Structured Discourse Reference to Individuals

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Abstract

The paper argues that discourse reference in natural language involves two equally important components with essentially the same interpretive dynamics, namely reference to *values*, i.e. (non-singleton) sets of objects, and reference to *structure*, i.e. the correlation / dependency between such sets, which is introduced and incrementally elaborated upon in discourse. To define and investigate structured discourse reference, a new dynamic system couched in classical (many-sorted) type logic is introduced which extends Compositional DRT (Muskens, 1996) with *plural* information states, i.e. information states are modeled as sets of variable assignments (following van den Berg, 1996), which can be can be represented as matrices with assignments (sequences) as rows. A plural info state encodes both values (the columns of the matrix store sets of objects) and structure (each row of the matrix encodes a correlation / dependency between the objects stored in it). Given the underlying type logic, compositionality at sub-clausal level follows automatically and standard techniques from Montague semantics (e.g. type shifting) become available. The idea that plural info states are semantically necessary is motivated by relative-clause donkey sentences with multiple instances of anaphora: mixed reading (weak & strong) sentences and sentences exemplifying donkey anaphora to structure.

1. Multiple Donkey Anaphora

The main goal of this paper is to argue that discourse reference in natural language involves two equally important components with essentially the same interpretive dynamics, namely reference to *values*, i.e. (non-singleton) sets of objects (individuals, events, times, propositions etc.), and reference to *structure*, i.e. *the correlation / dependency* between such sets that is introduced and incrementally elaborated upon in discourse.

The notion of structured discourse reference enables us to provide a precise compositional interpretation procedure for discourses involving complex descriptions of multiple related objects as, for example, the sentences in (1) and (2) below which contain multiple instances of donkey anaphora. Indefinites introduce a discourse referent (dref) u, u'' etc., represented by a superscript, while pronouns are anaphoric to a dref, represented by a subscript.

1. Every^{u} person who buys a$^{u'}$ book on amazon.com and has a$^{u''}$ credit card uses it$_{u''}$ to pay for it$_{u'}$.
2. Every^{u} boy who bought a$^{u'}$ Christmas gift for a$^{u''}$ girl in his class asked her$_{u''}$ deskmate to wrap it$_{u'}$.

Sentence (1) shows that singular donkey anaphora can refer to (non-singleton) sets of individuals (i.e. values), while (2) shows that singular donkey anaphora can refer to a dependency between such sets (i.e. structure). Let us examine them in turn.

Sentence (1) is a mixed weak & strong donkey sentence: it asserts that, for *every* book (strong) that any credit-card owner buys on amazon.com, there is *some* credit card (weak) that s/he uses to pay for the book. Note in particular that the credit card can vary from book to book, e.g. I can use my MasterCard to buy set theory books and my Visa to buy detective novels; that is, even the weak indefinite *a$^{u''}$ credit card* can introduce a non-singleton set.

For each buyer, the two sets of objects, i.e. all the purchased books and some of the credit cards, are correlated and the dependency between these sets – left implicit in the restrictor of the quantification – is specified in the nuclear scope: each book is correlated with the credit card that was used to pay for it.

The translation of sentence (1) in classical (static) first-order logic is provided in (3) below.

3. $\forall x(pers(x)\wedge\exists y(bk(y)\wedge buy(x,y))\wedge\exists z(card(z)\wedge hv(x,z))$
$\rightarrow \forall y'(bk(y')\wedge buy(x,y')$
$\rightarrow \exists z'(card(z')\wedge hv(x,z')\wedge use\_to\_pay(x,z',y'))))$

The challenge posed by sentence (1) is to compositionally derive its interpretation while allowing for: (**i**) the fact that the two donkey indefinites in the restrictor of the quantification receive two distinct readings (strong and weak respectively) and (**ii**) the fact that the value of the weak indefinite *a$^{u''}$ credit card* co-varies with / is dependent on the value of the strong indefinite *a$^{u'}$ book* although the strong indefinite cannot syntactically scope over the weak one, since both DP's are trapped in their respective VP-conjuncts.

Sentence (2) contains two instances of strong donkey anaphora: we are considering *every* Christmas gift and *every* girl. The restrictor introduces a dependency between the set of gifts and the set of girls: each gift is correlated with the girl it was bought for. The nuclear scope of the donkey quantification retrieves not only the two sets of objects, but also the structure associated with them, i.e. the dependency between them: each gift was wrapped by the deskmate of the girl that the gift was bought for. Thus, we have here donkey anaphora to structure in addition to donkey anaphora to values.

Importantly, the structure associated with the two sets, i.e. the dependency between gifts and girls, is *semantically* encoded and not pragmatically inferred: the correlation between the two sets is not left vague / underspecified and subsequently made precise based on various extra-linguistic factors. To see that the structure is semantically encoded, consider the following situation: suppose that John buys two gifts, one for Mary and the other for Helen; moreover, the two girls are deskmates. Intuitively, sentence (2) is true if John asked Mary to wrap Helen's gift and Helen to wrap Mary's gift and it is false if John asked each girl to wrap her own gift. But, if the relation between gifts and girls were vague / underspecified, we would predict that sentence (2) should be true even in the second situation[1,2].

2. Plural Compositional DRT (PCDRT)

To give a compositional account of sentences (1) and (2) above – and, in general, of discourses involving complex descriptions of multiple related objects –, I will introduce a new dynamic system couched in classical (many-sorted) type logic which extends Compositional DRT (CDRT – see Muskens, 1996) with: (**i**) plural information states and (**ii**) selective generalized quantification. The resulting system is dubbed Plural CDRT (PCDRT).

2.1. Plural Information States

The first technical innovation relative to CDRT is that, just as in Dynamic Plural Logic (van den Berg, 1996), information states *I*, *J* etc. are modeled as sets of variable assignments *i*, *j* etc.; such plural info states can be represented as matrices with assignments (sequences) as rows, as shown in (4) below.

4. Info State *I*	…	*u*	*u'*	…
i_1	…	x_1 (i.e. ui_1)	y_1 (i.e. $u'i_1$)	…
i_2	…	x_2 (i.e. ui_2)	y_2 (i.e. $u'i_2$)	…
i_3	…	x_3 (i.e. ui_3)	y_3 (i.e. $u'i_3$)	…
…	…	…	…	…

Values: the sets $\{x_1, x_2, x_3, \ldots\}$ and $\{y_1, y_2, y_3, \ldots\}$. **Structure**: the relation $\{<x_1, y_1>, <x_2, y_2>, <x_3, y_3>, \ldots\}$.

Plural info states encode discourse reference to both values and structure. The values are the sets of objects that are stored in the columns of the matrix, e.g. a dref *u* for individuals stores a set of individuals relative to a plural info state since *u* is assigned an individual by each assignment (i.e. row). The structure is *distributively* encoded in the rows of the matrix: for each assignment / row in the plural info state, the individual assigned to a dref *u* by that assignment is structurally correlated with the individual assigned to some other dref *u'* by the same assignment.

More precisely, we work with a Dynamic Ty2 logic, i.e. basically with Muskens' Logic of Change (Muskens, 1996), which is based on Gallin's Ty2 (Gallin, 1975). There are three basic types: type *t* (truth-values), type *e* (individuals; variables: *x*, *x'* etc.) and type *s* ('variable assignments'; variables: *i*, *j* etc.). A suitable set of axioms ensures that the entities of type *s* behave as variable assignments (see Muskens, 1996 and chapter **3** in Brasoveanu, 2007 for more details).

A dref for individuals *u* is a function of type *se* from 'assignments' i_s to individuals x_e (subscripts on terms indicate their type). Intuitively, the individual $u_{se}i_s$ is the individual that the 'assignment' *i* assigns to the dref *u*.

Dynamic info states *I*, *J* etc. are plural: they are sets of 'variable assignments', i.e. they are terms of type *st*. As shown in (4) above, an individual dref *u* stores a set of individuals with respect to a plural info state *I*, abbreviated as $uI := \{u_{se}i_s\colon i_s \in I_{st}\}$, i.e. *uI* is the image of the set of 'assignments' *I* under the function *u*.

2.2. DRS's in PCDRT

A sentence is interpreted as a Discourse Representation Structure (DRS), i.e. as a relation of type $(st)((st)t)$ between an input info state I_{st} and an output info state J_{st}.

As shown in (5) below, a DRS is represented as a [**new dref's** | **conditions**] pair, which abbreviates a term of type $(st)((st)t)$ that places two kinds of constraints on the output info state *J*: (**i**) *J* differs from the input info state *I* at most with respect to the **new dref's** and (**ii**) *J* satisfies all the **conditions**. An example is provided in (6) below.

5. [**new dref's** | **conditions**] := $\lambda I_{st}.\lambda J_{st}.\ I[\textbf{new dref's}]J \wedge \textbf{conditions}J$
6. $[u, u' \mid person\{u\}, book\{u'\}, buy\{u, u'\}] := \lambda I_{st}.\lambda J_{st}.\ I[u, u']J \wedge person\{u\}J \wedge book\{u'\}J \wedge buy\{u, u'\}J$.

DRS's of the form [**conditions**] that do not introduce new dref's are *tests* and they abbreviate terms of the form $\lambda I_{st}.\lambda J_{st}.\ I{=}J \wedge \textbf{conditions}J$, e.g. $[book\{u'\}] := \lambda I_{st}.\lambda J_{st}.\ I{=}J \wedge book\{u'\}J$. The definitions of conditions and new dref's are provided in the following two sections (**2.3** and **2.4**).

The PCDRT definition of truth is given in (7) below.

7. **Truth:** A DRS *D* (of type $(st)((st)t)$) is *true* with respect to an input info state I_{st} iff $\exists J_{st}(DIJ)$.

2.3. Atomic Conditions

Atomic conditions (e.g. lexical relations like $buy\{u, u'\}$) are sets of plural info states, i.e. they are terms of type $(st)t$, and they are *unselectively distributive* with respect to the plural info states they accept, where "unselective" is used in the sense of Lewis (1975). That is, atomic conditions universally quantify over 'variable assignments' – or cases, to use the terminology of Lewis (1975): an atomic condition accepts a plural info state *I* iff it accepts, in a pointwise manner, every single 'assignment' *i* in the info state *I*, as shown in (8) below. The first conjunct in (8), i.e. $I{\neq}\emptyset$, rules out the (degenerate) case when the universal quantification in the second conjunct $\forall i_s{\in}I(\ldots)$ (which encodes unselective distributivity) is vacuously satisfied.

[1] Note the similarity between example (2), which crucially involves the symmetric relation *deskmate*, and the 'indistinguishable participants' examples in (i) and (ii) below which also involve symmetric relations (the examples are due to Hans Kamp, Jan van Eijck and Irene Heim – see Heim, 1990, pp. 147-148):
(i) If a man shares an apartment with another man, he shares the housework with him.
(ii) If a bishop meets a bishop, he blesses him.

[2] The donkey sentence in (2) does not pose problems for CDRT (or classical DRT / FCS / DPL) – at least to the extent to which CDRT can provide a suitable analysis of possessive definite descriptions like *her deskmate*. However, as the remainder of this section will show, the donkey sentence in (2) is an important companion to the mixed reading donkey sentence in (1); it is only together that these two sentences provide an argument for extending CDRT with plural information states as opposed to a more conservative extension with discourse referents for sets.

8. **Atomic conditions**:
$R\{u_1, \ldots, u_n\} := \lambda I_{st}.\ I \neq \emptyset \wedge \forall i_s \in I(R(u_1 i, \ldots, u_n i))$,
for any non-logical constant R of type $e^n t$ [3].

An info state I satisfying condition $R\{u_1, \ldots, u_n\}$ can be intuitively depicted by a matrix like (9) below.

9. Info state I	…	u_1	…	u_n	…
i	…	$x_1(=u_1 i)$	…	$x_n(=u_n i)$	…
		$R(u_1 i, \ldots, u_n i)$, i.e. $R(x_1, \ldots, x_n)$			
i'	…	$x_1'(=u_1 i')$	…	$x_n'(=u_n i')$	…
i''	…	$x_1''(=u_1 i'')$	…	$x_n''(=u_n i'')$	…
…	…	…	…	…	…

Given unselective distributivity, the denotation of atomic conditions has a lattice-theoretic ideal structure.

10. $\mathfrak{I}$ is a *complete ideal without a bottom element* (abbreviated as *c-ideal*) with respect to the partial order induced by set inclusion $\subseteq$ on the set $\wp^+(\boldsymbol{D}_s^{\boldsymbol{M}})$, where $\wp^+(\boldsymbol{D}_s^{\boldsymbol{M}}) := \wp(\boldsymbol{D}_s^{\boldsymbol{M}}) \backslash \{\emptyset\}$ and $\wp(\boldsymbol{D}_s^{\boldsymbol{M}})$ is the power set of the domain of 'variable assignments' of the model $\boldsymbol{M}$, iff: (**i**) $\mathfrak{I} \subseteq \wp^+(\boldsymbol{D}_s^{\boldsymbol{M}})$; (**ii**) $\mathfrak{I}$ is closed under non-empty subsets and under arbitrary unions.
11. For any c-ideal $\mathfrak{I}$, $\mathfrak{I} = \wp^+(\cup \mathfrak{I})$, i.e. c-ideals are complete Boolean algebras without a bottom element.

The definition of atomic conditions in (8) above ensures that they always denote c-ideals (in the atomic lattice $\wp(\boldsymbol{D}_s^{\boldsymbol{M}})$). We can in fact characterize them in terms of the supremum of their denotation – as in (12) below.

12. **Atomic conditions as c-ideals**: For any constant R of type $e^n t$ and sequence of dref's $<u_1, \ldots, u_n>$, let $\mathbb{I}(R, <u_1, \ldots, u_n>) := \lambda i_s.\ R(u_1 i, \ldots, u_n i)$, abbreviated $\mathbb{I}^R$ whenever the sequence $<u_1, \ldots, u_n>$ can be recovered from context. Then, $R\{u_1, \ldots, u_n\} = \wp^+(\mathbb{I}^R)$ [4].

The fact that atomic conditions denote c-ideals endows the PCDRT notion of dynamic meaning with a range of desirable formal properties – see for example the simplified definition of DRS's in (17) below.

2.4. New Discourse Referents

Consider first the CDRT notion of random assignment of value to a dref u, symbolized as $[u]$ and defined as shown in (13) below (for more discussion, see Muskens, 1996 and chapter **3** in Brasoveanu, 2007).

13. $[u] := \lambda i_s.\lambda j_s.\ \forall v_{se}(\mathbf{udref}(v) \wedge v \neq u \rightarrow vi = vj)$

The problem raised by the definition of new dref introduction in PCDRT is how to generalize (13), which relates single 'variable assignments' i_s and j_s, to a relation between sets of 'variable assignments' (i.e. plural info states) I_{st} and J_{st}. The PCDRT definition of new dref introduction (or: random assignment of value to a dref u) is based on the pointwise definition in (13), as shown in (14) below[5].

14. **New dref's in PCDRT**: $[u] :=$
$\lambda I_{st}.\lambda J_{st}.\ \forall i_s \in I(\exists j_s \in J(i[u]j)) \wedge \forall j_s \in J(\exists i_s \in I(i[u]j))$

The definition in (14) treats the structure and value components of a plural info state in parallel, since we non-deterministically introduce both of them, namely: (**i**) some new values for u and, also, (**ii**) some new structure associating the u-values and the values of any other (previously introduced) dref's u', u'' etc.[6]

The fact that the PCDRT definition of new dref introduction treats the dynamics of value and structure in parallel distinguishes it from most dynamic systems based on plural info states, including van den Berg (1996), Krifka (1996) and Nouwen (2003), which only introduce values non-deterministically, while any newly introduced set of values is *deterministically* associated with a particular structure[7].

The explicit PCDRT distinction between the two informational components of an info state, i.e. values and structure, and their parallel treatment is motivated both empirically and theoretically.

Empirically, the definition in (14) enables us to account for mixed reading donkey sentences like (1) above. Recall that, intuitively, we want to allow the credit cards to vary from book to book. That is, we want the restrictor of the *every*-quantification in (1) to non-deterministically introduce some set of u''-cards and non-deterministically associate them with the u'-books and let the nuclear scope filter the non-deterministically assigned values and structure by requiring each u''-card to be used to pay for the corresponding u'-book.

Theoretically, the PCDRT definition in (14) is the natural generalization of the CDRT definition in (13) insofar as it preserves its formal properties, i.e., just as (13) defines $[u]$ as an equivalence relation of type $s(st)$ between 'variable assignments', (14) defines $[u]$ as an equivalence relation of type $(st)((st)t)$ between sets of 'variable assignments' (i.e. between plural info states).

Moreover, the fact that $[u]$ is an equivalence relation enables us to simplify the definition of DRS's provided in section **2.2** above as shown in (17) below.

15. **PCDRT dynamic conjunction**:
$D; D' := \lambda I_{st}.\lambda J_{st}.\ \exists H_{st}(DIH \wedge D'HJ)$.
16. $[u_1, \ldots, u_n] := [u_1]; \ldots; [u_n]$
17. **DRS's in terms of c-ideals over relations**: For any DRS $D := [u_1, \ldots, u_n \mid C_1, \ldots, C_m]$, where the conditions $C_1, \ldots, C_m$ are c-ideals, let $\mathbb{R}^D := \lambda i_s.\lambda j_s.\ i[u_1, \ldots, u_n]j \wedge j \in ((\cup C_1) \cap \ldots \cap (\cup C_m))$[8]. Then, $D := \lambda I_{st}.\lambda J_{st}.\ \exists \mathbb{R}_{s(st)} \neq \emptyset(I = \mathbf{Dom}(\mathbb{R}) \wedge J = \mathbf{Ran}(\mathbb{R}) \wedge \mathbb{R} \subseteq \mathbb{R}^D) = \lambda I_{st}.\lambda J_{st}.\ \exists \mathbb{R} \in \wp^+(\mathbb{R}^D)(I = \mathbf{Dom}(\mathbb{R}) \wedge J = \mathbf{Ran}(\mathbb{R}))$[9].

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[3] Where, following Muskens (1996), $e^n t$ is defined as the smallest set of types such that: (**i**) $e^0 t := t$ and (**ii**) $e^{m+1} t := e(e^m t)$.

[4] Convention: $\wp^+(\emptyset_{st}) = \emptyset_{(st)t}$.

[5] This definition is equivalent to the definition of random assignment in van den Berg (1994).

[6] Definition (14) can be informally paraphrased as: each input 'assignment' i has a $[u]$-successor output 'assignment' j and, vice-versa, each output 'assignment' j has a $[u]$-predecessor input 'assignment' i. This ensures that we preserve the values and structure associated with previously introduced dref's u', u'' etc.

[7] The definition of random assignment in van den Berg (1996) (see also Krifka, 1996 and Nouwen, 2003), which treats value non-deterministically and structure deterministically, has the basic format in (i) below. See chapter **5** in Brasoveanu (2007) for a detailed comparison between (i) and (14).
(i) $\{u\} := \lambda I_{st}.\lambda J_{st}.\ \exists X_{et} \neq \emptyset(J = \{j_s: \exists i_s \in I(i[u]j \wedge uj \in X)\})$.

[8] Where: $i[u_1, \ldots, u_n]j := i([u_1]; \ldots; [u_n])j$. In this case, however, we work with CDRT dynamic conjunction, defined as relation composition over terms of type $s(st)$, i.e. $[u]; [u'] := \lambda i_s.\lambda j_s.\ \exists h_s(i[u]h \wedge h[u']j)$, where $[u]$ and $[u']$ are of type $s(st)$.

[9] Where: $\mathbf{Dom}(\mathbb{R}) := \{i_s: \exists j_s(\mathbb{R}ij)\}$ and $\mathbf{Ran}(\mathbb{R}) := \{j_s: \exists i_s(\mathbb{R}ij)\}$.

With the basic dynamic system now in place, we can turn to the compositional interpretation of indefinites and generalized quantification in PCDRT.

2.5. Compositionality

Given the underlying type logic, compositionality at sub-clausal level follows automatically and standard techniques from Montague semantics (e.g. type shifting) become available.

In more detail, the compositional aspect of interpretation in an extensional Fregean/Montagovian framework is largely determined by the types for the (extensions of the) 'saturated' expressions, i.e. names and sentences. Let us abbreviate them as **e** and **t**.

An extensional static logic identifies **e** with *e* (individuals) and **t** with *t* (truth-values). The denotation of the noun *book* is of type (**et**), i.e. (*et*): *book* $\rightsquigarrow \lambda x_e.\ book_{et}(x)$. The generalized determiner *every* is of type (**et**)((**et**)**t**), i.e. (*et*)((*et*)*t*).

We go dynamic with respect to both value and structure by complicating the 'meta-types' **e** and **t**, i.e. by assigning finer-grained meanings to names and sentences. More precisely, PCDRT assigns the following dynamic types to the 'meta-types' **e** and **t**: **t** abbreviates (*st*)((*st*)*t*), i.e. a sentence is interpreted as a DRS, and **e** abbreviates *se*, i.e. a name is interpreted as a dref.

The denotation of the noun *book* is still of type (**et**), as shown in (18) below. A pronoun anaphoric to a dref *u* is interpreted as the Montagovian quantifier-lift of the dref *u* (resulting type: (**et**)**t**), as shown in (19).

18. *book* $\rightsquigarrow \lambda v_{\mathbf{e}}.\ [book\{v\}]$
 $\rightsquigarrow \lambda v_{\mathbf{e}}.\lambda I_{st}.\lambda J_{st}.\ I{=}J \wedge book\{v\}J$
19. $it_u \rightsquigarrow \lambda P_{\mathbf{et}}.P(u)$

Indefinite articles and generalized determiners have denotations of the expected type, i.e. (**et**)((**et**)**t**); these denotations are introduced in the following two sections, i.e. **2.6** and **2.7** respectively.

See Brasoveanu (2007) for the complete definition of: (**i**) the syntax of a fragment of English containing the multiple donkey sentences in (1) and (2) above and (**ii**) its corresponding PCDRT semantics (defined in terms of type-driven translation).

2.6. Weak/Strong Indefinites and Maximization

In PCDRT, indefinites non-deterministically introduce both values and structure, i.e. they introduce *structured sets* of individuals. Pronouns are anaphoric to such sets.

The weak / strong donkey ambiguity is attributed to the indefinite articles; this enables us to give a compositional account of the mixed reading (weak & strong) sentence in (1) because we *locally* decide for each indefinite article whether it receives a weak or a strong reading. The two basic meanings have the format in (20).

20. **weak indef**: $a^{\mathrm{wk}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [u];\ P(u);\ P'(u)$
 strong indef: $a^{\mathrm{str}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^{u}(P(u);\ P'(u))$

The only difference between a weak and a strong indefinite article is the presence vs. absence of a maximization operator **max**. We can therefore think of the singular indefinite article as *underspecified* with respect to the presence / absence of this operator: the decision to introduce it or not is made online depending on the discourse and utterance context of a particular donkey sentence – much like aspectual coercion[10] or the selection of a particular type for the denotation of an expression[11] are context-driven online processes.

The **max** operator, defined in (21) below, ensures that, after we process a strong indefinite, the output plural info state stores with respect to the dref *u* the *maximal* set of individuals satisfying both the restrictor dynamic property *P* and the nuclear scope dynamic property *P′*. In contrast, a weak indefinite will non-deterministically store *some* set of individuals satisfying its restrictor and nuclear scope.

21. $\mathbf{max}^{u}(D) :=$
 $\lambda I_{st}.\lambda J_{st}.\ ([u];\ D)IJ \wedge \forall K_{st}(([u];\ D)IK \rightarrow uK{\subseteq}uJ)$,
 where *D* is a DRS of type $\mathbf{t} := (st)((st)t)$

The first conjunct in (21) introduces *u* as a new dref (by [*u*]) and makes sure (by *D*) that each individual in *uJ* 'satisfies' *D*, i.e. *uJ* stores *only* individuals that 'satisfy' *D*. The second conjunct enforces the maximality requirement: any other set *uK* obtained by a similar procedure (i.e. any other set of individuals that 'satisfies' *D*) is included in *uJ*, i.e. *uJ* stores *all* the individuals that satisfy *D*. The DRS $\mathbf{max}^{u}(D)$ can be thought of as dynamic λ-abstraction over individuals: the 'abstracted variable' is the dref *u*, the 'scope' is the DRS *D* and the result of the 'abstraction' is a set of individuals *uJ* containing *all* and *only* the individuals that 'satisfy' *D*. Thus, **max** together with plural info states and unselective distributivity (contributed by atomic conditions) enables us to 'λ-abstract' over both values and structure.

I conclude this section by showing how we can simplify updates in which one **max** operator occurs within the scope of another – see (22) below. Updates of the form $\mathbf{max}^{u}(D;\ \mathbf{max}^{u'}(D'))$ occur fairly frequently in the PCDRT translations of natural language discourses (see for example sentence (2) above, which is translated in (26) below) – and they are difficult to grasp intuitively; (22) states that we can reduce **max** embeddings to (intuitively clearer) **max** sequences.

22. **Simplifying 'max-under-max' representations**: $\mathbf{max}^{u}(D;\ \mathbf{max}^{u'}(D')) = \mathbf{max}^{u}(D;\ [u'];\ D');\ \mathbf{max}^{u'}(D')$, if the following two conditions obtain: (**i**) *u* is not reintroduced in *D′* and (**ii**) *D′* is of the form $[u_1, \ldots, u_n \mid C_1, \ldots, C_m]$, where $C_1, \ldots, C_m$ are c-ideals[12].

2.7. Generalized Quantification

The PCDRT translation for generalized determiners, provided in (23) below, is of the expected type (**et**)((**et**)**t**).

23. $det^{u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [\mathbf{det}_{u}(P(u),\ P'(u))]$

The $\mathbf{det}_{u}(P(u),\ P'(u))$ condition, which effectively contains the PCDRT notion of dynamic selective generalized quantification, is defined in (24) below.

24. $\mathbf{det}_{u}(D,\ D') := \lambda I_{st}.\ I{\neq}\emptyset \wedge \mathbf{DET}(u[DI],\ u[(D;\ D')I])$,
 where *D* and *D′* are DRS's of type $\mathbf{t} := (st)((st)t)$,
 $u[DI] := \cup\{uJ\!:\ ([u \mid \mathbf{unique}\{u\}];\ D)IJ\}$,

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[10] For example, the iterative interpretation of: (**i**) *John sent a letter to the company for years* or (**ii**) *The light is flashing*.

[11] For example, proper names are type-lifted when they are conjoined with generalized quantifiers.

[12] See chapter **5** in Brasoveanu (2007) for the proof.

unique$\{u\} := \lambda I_{st}.\ I \neq \emptyset \wedge \forall i_s \in I \forall i'_s \in I(ui=ui')$ and **DET** is the corresponding static determiner.

The inner workings of definition (24) will become clearer in the following section where we work through the PCDRT analyses of the donkey examples in (1) and (2) above. See Brasoveanu (2007) for more discussion and for a comparison with alternative dynamic definitions of generalized quantification.

3. Multiple Donkey Anaphora in PCDRT

The PCDRT analysis of the mixed reading (weak & strong) donkey sentence in (1) is provided in (25) below.

Informally, the update in (25) can be described as follows. After the input info state is updated with the restrictor of the quantification in (1), we will be in a plural info state that stores, for each *u*-person that is a book buyer and a card owner: (**i**) the maximal set of purchased books, stored relative to the dref *u'* (since the indefinite *a*$^{\mathrm{str}:u'}$ *book* is strong), (**ii**) some non-deterministically introduced set of credit cards, stored relative to the dref *u''* (since the indefinite *a*$^{\mathrm{wk}:u''}$ *credit card* is weak) and, finally, (**iii**) some non-deterministically introduced structure correlating the *u'*-values and the *u''*-values.

The nuclear scope of the quantification in (1) is anaphoric to both values and structure: we test that the non-deterministically introduced values for *u''* and the non-deterministically introduced structure associating *u''* and *u'* satisfy the nuclear scope condition, i.e. we test that, for each 'assignment' in the info state, the *u''*-card stored in that 'assignment' is used to pay for the *u'*-book stored in the same 'assignment'. That is, the nuclear scope elaborates on the structure (i.e. the dependency between *u''* and *u'*) non-deterministically introduced in the restrictor of the quantification.

Note that the pseudo-scopal relation between the strong indefinite *a*$^{\mathrm{str}:u'}$ *book* and the weak indefinite *a*$^{\mathrm{wk}:u''}$ *credit card* ("pseudo" because, by the Coordinate Structure Constraint, the strong indefinite cannot syntactically take scope over the weak indefinite) emerges as a consequence of the fact that PCDRT uses plural information states, which store and pass on information about both the objects and the dependencies between them that are introduced and elaborated upon in discourse.

As (26) below shows, the PCDRT analysis of sentence (2) is largely parallel to the analysis of sentence (1).

By the time we are done processing the restrictor of the quantification in (2), we will be in a plural info state that stores both values and structure, i.e., for each particular *u*-boy, we will have: (**i**) the maximal set of gifts that the boy bought for some girl, stored relative to dref *u'*, (**ii**) the maximal set of girls for which the boy bought a gift, stored relative to dref *u''*, and (**iii**) the structure associating the *u'*-values and the *u''*-values, i.e., for each 'assignment' in the plural info state, the *u'*-gift stored in that 'assignment' was bought for the *u''*-girl stored in the same 'assignment'.

When we process the nuclear scope of the quantification in (2), we are anaphoric to both values and structure: we require each 'assignment' in the plural info state to be such that the deskmate of the *u''*-girl in that 'assignment' was asked to wrap the *u'*-gift in the same 'assignment'. Thus, yet again, the nuclear scope elaborates on the structured dependency between the two sets of values (gifts and girls) introduced in the restrictor.

The reader can check that, based on the PCDRT definition of truth in (7) above, the compositionally derived representations in (25) and (26) below assign the intuitively correct truth-conditions to (1) and (2)[13].

25. $[\mathbf{every}_u([pers\{u\}];\ \mathbf{max}^{u'}([bk\{u'\}, buy\{u, u'\}]);$
$[u'' \mid card\{u''\}, hv\{u, u''\}],$
$[use\_to\_pay\{u, u', u''\}])]$

26. $[\mathbf{every}_u([boy\{u\}];\ \mathbf{max}^{u'}([gift\{u'\}];$
$\mathbf{max}^{u''}([girl\{u''\}, buy\_for\{u, u', u''\}])),$
$\mathbf{max}^{u'''}([deskmate\{u'''\}, of\{u''', u''\}]);$
$[\mathbf{unique}_{u''}\{u'''\}];\ [ask\_to\_wrap\{u, u''', u'\}])]$

In particular, the PCDRT truth-conditions for (1) are basically identical to the classical (static) first-order formula in (3) above.

4. Comparison with Other Approaches

PCDRT differs from previous dynamic and static approaches in three general respects.

The first difference is conceptual: PCDRT explicitly embodies the idea that reference to *structure* is as important as reference to *value* and that the two should be treated in parallel (see the definition of dref introduction in section **2.4** above).

The PCDRT analysis of reference to structure as *discourse* reference to structure, i.e. in terms of plural info states, contrasts with the analysis of reference to structure by means of (dref's for) choice and / or Skolem functions. Although such functions could be used to capture (donkey) anaphora to structure, they would have variable arity depending on how many simultaneous anaphoric connections there are. That is, the arity of the functions is determined by the discourse context. It is therefore preferable to encode this context dependency in the database that stores discourse information, i.e. in the info state (as PCDRT does), and not in the representation of a lexical item, i.e. in the pronoun and / or its antecedent.

The second difference is empirical: the motivation for plural information states is provided by *singular* and *intra-sentential* donkey anaphora, in contrast to the previous literature which relies on *plural* and *cross-sentential* anaphora (see for example van den Berg, 1996, Krifka, 1996 and Nouwen, 2003 among others).

Intra-sentential donkey anaphora to structure provides a much stronger argument for the idea that plural info states are *semantically* necessary. To see this, consider anaphora to value first: a pragmatic account is plausible for cases of cross-sentential anaphora (e.g. in *A man came in. He sat down*, the pronoun *he* can be taken to refer to whatever man is pragmatically brought to salience by the use of an indefinite in the first sentence), but less plausible for cases of intra-sentential donkey anaphora (no

[13] The possessive *her*$_{u''}$$^{u'''}$ *deskmate* in (2) is analyzed as $\mathbf{max}^{u'''}([deskmate\{u'''\}, of\{u''', u''\}]);\ [\mathbf{unique}_{u''}\{u'''\}]$, i.e. as a Russellian definite description that contributes both existence (since we introduce the dref *u'''*) and uniqueness (relativized to *u''*-girls). The *u'''*-uniqueness is a consequence of combining the **max** operator (with scope only over the restrictor of the possessive; cf. the scope of **max** in strong indefinites) and the **unique** condition. See Brasoveanu (2007) for more details.

particular donkey is brought to salience in *Every farmer who owns a donkey beats it*).

Similarly, a pragmatic account of anaphora to structure is plausible for cases of cross-sentential anaphora like *Every man saw a woman. They greeted them.* This discourse asserts that every man greeted the woman/women that he saw, i.e. the greeting structure is the same as the seeing structure – but the identity of structure might be a pragmatic addition to semantic values that are unspecified for structure (e.g. the second sentence *They greeted them* could be interpreted cumulatively). However, a pragmatic approach is less plausible for cases of intra-sentential donkey anaphora to structure instantiated by (2) above.

Third, PCDRT takes the research program in Muskens (1996) of constructing theories and formal systems that unify different frameworks, e.g. Montague semantics and dynamic semantics, one step further: PCDRT unifies in classical type logic the static, compositional analysis of generalized quantification in Montague semantics and van den Berg's Dynamic Plural Logic. The unification is a non-trivial task due to certain peculiarities of Dynamic Plural Logic, e.g. the fact that its underlying logic is partial and the fact that it conflates discourse-level plurality (i.e. plural info states) and domain-level plurality (i.e. non-atomic individuals).

Moreover, the type logic that underlies PCDRT can be extended in the usual way with additional sorts for eventualities, times and possible worlds, which enables us to account for temporal and modal anaphora and quantification in a way that is *parallel* to the account of individual-level anaphora and quantification (see Stone, 1997 among others for arguments that such a parallel treatment is desirable). See Brasoveanu (2006, 2007) for more discussion – and for an account of quantificational and modal subordination as structured anaphora to quantifier domains, which extends the present account of multiple donkey anaphora.

Turning now to the analysis of weak / strong donkey ambiguities, three basic strategies have been pursued in the previous literature. The weak / strong ambiguity (or, to use a more neutral term: the weak / strong variation) is located: (**i**) at the level of selective generalized determiners (e.g. *every* in (1) and (2) above); this is what most dynamic approaches do; (**ii**) at the level of donkey pronouns (e.g. *it* and *her* in (1) and (2)); this is what we expect E-type approaches to do (see, for example, Lappin & Francez, 1994); (**iii**) at the level of donkey indefinites; this is what van den Berg (1996) and PCDRT do.

I will only outline here the argument against approaches that locate the weak / strong ambiguity at the level of donkey pronouns; for a more detailed comparison, see chapter **5** in Brasoveanu (2007). The argument relies on mixed reading DP-conjunction donkey sentences like the ones in (27) and (28)[14] below.

27. (The newspaper claims that, based on the most recent statistics,)
    Every^{u} company that hired a$^{str:u'}$ Moldavian man, but no$^{u''}$ company that hired a$^{wk:u'}$ Transylvanian man promoted him$_{u'}$ within two weeks of hiring.
28. (Imagine a Sunday fair where people come to sell their puppies before they get too old and where the entrance fee is one dollar. The fair has two strict rules: all the puppies need to be checked for fleas at the gate and the one dollar bills need to be checked for authenticity because of the many faux-monnayeurs in the area. So:)
    Everyone^{u} who has a$^{str:u'}$ puppy and everyone$^{u''}$ who has a$^{wk:u'}$ dollar brings it$_{u'}$ to the gate to be checked.

Both (27) and (28) are mixed reading sentences that contain (at least overtly) two donkey indefinites and only one donkey pronoun. The simplest hypothesis is therefore that the two distinct readings are in fact due to the two indefinites and not to the pronoun.

[14] Example (28) is based on an example due to Sam Cumming, revised based on Klaus von Heusinger's and Hans Kamp's suggestions (p.c.).

5. References

Brasoveanu, A. (2007). Structured Nominal and Modal Reference. PhD dissertation, Rutgers University.

Gallin, D. (1975). *Intensional and Higher-Order Modal Logic with applications to Montague semantics*. North-Holland Mathematics Studies.

Heim, I. (1990). E-Type Pronouns and Donkey Anaphora. In *Linguistics and Philosophy* 19, pp. 137-177.

Krifka, M. (1996). Parametric Sum Individuals for Plural Anaphora. In *Linguistics and Philosophy* 19, pp. 555-598.

Lappin, S. & N. Francez 1994. E-type Pronouns, I-sums and Donkey Anaphora. In *Linguistics and Philosophy* 17, pp. 391-428.

Lewis, D. (1975). Adverbs of Quantification. In *Formal Semantics of Natural Language*, E. Keenan (ed.), Cambridge: Cambridge University Press, pp. 3-15.

Muskens, R. (1996). Combining Montague Semantics and Discourse Representation. In *Linguistics and Philosophy* 19, pp. 143-186.

Nouwen, R. (2003). *Plural Pronominal Anaphora in Context*. PhD dissertation, University of Utrecht.

Stone, M. (1997). *The Anaphoric Parallel between Modality and Tense*. IRCS TR 97-06, Pennsylvania.

Van den Berg, M. (1994). A Direct Definition of Generalized Dynamic Quantifiers. In the *Proceedings of the 9th Amsterdam Colloquium*.

Van den Berg, M. (1996). *Some aspects of the Internal Structure of Discourse*. PhD dissertation, University of Amsterdam.

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