in this Letter is to study how the system evolves when the dynamics is not governed by the standard (unitary) Schrödinger equation, but by the Lindblad-type dynamics:

$$\begin{aligned} \frac{d\rho}{dt} &= -\frac{i}{\hbar}[H, \rho] - \frac{1}{2}\eta[q, [q, \rho]] \\ &= -\frac{i}{\hbar}[H, \rho] - \frac{1}{2}\eta\sigma^2[b + b^\dagger, [b + b^\dagger, \rho]], \end{aligned} \quad (5)$$

where $q = \sigma(b + b^\dagger)$ is the position operator associated with the center of mass of the mirror. The relevance of an analysis of this kind lies in the fact that Eq. (5) represents a cornerstone both for the theory of open quantum systems [5] and for collapse models [6]. In the first case, it describes the reduced evolution of a quantum particle interacting with a gas, by combining the free quantum dynamics and the scattering with the particle flux. In the second case, it represents the statistical evolution of a wavefunction undergoing a spontaneous stochastic localization process in space. Our problem is to solve the dynamics represented by Eq. (5), so as to calculate f .

First, two of us (A.B. and E.I.) set up the following linear stochastic equation that unravels Eq. (5):

$$d|\psi_t\rangle = \left[-\frac{i}{\hbar}Hdt + \sqrt{\eta}q dW_t - \frac{\eta}{2}q^2dt \right] |\psi_t\rangle, \quad (6)$$

where W_t is a standard Wiener process defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. It is easy to show that f can be written as follows:

$$f = \int_{-\infty}^{+\infty} \mathbb{E}_{\mathbb{P}} [\langle x|\phi_t^A\rangle_m \langle \phi_t^B|x\rangle] dx, \quad (7)$$

where $\mathbb{E}_{\mathbb{P}}[\dots]$ denotes the stochastic average with respect to $\mathbb{P}$ and $|\phi_t^A\rangle_m$ and $|\phi_t^B\rangle_m$ are two wavefunctions for the mirror; they are solutions of Eq. (6) when H is replaced by the reduced Hamiltonians H^A and H^B respectively, with H^A the effective mirror Hamiltonian acting when the photon passes through interferometer arm A, and with H^B the corresponding effective mirror Hamiltonian acting when the photon passes through arm B:

$$H^A = \hbar\omega_m b^\dagger b - G(b + b^\dagger) \quad H^B = \hbar\omega_m b^\dagger b. \quad (8)$$

Thus the problem reduces to that of finding the two solutions of Eq. (6), given the two initial conditions $|\psi_0^A\rangle_m = |\psi_0^B\rangle_m = |0\rangle_m$.

The other author (S.L.A.) then found a way to work directly from Eq. (5). Defining an off-diagonal density matrix ρ_{OD} acting in the mirror Hilbert subspace by

${}_A\langle 1|{}_B\langle 0|\rho|0\rangle_A|1\rangle_B = \frac{1}{2}\rho_{OD}$, so that the factor f introduced above is $\text{Tr}_m \rho_{OD}$, one can project out from Eq. (5) the evolution equation for ρ_{OD} ,

$$\begin{aligned} \frac{d\rho_{OD}(t)}{dt} &= -\frac{i}{\hbar}H^A\rho_{OD}(t) + \frac{i}{\hbar}\rho_{OD}(t)H^B \\ &\quad - \frac{1}{2}\eta\sigma^2[b + b^\dagger, [b + b^\dagger, \rho_{OD}(t)]], \end{aligned} \quad (9)$$

By a combined use of the interaction picture, the generalized Baker–Hausdorff formula, and cyclic permutation under the trace Tr_m , one gets directly $\text{Tr}_m \rho_{OD}$ and thus f .

Details of both methods of calculation have been given in [7]; we give here only the result, which is

$$f = \text{Tr}_m \rho_{OD}(t) = e^{i\kappa^2(\omega_m t - \sin \omega_m t) - \kappa^2(1 - \cos \omega_m t)} \times e^{-\frac{3}{16}\eta\ell^2(t - \frac{4}{3}\frac{\sin \omega_m t}{\omega_m} + \frac{\sin 2\omega_m t}{6\omega_m})}, \quad (10)$$

where we have defined $\ell = 4\kappa\sigma$ (so that ℓ is the maximum mirror center of mass displacement in the state $|\alpha_t\rangle$). The first term of f in Eq. (10) corresponds to the standard quantum result — compare with Eq. (4) — while the second term gives the correction due to the non-unitary part of the evolution: as expected, it induces a damping of the off-diagonal elements of ρ_p , which increases in time.

The above result should be compared with the “back of the envelope” calculation one can make by simply multiplying the standard formula (4) for f by the factor $e^{-\frac{1}{2}\eta\ell^2 t}$ coming from Eq. (5) when the unitary part of the evolution is omitted and one takes into account the largest distance between the two states $|0\rangle_m$ and $|\alpha_t\rangle_m$ of the mirror. This heuristic expression is remarkably close to Eq. (10). We have learned that such a “back of the envelope” estimate has been made earlier by Philip Pearle [8] and applied to the Marshall *et al.* experiment, with results in agreement with ours below. Our formula of Eq. (10) goes beyond this heuristic estimate by giving the exact time dependence of f .

We finally note that the exact solubility of the Marshall *et al.* model suggests that there may be further exact results associated with the decoherent oscillator, and that this turns out to be the case [9]. We now discuss two applications of our result.

Application 1: test for collapse models. There are two principal proposals for modifications to quantum superpositions of states with sufficiently large center of mass displacement. Penrose [10] has suggested that when two macroscopic states are displaced so that their center of mass wave packets no longer overlap, then new effects associated with quantum gravity will destroy the coherent superposition. On the other hand, there are the so called collapse models for spontaneous stochastic state vector reduction [6]. As distinct from Penrose’s proposal, collapse models require a much larger center of mass displacement, typically taken as of order 10^{-5} cm, for quantum superpositions to be destroyed.

Experiments performed to date fail by at least eleven orders of magnitude [11] to rule out the above proposals for modifications to standard quantum theory. On the other hand, in the proposed experiment of Marshall *et al* [3] — developed to test Penrose’s proposal — the mirror center of mass wave packet has a width $\sigma \sim 10^{-11}$ cm, and the two states of the mirror that are superposed are displaced by at least the wave packet width; in this circumstance, quantum mechanics predicts full maintenance of coherence, whereas the Penrose’s proposal motivates a search for loss of coherence as evidenced by reduced visibility of the interference fringes.

The quantity measured in the experiment is the maximum interference visibility $\nu(t)$, which is equal to the magnitude of f ; thus, under standard quantum mechanical evolution of the state — see Eq. (4) — one has for the time dependence of the visibility

$$\nu(t) = e^{-\kappa^2(1-\cos \omega_m t)}. \quad (11)$$

The strategy to test the macroscopic superposition of the mirror then goes as follows. One measures the photon’s visibility after one period $T = 2\pi/\omega_m$ of the mirror’s motion: if it is close to 1, then no collapse of the mirror’s wavefunction has occurred; if on the contrary it is significantly smaller than 1, a spontaneous collapse process is present which reduces the superposition to one of its two terms. Of course, one must keep control of all sources of environmental decoherence, which tend to lower the observed visibility.

We now analyze the experiment within the context of collapse models; more specifically, we consider the QMUPL model of wave function collapse [6], which represents the small displacement Taylor expansion of the GRW and CSL models [7]. This model is described by the following stochastic differential equation

$$d|\psi_t\rangle = \left[-\frac{i}{\hbar} H dt + \sqrt{\eta}(q - \langle q \rangle_t) dW_t - \frac{\eta}{2}(q - \langle q \rangle_t)^2 dt \right] |\psi_t\rangle, \quad (12)$$

where H is given by Eq. (1), and $\langle q \rangle_t \equiv \langle \psi_t | q | \psi_t \rangle$ is the quantum mechanical expectation of the position operator q . Using the rules of the Itô calculus, the density matrix evolution corresponding to Eq. (12) is

$$d\hat{\rho} = -\frac{i}{\hbar}[H, \hat{\rho}]dt - \frac{1}{2}\eta[q, [\hat{\rho}, q]]dt + \sqrt{\eta}[\hat{\rho}, [\hat{\rho}, q]]dW_t. \quad (13)$$

Since to observe interference fringes experimentally requires passing an ensemble of identically prepared photons through the apparatus, the relevant density matrix in the stochastic case is the ensemble expectation $\rho = E[\hat{\rho}]$, which obeys Eq. (5). Accordingly, we can use Eq. (10) to compute the visibility, and we get [12]:

$$\nu(t) = e^{-\kappa^2(1-\cos \omega_m t)} \times e^{-\frac{3}{16}\eta\ell^2\left(t - \frac{4}{3}\frac{\sin \omega_m t}{\omega_m} + \frac{\sin 2\omega_m t}{6\omega_m}\right)}. \quad (14)$$

This shows how the quantum mechanical visibility of Eq. (11) is modified by stochastic reduction in the QMUPL model.

After one mirror oscillation has been completed, at time $T = 2\pi/\omega_m$, the visibility predicted by Eq. (14) is damped by a factor $e^{-\Lambda}$, with

$$\Lambda = (3/16)\eta\ell^2(2\pi/\omega_m), \quad (15)$$

while standard quantum mechanics (when no external sources of noise are present) predicts it to be zero. We must now address the question of the value of the stochasticity parameter η ; we focus our attention on the QMUPL, GRW, and CSL collapse models, which give considerably different values for η .

In the GRW model, one has $\eta = N\eta_0$, with N the number of nucleons in the mirror and with η_0 obtained by comparison of Eq. (5) with the small displacement Taylor expansion of Eq. (6.12) of Bassi and Ghirardi’s review [6]. In terms of the parameters $\lambda = 10^{-16}\text{s}^{-1}$ and $\alpha = 10^{10}\text{cm}^{-2}$ of GRW, this gives $\eta_0 = \frac{1}{2}\lambda\alpha \sim 0.5 \times 10^{-2}\text{s}^{-1}\text{m}^{-2}$, which for $N \sim 3 \times 10^{15}$ nucleons in the mirror, gives $\eta \sim 10^{13}\text{s}^{-1}\text{m}^{-2}$. This is also the value used in the QMUPL model.

In the CSL model, one can calculate η from the small displacement Taylor expansion of Eq. (8.23) of Bassi and Ghirardi [6], details of which are given in [7]. The result of this calculation, in terms of the parameters $\gamma \sim 10^{-30}\text{cm}^3\text{s}^{-1}$ and $\alpha = 10^{10}\text{cm}^{-2}$ of CSL, together with the nucleon density $D = 10^{24}\text{cm}^{-3}$ and the side length $S = 10^{-3}\text{cm}$ of the cubical mirror, is

$$\eta = \gamma S^2 D^2 \left(\frac{\alpha}{\pi}\right)^{\frac{1}{2}} \sim 0.6 \times 10^{21}\text{s}^{-1}\text{m}^{-2}. \quad (16)$$

Thus the CSL model gives an estimate of η larger than that of the QMUPL and QMSL models by a factor of 10^8 , and so we continue the analysis using this more conservative value of η . Taking $\kappa \sim 1/4$, so that the mirror excursion ℓ is equal to its center of mass wave packet spread $\sigma \sim 10^{-13}$ m, and with $2\pi/\omega_m = 2 \times 10^{-3}\text{s}$, we get from Eqs. (15) and (16) the result $\Lambda \sim 0.2 \times 10^{-8}$, indicating that according to the CSL model, coherence is maintained to an accuracy of better than one part in 10^8 . Put another way, if the Marshall *et al* experiment were to observe maintenance of coherence to 0.2 percent accuracy, it would set only the weak bound $\gamma < 10^{-24}\text{cm}^3\text{s}^{-1}$ on the CSL model stochasticity parameter. This would be considerably better than the bound $\gamma < 10^{-19}\text{cm}^3\text{s}^{-1}$ set [11] by fullerene diffraction experiments, but is still six orders of magnitude away from a decisive test of the CSL model. Thus, while providing a useful test of the Penrose model, the Marshall *et al* experiment will leave very much open the question of whether the laws of physics allow coherent superpositions of macroscopic states.

Application 2: test for environmental decoherence. We mention a second interesting application of our result, related to decoherence tests. As previously discussed, the ultimate aim of Marshall *et al* experiment is to test

quantum superpositions of macroscopic systems; in order to do this, one has to isolate such a system from external noise, which tends to lower the photon's visibility.

On the contrary, one can modify the experiment to deliberately allow the mirror to interact with the environment, so as to intentionally give a value of η large enough to produce an observable contribution from the second factor in Eq. (14). The exact time dependence of Eqs. (10) and (14) would then give a test of the en-

vironmental decoherence model of Eq. (5) and of the assumptions [5] underlying this model.

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