Cracking an Egg:
An Exercise in Commonsense Reasoning

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*** DRAFT ***

1 Introduction

This note is about a problem contributed by Ernie Davis to the Common Sense 98 Problem Page (`http://www.dcs.qmw.ac.uk/conferences/CS98/problems/Davis2.html`). The problem is to characterize the following:

> A cook is cracking a raw egg against a glass bowl. Properly performed, the impact of the egg against the edge of the bowl will crack the egg shell in half. Holding the egg over the bowl, the cook will then separate the two halves of the shell with his fingers, enlarging the crack, and the contents of the egg will fall gently into the bowl. The end result is that the entire contents of the egg will be in the bowl, with the yolk unbroken, and that the two halves of the shell are held in the cook's fingers.
>
> What happens if: The cook brings the egg to impact very quickly? Very slowly? The cook lays the egg in the bowl and exerts steady pressure with his hand? The cook, having cracked the egg, attempts to peel it off its contents like a hard-boiled egg? The bowl is made of looseleaf paper? of soft clay? The bowl is smaller than the egg? The bowl is upside down? The cook tries

this procedure with a hard-boiled egg? With a coconut? With an M&M?

The postulates introduced below are designed to formalize the scenario described in the first paragraph of the problem, with an eye to some of the modifications mentioned in the second.

This is a report on work in progress. The adequacy of this formalization needs to be further investigated. Computational experiments related to this formalization of the egg domain are described by Norman McCain and Hudson Turner in `http://www.cs.utexas.edu/users/hudson/eggnote.html`.

2 Stating the Problem More Precisely

We will represent time by subscripts $0, \ldots, T$, where T is the maximum length of the scenarios we are interested in. For any truth-valued fluent P and any nonnegative integer $t \leq T$, the predicate constant P_t will represent the value of P at time t. For any action symbol A and any nonnegative integer $t < T$, the predicate constant A_t will express that A is executed between the times t and $t+1$.

The "initial conditions" of the egg cracking problem are expressed by the formulas:

$$\begin{gathered} \mathit{IsEgg}(E),\ \mathit{Raw}_0(E), \\ \mathit{InHands}_0(E),\ \neg\mathit{Broken}_0(\mathit{Shell}(E)),\ \neg\mathit{Broken}_0(\mathit{Yolk}(E)), \\ \mathit{IsBowl}(B),\ \mathit{Glass}(B),\ \neg\mathit{Small}(B),\ \mathit{Upright}_0(B). \end{gathered} \tag{1}$$

Here *Raw*, *InHands*, *Broken* and *Upright* are fluent symbols.

The terms E, $\mathit{Shell}(E)$, $\mathit{Yolk}(E)$ and B in these formulas represent "physical objects"; later on, we will introduce terms of two other sorts. An egg is considered a physical object even after its parts have been separated. Accordingly, when we talk below about the "locations" of physical objects, we will not assume that any physical object has a location. After the separation of the two halves of the shell, the egg is not "located," and neither is the shell.

Note that the assumptions in the second line of (1) are not stated in the problem explicitly. But we will need them.

Between the times 0 and 1, the cook strikes the egg against the bowl; he brings it to impact at a proper speed—neither too quickly nor too slowly:

$$\mathit{StrikeAgainst}_0(E, B, \mathit{Properly}). \tag{2}$$

Here *StrikeAgainst* is an action symbol, and *Properly* is a term of a new sort—it represents a "mode of bringing to impact." In combination with the Theory of Egg Cracking (*TEC*) presented in Section 3, axioms (1) and (2) are expected to imply that at time 1 the shell is broken, but the yolk is not:

$$Broken_1(Shell(E)),\ \neg Broken_1(Yolk(E)). \tag{3}$$

Between the times 1 and 2, the cook places the egg over the bowl:

$$Move_1(E, Over(B)). \tag{4}$$

Here *Move* is an action symbol, and $Over(B)$ is a term of a new sort—it represents a "location." We also need to assume that the egg did not hit an obstacle while it was being moved to the position over the bowl or during the next time period, because this could have broken the yolk:

$$\neg StrikeAgainst_t(E, x, mode) \qquad (t = 1, 2). \tag{5}$$

Here x and *mode* are variables of appropriate sorts. We would like to show that facts similar to (3) hold at time 2, and, in addition, that the egg and its parts are located at time 2 over the bowl:

$$\begin{gathered} Broken_2(Shell(E)),\ \neg Broken_2(Yolk(E)), \\ At_2(E, Over(B)),\ At_2(Shell(E)), Over(B)), \\ At_2(Yolk(E)), Over(B)),\ At_2(White(E)), Over(B)). \end{gathered} \tag{6}$$

Here *At* is a fluent symbol.

Between the times 2 and 3, the cook separates the two halves of the shell:

$$SeparateHalfShells_2(E). \tag{7}$$

Here *SeparateHalfShells* is an action symbol. At time 3, the egg and its shell and not located anymore. But the formulas in the first and third lines of (6) are expected to extend to time 3; also, we want to be able to show that at that time the two halves of the shell are separated:

$$\begin{gathered} Broken_3(Shell(E)),\ \neg Broken_3(Yolk(E)), \\ At_3(Yolk(E)), Over(B)),\ At_3(White(E)), Over(B)), \\ SeparatedShell_3(E). \end{gathered} \tag{8}$$

Here *SeparatedShell* is a fluent symbol. Furthermore, we want to be able to prove that between the times 3 and 4 the contents of the egg fall into the bowl, with the yolk unbroken, and that the two halves of the shell remain in the cook's hands:

$$\begin{array}{c} InBowl_4(Yolk(E), B),\ InBowl_4(White(E), B), \\ \neg Broken_4(Yolk(E)), \\ InLeftHand_4(LeftHalfShell(E)), \\ InRightHand_4(RightHalfShell(E)). \end{array} \tag{9}$$

Here *InBowl*, *InLeftHand* and *InRightHand* are fluent symbols.

Most of the intended conclusions (3), (6), (8) and (9) are verified in Section 4.

3 A Formal Theory of Egg Cracking

Nonmonotonic Reasoning

One of the issues we need to address in the design of *TEC* is the frame problem. We should be able to prove, for instance, that the yolk does not disappear from inside the shell while the egg is struck against the bowl, and that the bowl does not turn upside down while the egg is being placed above it. This calls for the use of nonmonotonic reasoning.

Another application of nonmonotonic reasoning in this formalization is related to the idea of a "supported" physical object. (One of the relevant facts is that an unsupported located object is brought to motion by the force of gravity.) This idea will be characterized using the closed-world assumption: we will give some sufficient conditions for supportedness and then add that, by default, a located object is not supported.

The nonmonotonic logic employed here is causal logic in the sense of [Lifschitz, 1997]. (That paper extends the system of [McCain and Turner, 1997]. It is available at `http://www.cs.utexas.edu/users/vl/mypapers/causal.ps`). The postulates of a causal theory are rules of the form $F \leftarrow G$, where F and G are formulas. Such a rule expresses that there is a cause for F if G holds. The Theory of Egg Cracking will be defined as a causal theory.

The frame problem is solved in *TEC* by postulating the inertia rules

$$\begin{array}{c} P_{t+1}(v) \leftarrow P_{t+1}(v) \wedge P_t(v), \\ \neg P_{t+1}(v) \leftarrow \neg P_{t+1}(v) \wedge \neg P_t(v) \end{array} \tag{10}$$

for the fluent symbols P designated as "inertial" and for all $t < T$, where v is a tuple of distinct variables of appropriate sorts. (The idea is due to

McCain and Turner [1997].) These rules say that if $P(v)$ has not changed its truth value then there is a cause for this. Among the fluent symbols in the language of *TEC*, all except *InHands*, *InBowl* and *Supported* are inertial. Thus the set of postulates of *TEC* includes, for instance, the rules

$$\begin{gathered} \mathit{Raw}_{t+1}(x) \leftarrow \mathit{Raw}_{t+1}(x) \wedge \mathit{Raw}_t(x), \\ \neg\mathit{Raw}_{t+1}(x) \leftarrow \neg\mathit{Raw}_{t+1}(x) \wedge \neg\mathit{Raw}_t(x). \end{gathered}$$

The reason why we chose not to postulate inertia for the fluents *InBowl* and *InHands* is that *TEC* includes explicit definitions for them—formulas (11) and (12) below. The fluent *Supported*, as mentioned above, will be characterized using the closed-world assumption.

To describe a causal theory, one needs to specify, in addition to the postulates of the theory, which nonlogical constants of the underlying language are "explainable." The explainable symbols of *TEC* are the predicate constants P_t for all fluent symbols P and time instants t except for those in which P is inertial and $t = 0$. (The causal rules of *TEC* are not sufficient for explaining the initial values of inertial fluents.)

We identify any formula F with the rule $\mathit{False} \leftarrow F$. In view of this convention, a formula can be used as a postulate of a causal theory. For instance, the first formula in (1) is shorthand for the rule

$$\mathit{False} \leftarrow \neg\mathit{IsEgg}(E).$$

In addition to (10), *TEC* has the following postulates.

Hands, Bowls and Shells

To be in hands means to be in the left hand and in the right hand:

$$\mathit{InHands}_t(x) \equiv [\mathit{InLeftHand}_t(x) \wedge \mathit{InRightHand}_t(x)] \leftarrow \mathit{True}. \tag{11}$$

To be in a bowl means to be located on its upper surface while the bowl is in the upright position:

$$\mathit{InBowl}_t(x, y) \equiv [\mathit{IsBowl}(y) \wedge \mathit{Upright}_t(y) \wedge \mathit{At}_t(x, \mathit{Top}(y))] \leftarrow \mathit{True}. \tag{12}$$

An object cannot be in hands and inside a shell at the same time:

$$\begin{gathered} \neg\mathit{Inside}_t(x, \mathit{Shell}(y)) \leftarrow \mathit{InLeftHand}_t(x) \wedge \mathit{HasShell}(y), \\ \neg\mathit{Inside}_t(x, \mathit{Shell}(y)) \leftarrow \mathit{InRightHand}_t(x) \wedge \mathit{HasShell}(y), \\ \neg\mathit{InLeftHand}_t(x) \leftarrow \mathit{Inside}_t(x, \mathit{Shell}(y)) \wedge \mathit{HasShell}(y), \\ \neg\mathit{InRightHand}_t(x) \leftarrow \mathit{Inside}_t(x, \mathit{Shell}(y)) \wedge \mathit{HasShell}(y). \end{gathered} \tag{13}$$

An object cannot be inside two different shells at the same time:

$$\begin{aligned}\neg Inside_t(x, Shell(z)) \leftarrow\ & Inside_t(x, Shell(y)) \\ & \wedge HasShell(y) \wedge HasShell(z) \wedge y \neq z.\end{aligned} \tag{14}$$

If the shell of an object is not broken then the parts of the shell are not separated:

$$\neg SeparatedShell_t(x) \leftarrow HasShell(x) \wedge \neg Broken_t(Shell(x)). \tag{15}$$

An object cannot be positioned both upright and upside down at the same time:

$$\begin{aligned}\neg UpsideDown_t(x) &\leftarrow Upright_t(x), \\ \neg Upright_t(x) &\leftarrow UpsideDown_t(x).\end{aligned} \tag{16}$$

Glass is hard; soft clay is not:

$$\begin{aligned}Glass(x) &\supset Hard(x), \\ SoftClay(x) &\supset \neg Hard(x).\end{aligned} \tag{17}$$

Eggs and Coconuts

Eggs and coconuts have shells:

$$\begin{aligned}IsEgg(x) &\supset HasShell(x), \\ IsCoconut(x) &\supset HasShell(x).\end{aligned} \tag{18}$$

An egg's shell is fragile, and a coconut's shell is not:

$$\begin{aligned}IsEgg(x) &\supset Fragile(Shell(x)), \\ IsCoconut(x) &\supset \neg Fragile(Shell(x)).\end{aligned} \tag{19}$$

If the shell of an egg is not broken then the contents are inside the shell:

$$\begin{aligned}Inside_t(Yolk(x), Shell(x)) \wedge Inside_t(White(x), Shell(x)) \leftarrow & \\ IsEgg(x) \wedge \neg Broken_t(Shell(x)).&\end{aligned} \tag{20}$$

A hard-boiled egg is not raw:

$$\neg Raw_t(x) \leftarrow IsEgg(x) \wedge HardBoiled_t(x). \tag{21}$$

Supportedness

An object that is held in hands is supported:

$$\begin{aligned} \mathit{Supported}_t(x) &\leftarrow \mathit{InLeftHand}_t(x), \\ \mathit{Supported}_t(x) &\leftarrow \mathit{InRightHand}_t(x). \end{aligned} \tag{22}$$

An object in a bowl is supported:

$$\mathit{Supported}_t(x) \leftarrow \mathit{InBowl}_t(x, y). \tag{23}$$

If parts of a shell are not separated then an object inside the shell is supported:

$$\mathit{Supported}_t(x) \leftarrow \mathit{HasShell}(y) \wedge \neg \mathit{SeparatedShell}_t(y) \wedge \mathit{Inside}_t(x, \mathit{Shell}(y)). \tag{24}$$

Otherwise, objects are unsupported:

$$\neg \mathit{Supported}_t(x) \leftarrow \neg \mathit{Supported}_t(x). \tag{25}$$

Locations

Moving an object x to a location l causes x to be at l:

$$\mathit{At}_{t+1}(x, l) \leftarrow \mathit{Move}_t(x, l) \qquad (t < T). \tag{26}$$

If an object is at location l then it is not at any location other than l:

$$\neg \mathit{At}_t(x, l') \leftarrow \mathit{At}_t(x, l) \wedge l \neq l'. \tag{27}$$

An object can be moved only when it is held in a hand:

$$\mathit{Move}_t(x, l) \supset \mathit{InLeftHand}_t(x) \vee \mathit{InRightHand}_t(x). \tag{28}$$

The shell of a located object is where the object is:

$$\mathit{At}_t(\mathit{Shell}(x), l) \leftarrow \mathit{HasShell}(x) \wedge \mathit{At}_t(x, l). \tag{29}$$

An object inside a shell is where the shell is:

$$\mathit{At}_t(x, l) \leftarrow \mathit{HasShell}(y) \wedge \mathit{Inside}_t(x, \mathit{Shell}(y)) \wedge \mathit{At}_t(\mathit{Shell}(y), l). \tag{30}$$

An unsupported object will not stay where it is:

$$\neg \mathit{At}_{t+1}(x, l) \leftarrow \neg \mathit{Supported}_t(x) \wedge \mathit{At}_t(x, l) \qquad (t < T). \tag{31}$$

An unsupported object inside a shell will not stay there:

$$\neg Inside_{t+1}(x, Shell(y)) \leftarrow \neg Supported_t(x) \wedge HasShell(y) \\ \wedge Inside_t(x, Shell(y)) \qquad (t < T). \tag{32}$$

An unsupported object located over a surface that is not too small will fall on it:

$$At_{t+1}(x, Top(y)) \leftarrow \neg Supported_t(x) \wedge At_t(x, Over(y)) \\ \wedge \neg Small(y). \qquad (t < T). \tag{33}$$

Cracking Shells

One object can be struck against another in three different ways:

$$mode = Properly \vee mode = Quickly \vee mode = Slowly. \tag{34}$$

Striking an object whose shell is fragile against a hard object, if not performed too slowly, will break the shell:

$$Broken_{t+1}(Shell(x)) \leftarrow HasShell(x) \wedge Fragile(Shell(x)) \wedge Hard(y) \\ \wedge mode \neq Slowly \wedge StrikeAgainst_t(x, y, mode) \tag{35}$$

$(t < T)$. Striking a raw egg against a hard object too quickly will break the yolk:

$$Broken_{t+1}(Yolk(x)) \leftarrow IsEgg(x) \wedge Raw_t(x) \wedge Hard(y) \\ \wedge StrikeAgainst_t(x, y, Quickly). \tag{36}$$

An object can be struck against another only if it is held in a hand:

$$StrikeAgainst_t(x, y, mode) \supset InLeftHand_t(x) \vee InRightHand_t(x). \tag{37}$$

Two actions of this kind cannot be performed with the same object simultaneously:

$$StrikeAgainst_t(x, y_1, mode_1) \wedge StrikeAgainst_t(x, y_2, mode_2) \\ \supset y_1 = y_2 \wedge mode_1 = mode_2. \tag{38}$$

Separating the Parts of a Shell

After separating the two halves of an egg shell, each half stays in one of the hands of the agent, but the egg is not in his hands anymore. Also, the egg and the shell are not located anymore. This is expressed by the rule

$$\begin{array}{lcl} \mathit{SeparatedShell}_{t+1}(x) & & \\ \wedge \mathit{InLeftHand}_{t+1}(\mathit{LeftHalfShell}(x)) & & \\ \wedge \neg \mathit{InRightHand}_{t+1}(\mathit{LeftHalfShell}(x)) & & \\ \wedge \mathit{InRightHand}_{t+1}(\mathit{RightHalfShell}(x)) & & \\ \wedge \neg \mathit{InLeftHand}_{t+1}(\mathit{RightHalfShell}(x)) & & \\ \wedge \neg \mathit{InLeftHand}_{t+1}(x) & & \\ \wedge \neg \mathit{InRightHand}_{t+1}(x) & & \\ \wedge \neg \mathit{At}_{t+1}(x,l) & & \\ \wedge \neg \mathit{At}_{t+1}(\mathit{Shell}(x),l) & \leftarrow & \mathit{IsEgg}(x) \\ & & \wedge \mathit{SeparateHalfShells}_t(x) \end{array} \tag{39}$$

$(t < T)$. The two halves of the shell of an object can be separated only if the object is held in hands and has a shell which is broken:

$$\mathit{SeparateHalfShells}_t(x) \supset \mathit{InHands}_t(x) \wedge \mathit{HasShell}(x) \wedge \mathit{Broken}_t(\mathit{Shell}(x)) \tag{40}$$

$(t < T)$. The action cannot be performed on a hard-boiled egg:

$$\mathit{IsEgg}(x) \wedge \mathit{HardBoiled}_t(x) \supset \neg \mathit{SeparateHalfShells}_t(x) \tag{41}$$

$(t < T)$.

Unique Names Assumption

Finally, the Theory of Egg Cracking includes a number of unique names axioms. In a many-sorted language, if $f_1, \ldots, f_m$ are function constants with values of the same sort then

$$\mathit{UNA}[f_1, \ldots, f_m]$$

stands for the set consisting of the following formulas:

- for every pair i, j such that $1 \leq i < j \leq m$, the formula saying that the ranges of f_i and f_j are disjoint, that is,

  $$f_i(v) \neq f_j(w),$$

  where v, w are disjoint tuples of distinct variables of appropriate sorts, and

- for every f_i whose arity is not 0 (in other words, which is not an object constant), the formula saying that it is one-to-one, that is,

$$f_i(v) = f_i(w) \supset v = w,$$

where v, w are disjoint tuples of distinct variables of appropriate sorts, and $v = w$ is understood as the conjunction of equalities among the corresponding members of the tuples v and w.

The set of postulates of TEC includes the unique names of assumptions for all function symbols of all three sorts:

$$\begin{gathered} UNA[B, E, Shell, LeftHalfShell, RightHalfShell, Yolk, White], \\ UNA[Over, Top], \\ UNA[Properly, Quickly, Slowly]. \end{gathered} \tag{42}$$

These axioms guarantee, for instance, that the yolk of an egg is not identical to the shell of the same egg or of another egg, that being located over a bowl is not the same as being in the bowl, etc.

This is the end of the list of postulates of TEC.

4 Proving Theorems about Egg Cracking

At Time 1, the Shell Is Broken

From (1) and (17)–(19),

$$HasShell(E),\ \ Fragile(Shell(E)),\ \ Hard(B). \tag{43}$$

By the unique names assumption (42),

$$Properly \neq Slowly. \tag{44}$$

From 2, (43) and (44), using the material implication corresponding to rule (35), that is,

$$\begin{aligned} &HasShell(x) \wedge Fragile(Shell(x)) \wedge Hard(y) \\ &\wedge mode \neq Slowly \wedge StrikeAgainst_t(x, y, mode) \\ &\qquad\qquad\qquad\qquad \supset Broken_{t+1}(Shell(x)), \end{aligned}$$

we can derive the first of formulas (3):

$$Broken_1(Shell(E)).$$

At Time 2, the Egg Is over the Bowl

The formula

$$At_2(E, Over(B)) \tag{45}$$

follows from (4) and the material implication corresponding to rule (26). This is one of the expected conclusions (6).

The Completion Method

Some theorems of a causal theory can be proved by applying classical logic to the material implications corresponding to the rules of the theory, as in the two proofs above. But to use this method only would mean to disregard the nonmonotonic component of the theory. Sometimes using the material implications is insufficient.

Consider the following modification of *TEC*. First, we replace the rules (11) and (12) that define *InHands* and *InBowl* by the corresponding formulas

$$\begin{array}{r} InHands_t(x) \equiv InLeftHand_t(x) \wedge InRightHand_t(x), \\ InBowl_t(x,y) \equiv IsBowl(y) \wedge Upright_t(y) \wedge At_t(x, Top(y)), \end{array} \tag{46}$$

and reclassify the predicates $InHands_t$, $InBowl_t$ as unexplainable. Second, we break rule (20) into two rules, one for each conjunctive term in the head:

$$\begin{array}{r} Inside_t(Yolk(x), Shell(x)) \leftarrow IsEgg(x) \wedge \neg Broken_t(Shell(x)), \\ Inside_t(White(x), Shell(x)) \leftarrow IsEgg(x) \wedge \neg Broken_t(Shell(x)), \end{array}$$

and similarly break (39) into 9 rules. It can be proved that the new causal theory TEC' is equivalent to *TEC*. On the other hand, this theory is "definite," so that the literal completion method ([McCain and Turner, 1997], [Lifschitz, 1997]) is applicable to it. We can prove theorems in *TEC* by deriving them in classical logic from the completion sentences for TEC'.

The Bowl Doesn't Change Its Orientation

As an example of the use of this method, we will prove the theorem of *TEC* which asserts that an object in the upright position remains in this position forever. (This peculiar conclusion is possible because the language of *TEC* includes no actions that could change the orientation of an object. In a more realistic formalization of the egg cracking domain that includes the action of turning an object upside down, we would need to postulate that this action is not applied to the bowl B, at least until time 4.)

The only rule of TEC' with positive *UpsideDwon* in the head is the inertia rule

$$UpsideDown_{t+1}(x) \leftarrow UpsideDown_{t+1}(x) \wedge UpsideDown_t(x).$$

Consequently, the positive completion formula for $UpsideDown_{t+1}$ is (the universal closure of)

$$UpsideDown_{t+1}(x) \equiv UpsideDown_{t+1}(x) \wedge UpsideDown_t(x),$$

which is equivalent to

$$UpsideDown_{t+1}(x) \supset UpsideDown_t(x). \tag{47}$$

Furthermore, the rules of TEC' with negative *Upright* in the head are the inertia rule

$$\neg Upright_{t+1}(x) \leftarrow \neg Upright_{t+1}(x) \wedge \neg Upright_t(x)$$

and the second of rules (16). Consequently, the negative completion formula for $Upright_{t+1}$ is

$$\begin{aligned}\neg Upright_{t+1}(x) \equiv\ & [\neg Upright_{t+1}(x) \wedge \neg Upright_t(x)] \\ & \vee UpsideDown_{t+1}(x),\end{aligned}$$

which entails

$$\neg Upright_{t+1}(x) \supset \neg Upright_t(x) \vee UpsideDown_{t+1}(x).$$

Using (47), we conclude:

$$Upright_t(x) \wedge \neg UpsideDown_t(x) \supset Upright_{t+1}(x).$$

In combination with (the material implication corresponding to) the first of rules (16), this formula entails

$$Upright_t(x) \supset Upright_{t+1}(x).$$

Since one of the initial conditions (1) is $Upright_0(B)$, we conclude that, for all $t \leq T$, the formula

$$Upright_t(B) \tag{48}$$

is a theorem of $TEC \cup (1)$.

The Yolk Is Unbroken

The rules of TEC' with positive *Broken* in the head are the inertia rule

$$Broken_{t+1}(x) \leftarrow Broken_{t+1}(x) \wedge Broken_t(x)$$

and rules (35) and (36). Consequently, the positive completion formula for $Broken_{t+1}$ is (the universal closure of)

$$\begin{aligned} &Broken_{t+1}(x) \\ &\quad \equiv [Broken_{t+1}(x) \wedge Broken_t(x)] \\ &\qquad \vee \exists y, z, mode[x = Shell(z) \wedge HasShell(z) \wedge Fragile(Shell(z)) \\ &\qquad\qquad \wedge Hard(y) \wedge mode \neq Slowly \wedge StrikeAgainst_t(z, y, mode)] \\ &\qquad \vee \exists yz[x = Yolk(z) \wedge IsEgg(z) \wedge Raw_t(z) \wedge Hard(y) \\ &\qquad\qquad \wedge StrikeAgainst_t(z, y, Quickly)]. \end{aligned}$$

Using the unique names assumption (42), we conclude:

$$\begin{aligned} &Broken_{t+1}(Yolk(u)) \\ &\qquad \supset Broken_t(Yolk(u)) \vee \exists y StrikeAgainst_t(u, y, Quickly). \end{aligned} \tag{49}$$

This theorem of *TEC* tells us that the only way to break a yolk is to quickly strike the egg against some object. (The possibility of exerting "steady pressure" on the egg, mentioned in the second paragraph of Davis's problem, is not included in this formalization.)

According to one of the initial conditions (1), the yolk is not broken at time 0:

$$\neg Broken_0(Yolk(E)). \tag{50}$$

We can now verify that the same is the case at times 1–4 as well. First note that (2) and (38), in combination with the unique names assumption (42), imply

$$\neg StrikeAgainst_0(E, y, Quickly). \tag{51}$$

From (49)–(51),

$$\neg Broken_1(Yolk(E)). \tag{52}$$

This is the second of formulas (3); we have shown that it is a theorem of $TEC \cup (1) \cup (2)$. Furthermore, from (5), (49) and (52),

$$\neg Broken_2(Yolk(E)),\ \neg Broken_3(Yolk(E)). \tag{53}$$

From (1), (7) and (39),

$$\neg InLeftHand_3(E) \wedge \neg InRightHand_3(E),$$

which implies, in combination with (37):

$$\neg StrikeAgainst_3(E, y, mode). \tag{54}$$

From (49), (53) and (54),

$$\neg Broken_4(Yolk(E)). \tag{55}$$

We have derived (53) and (55) from the postulates of TEC combined with assumptions (1), (2), (5) and (7).

The Cook Never Holds the Yolk or White in His Hands

The rules of TEC' with positive $InLeftHand$ in the head are the inertia rule

$$InLeftHand_{t+1}(x) \leftarrow InLeftHand_{t+1}(x) \wedge InLeftHand_t(x)$$

and one of the rules obtained by splitting (39):

$$InLeftHand_{t+1}(LeftHalfShell(x)) \leftarrow IsEgg(x) \wedge SeparateHalfShells_t(x).$$

Consequently, the positive completion formula for $InLeftHand_{t+1}$ is

$$\begin{aligned} &InLeftHand_{t+1}(x) \\ &\quad \equiv [InLeftHand_{t+1}(x) \wedge InLeftHand_t(x)] \\ &\quad\quad \vee \exists y[x = LeftHalfShell(y) \wedge IsEgg(y) \wedge SeparateHalfShells_t(y)]. \end{aligned}$$

Using the unique names assumption (42), we conclude:

$$InLeftHand_{t+1}(Yolk(x)) \equiv InLeftHand_{t+1}(Yolk(x)) \wedge InLeftHand_t(Yolk(x))$$

or, equivalently,

$$InLeftHand_{t+1}(Yolk(x)) \supset InLeftHand_t(Yolk(x)). \tag{56}$$

This theorem of TEC tells us that a yolk can be in the cook's left hand at time $t+1$ only if it is there at time t. In a similar way we can prove

$$InRightHand_{t+1}(Yolk(x)) \supset InRightHand_t(Yolk(x)). \tag{57}$$

From (1) and (20),

$$Inside_0(Yolk(E), Shell(E)). \tag{58}$$

Using (43) and the last two rules of (13), we conclude:

$$\neg InLeftHand_0(Yolk(E)),\ \neg InRightHand_0(Yolk(E)).$$

In combination with (56) and (57), these formulas allow us to derive in $TEC \cup (1)$, for all $t \leq T$:

$$\neg InLeftHand_t(Yolk(E)),\ \neg InRightHand_t(Yolk(E)). \tag{59}$$

In a similar way, we can derive

$$\neg InLeftHand_t(White(E)),\ \neg InRightHand_t(White(E)). \tag{60}$$

The Yolk and White Never Get Inside a Wrong Shell

The rules of TEC' with positive *Inside* in the head are the inertia rule

$$Inside_{t+1}(x,y) \leftarrow Inside_{t+1}(x,y) \wedge Inside_t(x,y)$$

and the two rules obtained by splitting (20). Consequently, the positive completion formula for $Inside_{t+1}$ is (the universal closure of)

$$\begin{aligned}
&Inside_{t+1}(x,y)\\
&\quad \equiv [Inside_{t+1}(x,y) \wedge Inside_t(x,y)]\\
&\quad\ \vee \exists z[x = Yolk(z) \wedge y = Shell(z) \wedge IsEgg(z) \wedge \neg Broken_{t+1}(Shell(z))]\\
&\quad\ \vee \exists z[x = White(z) \wedge y = Shell(z) \wedge IsEgg(z) \wedge \neg Broken_{t+1}(Shell(z))],
\end{aligned}$$

which entails

$$\begin{aligned}
&Inside_{t+1}(Yolk(u), Shell(v))\\
&\qquad\qquad \supset Inside_t(Yolk(u), Shell(v))\\
&\qquad\qquad\quad \vee \exists z[Yolk(u) = Yolk(z) \wedge Shell(v) = Shell(z)]\\
&\qquad\qquad\quad \vee \exists z[Yolk(u) = White(z)].
\end{aligned}$$

By the unique names assumption (42), the last disjunctive term here can be dropped, and the second disjunctive term can be replaced by

$$\exists z[u = z \wedge v = z],$$

which is equivalent to $u = v$. Consequently, we get:

$$Inside_{t+1}(Yolk(u), Shell(v)) \wedge u \neq v \supset Inside_t(Yolk(u), Shell(v)). \tag{61}$$

This theorem of TEC tells us that a yolk can be inside of a wrong shell at time $t+1$ only if it was there at time t.

From (14), (43) and (58),

$$HasShell(v) \wedge E \neq v \supset \neg Inside_0(Yolk(E), Shell(v)).$$

Using (61), we conclude that, for every $t \leq T$, the formula

$$HasShell(v) \wedge E \neq v \supset \neg Inside_t(Yolk(E), Shell(v)) \tag{62}$$

is a theorem of $TEC \cup (1)$.

In a similar way, we can derive

$$HasShell(v) \wedge E \neq v \supset \neg Inside_t(White(E), Shell(v)) \tag{63}$$

At First, the Two Halves of the Shell Are Not Separated

The rules of TEC' with positive *SeparatedShell* in the head are the inertia rule

$$SeparatedShell_{t+1}(x) \leftarrow SeparatedShell_{t+1}(x) \wedge SeparatedShell_t(x)$$

and one of the rules obtained by splitting (39):

$$SeparatedShell_{t+1}(x) \leftarrow IsEgg(x) \wedge SeparateHalfShells_t(x).$$

Consequently, the positive completion formula for $SeparatedShell_{t+1}$ is

$$\begin{aligned} SeparatedShell_{t+1}(x) \equiv\ & [SeparatedShell_{t+1}(x) \wedge SeparatedShell_t(x)] \\ & \vee [IsEgg(x) \wedge SeparateHalfShells_t(x)], \end{aligned}$$

which implies

$$\begin{aligned} & SeparatedShell_{t+1}(x) \\ & \quad \supset SeparatedShell_t(x) \vee [IsEgg(x) \wedge SeparateHalfShells_t(x)]. \end{aligned} \tag{64}$$

Using (40), we conclude:

$$SeparatedShell_{t+1}(x) \supset SeparatedShell_t(x) \vee Broken_t(Shell(x))$$

or, equivalently,

$$\neg SeparatedShell_t(x) \wedge SeparatedShell_{t+1}(x) \supset Broken_t(Shell(x)). \tag{65}$$

This theorem of TEC tells us that the two halves of a shell can become separated between the times t and $t+1$ only if the shell is broken at time t.

On the other hand, from (39) and (64),

$$SeparatedShell_{t+1}(x) \supset SeparatedShell_t(x) \vee \neg At_{t+1}(x,l)$$

or, equivalently,

$$\neg SeparatedShell_t(x) \wedge SeparatedShell_{t+1}(x) \supset \neg At_{t+1}(x,l). \tag{66}$$

This theorem of TEC tells us that if the two halves of a shell become separated between the times t and $t+1$ then the egg is not located at time $t+1$.

From (1), (15) and (43),

$$\neg SeparatedShell_0(E). \tag{67}$$

Using (1) and (65), we conclude:

$$\neg SeparatedShell_1(E). \tag{68}$$

This formula has been derived from the postulates of $TEC \cup (1)$.

Furthermore, from (45), (66) and (68),

$$\neg SeparatedShell_2(E). \tag{69}$$

This formula has been derived from the postulates of $TEC \cup (1) \cup (4)$.

At First, the Yolk is Inside and Supported

The rules of TEC' with negative $Inside$ in the head are the inertia rule

$$\neg Inside_{t+1}(x,y) \leftarrow \neg Inside_{t+1}(x,y) \wedge \neg Inside_t(x,y),$$

the first two of rules (13), and rules (14) and (32). Consequently, the negative completion formula for $Inside_{t+1}$ is (the universal closure of)

$$\begin{aligned}
&\neg Inside_{t+1}(x,y)\\
&\quad \equiv [\neg Inside_{t+1}(x,y) \wedge \neg Inside_t(x,y)]\\
&\qquad \vee \exists z[y = Shell(z) \wedge InLeftHand_{t+1}(x) \wedge HasShell(z)]\\
&\qquad \vee \exists z[y = Shell(z) \wedge InRightHand_{t+1}(x) \wedge HasShell(z)]\\
&\qquad \vee \exists zu[y = Shell(z) \wedge Inside_{t+1}(x, Shell(u))\\
&\qquad\qquad\qquad \wedge HasShell(u) \wedge HasShell(z) \wedge u \neq z]\\
&\qquad \vee \exists z[y = Shell(z) \wedge \neg Supported_t(x)\\
&\qquad\qquad\qquad \wedge HasShell(z) \wedge Inside_t(x, Shell(z))],
\end{aligned}$$

which implies

$$
\begin{aligned}
&\neg Inside_{t+1}(x,y)\\
&\quad \supset \neg Inside_t(x,y) \vee InLeftHand_{t+1}(x) \vee InRightHand_{t+1}(x)\\
&\qquad \vee \exists zu[y = Shell(z) \wedge Inside_{t+1}(x, Shell(u)) \wedge HasShell(u) \wedge u \neq z]\\
&\qquad \vee \neg Supported_t(x).
\end{aligned}
$$

By the unique names assumption (42),

$$Shell(E) = Shell(z) \supset E = z;$$

using this fact, we further conclude:

$$
\begin{aligned}
&\neg Inside_{t+1}(x, Shell(E))\\
&\quad \supset \neg Inside_t(x, Shell(E)) \vee InLeftHand_{t+1}(x) \vee InRightHand_{t+1}(x)\\
&\qquad \vee \exists u[Inside_{t+1}(x, Shell(u)) \wedge HasShell(u) \wedge u \neq E]\\
&\qquad \vee \neg Supported_t(x).
\end{aligned}
$$

By (59), it follows that

$$
\begin{aligned}
&\neg Inside_{t+1}(Yolk(E), Shell(E))\\
&\quad \supset \neg Inside_t(Yolk(E), Shell(E))\\
&\qquad \vee \exists u[Inside_{t+1}(Yolk(E), Shell(u)) \wedge HasShell(u) \wedge u \neq E]\\
&\qquad \vee \neg Supported_t(Yolk(E)).
\end{aligned}
$$

By (61), the second disjunctive term can be dropped:

$$
\begin{aligned}
&\neg Inside_{t+1}(Yolk(E), Shell(E))\\
&\qquad\qquad \supset \neg Inside_t(Yolk(E), Shell(E)) \vee \neg Supported_t(Yolk(E)).
\end{aligned}
$$

This is equivalent to

$$
\begin{aligned}
&Inside_t(Yolk(E), Shell(E)) \wedge Supported_t(Yolk(E))\\
&\qquad\qquad \supset Inside_{t+1}(Yolk(E), Shell(E)).
\end{aligned}
\tag{70}
$$

Using the last conclusion, we will show now that the yolk remains inside the shell at times 1–3 (we already know from (58) that it is there at time 0). First observe that

$$Supported_0(Yolk(E)) \tag{71}$$

is a consequence of (24), (43), (58) and (67). From (58), (70) and (71),

$$Inside_1(Yolk(E), Shell(E)). \tag{72}$$

The formula

$$Supported_1(Yolk(E)) \tag{73}$$

is a consequence of (24), (43), (68) and (72). From (70), (72) and (73),

$$Inside_2(Yolk(E), Shell(E)). \tag{74}$$

The formula

$$Supported_2(Yolk(E)) \tag{75}$$

is a consequence of (24), (43), (69) and (74). Finally, from (70), (74) and (75),

$$Inside_3(Yolk(E), Shell(E)). \tag{76}$$

We have derived formulas (71)–(76) from the postulates of $TEC \cup (1)$.

At Time 3, the Yolk and White Are over the Bowl

At time 2, the yolk, white and shell are over the bowl:

$$\begin{gathered} At_2(Yolk(E), Over(B)),\ At_2(White(E), Over(B)), \\ At_2(Shell(E), Over(B)). \end{gathered} \tag{77}$$

These formulas are entailed by (29), (43), (74) and (45). At time 3, the egg and its shell are not located: from (7) and (39),

$$\neg At_3(E, l),\ \neg At_3(Shell(E), l). \tag{78}$$

We will show, however, that the first two of conclusions (77) can be extended to time 3.

The rules of TEC with positive At in the head are the inertia rule

$$At_{t+1}(x, l) \leftarrow At_{t+1}(x, l) \wedge At_t(x, l)$$

and rules (26), (29), (30) and (33). Consequently, the positive completion formula for At_{t+1} is

$$\begin{aligned} &At_{t+1}(x, l) \\ &\quad \equiv [At_{t+1}(x, l) \wedge At_t(x, l)] \\ &\quad\quad \vee Move_t(x, l) \\ &\quad\quad \vee \exists y[x = Shell(y) \wedge HasShell(y) \wedge At_{t+1}(y, l)] \\ &\quad\quad \vee \exists y[HasShell(y) \wedge Inside_{t+1}(x, Shell(y)) \wedge At_{t+1}(Shell(y), l)] \\ &\quad\quad \vee \exists y[\neg Supported_t(x) \wedge At_t(x, Over(y)) \wedge \neg Small(y)]. \end{aligned}$$

In combination with (28), this formula implies

$$\begin{array}{l} At_{t+1}(x,l) \\ \quad \supset At_t(x,l) \\ \quad\quad \vee[\mathit{InLeftHand}_t(x) \vee \mathit{InRightHand}_t(x)] \\ \quad\quad \vee\exists y[x = \mathit{Shell}(y)] \\ \quad\quad \vee\exists y[\mathit{HasShell}(y) \wedge \mathit{Inside}_{t+1}(x,\mathit{Shell}(y)) \wedge At_{t+1}(\mathit{Shell}(y),l)] \\ \quad\quad \vee\neg \mathit{Supported}_t(x). \end{array}$$

We can conclude, in particular, using the unique names assumption (42), (59) and (75):

$$\begin{array}{l} At_3(\mathit{Yolk}(E),l) \\ \quad \supset At_2(\mathit{Yolk}(E),l) \\ \quad\quad \vee\exists y[\mathit{HasShell}(y) \wedge \mathit{Inside}_3(\mathit{Yolk}(E),\mathit{Shell}(y)) \wedge At_3(\mathit{Shell}(y),l)]. \end{array}$$

By (62), it follows that

$$\begin{array}{l} At_3(\mathit{Yolk}(E),l) \\ \quad \supset At_2(\mathit{Yolk}(E),l) \\ \quad\quad \vee\exists y[E = y \wedge At_3(\mathit{Shell}(y),l)]. \end{array}$$

By the second of formulas (78), the last disjunctive term can be dropped:

$$At_3(\mathit{Yolk}(E),l) \supset At_2(\mathit{Yolk}(E),l). \tag{79}$$

The rules of *TEC* with negative At in the head are the inertia rule

$$\neg At_{t+1}(x,l) \leftarrow \neg At_{t+1}(x,l) \wedge \neg At_t(x,l)$$

and rules (27) and (31). Consequently, the negative completion formula for At_{t+1} is

$$\begin{array}{l} \neg At_{t+1}(x,l) \\ \quad \equiv [\neg At_{t+1}(x,l) \wedge \neg At_t(x,l)] \\ \quad\quad \vee\exists l'[At_{t+1}(x,l') \wedge l' \neq l] \\ \quad\quad \vee[\neg \mathit{Supported}_t(x) \wedge At_t(x,l)], \end{array}$$

from which we can derive using (79):

$$\begin{array}{l} \neg At_3(\mathit{Yolk}(E),\mathit{Over}(B)) \\ \quad \supset \neg At_2(\mathit{Yolk}(E),\mathit{Over}(B)) \\ \quad\quad \vee\exists l'[At_2(\mathit{Yolk}(E),l') \wedge l' \neq \mathit{Over}(B)] \\ \quad\quad \vee\neg \mathit{Supported}_2(\mathit{Yolk}(E)). \end{array}$$

By (27) and (77), the second disjunctive term in the consequent implies the first; consequently, it can be dropped. The first disjunctive term contradicts (77). The third disjunctive term contradicts (75). We conclude that

$$At_3(\mathit{Yolk}(E), \mathit{Over}(B)). \tag{80}$$

The formula

$$At_3(\mathit{White}(E), \mathit{Over}(B)), \tag{81}$$

is proved in a similar way.

At Time 3, the Yolk and White Are Not Supported

To prove that the yolk is not supported at time 3, consider the positive completion formula for $\mathit{Supported}_t$:

$$\begin{aligned} \mathit{Supported}_t(x) \equiv\ & \mathit{InLeftHand}_t(x) \vee \mathit{InRightHand}_t(x) \\ & \vee \exists y[\mathit{InBowl}_t(x,y) \wedge \neg \mathit{Small}(y)] \\ & \vee \exists y[\mathit{HasShell}(y) \wedge \neg \mathit{SeparatedShell}_t(y) \wedge \mathit{Inside}_t(x, \mathit{Shell}(y))]. \end{aligned}$$

In combination with (46) and (59), this formula implies

$$\begin{aligned} & \mathit{Supported}_t(\mathit{Yolk}(E)) \\ & \quad \supset \exists y At_t(\mathit{Yolk}(E), \mathit{Top}(y)) \\ & \quad\ \vee \exists y[\mathit{HasShell}(y) \wedge \neg \mathit{SeparatedShell}_t(y) \wedge \mathit{Inside}_t(\mathit{Yolk}(E), \mathit{Shell}(y))]. \end{aligned}$$

By (62), the formula in the brackets implies $E = y$, so that we can conclude:

$$\begin{aligned} & \mathit{Supported}_t(\mathit{Yolk}(E)) \\ & \qquad \supset \exists y At_t(\mathit{Yolk}(E), \mathit{Top}(y)) \vee \neg \mathit{SeparatedShell}_t(E). \end{aligned} \tag{82}$$

By the unique names assumption (42),

$$\mathit{Top}(y) \neq \mathit{Over}(B).$$

In combination with (27) and (80), this formula implies

$$\neg At_3(\mathit{Yolk}(E), \mathit{Top}(y)). \tag{83}$$

On the other hand, from (1), (7) and (39) we conclude:

$$\mathit{SeparatedShell}_3(E). \tag{84}$$

From (82)–(84),

$$\neg \mathit{Supported}_3(\mathit{Yolk}(E)). \tag{85}$$

Similarly, we can prove

$$\neg \mathit{Supported}_3(\mathit{White}(E)). \tag{86}$$

The Contents of the Egg Fall into the Bowl

From (1), (33), (80) and (85),

$$At_4(\mathit{Yolk}(E), \mathit{Top}(B)).$$

In combination with (1), (46) and (48), this formula implies one of the desired conclusions (9):

$$\mathit{InBowl}_4(\mathit{Yolk}(E), B).$$

The proof of

$$\mathit{InBowl}_4(\mathit{White}(E), B)$$

is similar.

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