

Zhang, Yitang's life at Purdue(Jan 1985-1991) (Aug. 2013)(revised in bold face. 2018)

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Dr. Zhang Yitang made a major advancement to the twin prime conjecture as verified by Prof. H. Iwaniec, a famous number theorist. This is a historic result. I congratulate Dr. Zhang, Yitang.

The concept of *prime nummbers* started with Greek mathematics. Euclid shown that there were infinitely many primes. We may view the integers as houses built on the integer spots on the real line, and put a light in every prime number houses. Then infinitely many houses are lighted (Euclidean theorem). What are the relations of all lighted houses? The most nature conjecture is that the prime numbers appear randomly. There is a surprising conjecture (The twin prime conjecture) states that there are infinitely many pairs $(p, p+2)$ of houses are lighted. We can not find any trace of it in Euclidean books. Any way the conjecture might be thousand years old. If the twin prime conjecture is correct, then we may conclude that the integers are not constructed randomly. How strange. What Dr Zhang proved is that there is an n less than or equal to 70,000,000 such that $p, p+n$ are lighted as primes.

Some people are curious about Yitang's life as a graduate student at Purdue University. As the thesis adviser of Dr. Zhang, I will share my memories of him.

1 China to USA

In the year 1984, a famous mathematician Prof S.S.Chern called Prof W.Y.Hsiang, Prof H.H.Wu of Berkeley, Prof Y.T.Siu of Harvard, Prof J.Morgan of Columbia and me to go to Beijing, China to teach a special summer school with graduate students the very best all over China. I was greatly honored by Prof Chern's call and worked a whole summer for Prof Chern's pet project and met several important young futere mathematicians as Dr S.W. Zhang, Dr Yitang Zhang, Dr. M.Chen etc.

Certainly, we tried to move those talented graduate students to USA. People outside China knew nothing about their qualities. At that time, I was in the graduate committee of Math Department. I used my credit to convince my colleagues to admit 10 of them to Purdue. Once they showed up and did excellent in the qualifying exams, and all doubts dispersed. After they showed up I advised them to various professors at Purdue, then there is only one left with name Zhang, Yitang.

By the recommendations of Prof. Ding, Shihsun (an algebraist, President of Peking University) and Prof. Deng, D.G. (Chairman of Department of Mathematics, Peking University), Mr. Zhang was admitted to Purdue University as a graduate student in Jan, 1985. Prof. Ding, Shihsun specifically requested me to take care of Mr. Zhang. When he arrived, we had a cordial talk. Yitang expressed his desire to work in the field of Algebraic Geometry. I nodded while remembering the call of French mathematician Jean Dieudonné in the international conference at Purdue (it was to celebrate Louis de Branges' result on Bieberbach's conjecture). Dieudonné said "You should go to the field of algebraic geometry – that is the future of mathematics" to several hundred analysts. Yitang also mentioned that he wanted to study under my guidance. Knowing **(it was improper to insert this phrase here. The fact was that I did not know about Zhang, Yitang personally, I found out a year after Zhang arrived when I saw Prof C.L Zhao of Peking University who visited me for a semester, Prof Zhao and Yitang greeted to each other. I remarked to Prof Zhao that Mr Zhang was a student of me and casually mentioned that Yitang was a student of Prof Ding (an algebraist). To my surprise, Prof Zhao said: "Yitang was not a student of Prof Ding, Yitang was a student in number theory under Prof Pan."** I wondered sometimes that why Yitang kept this fact from me? **Prof C.L. Zhao is my witness about this episode.** that he had one published paper, and some training in the field of Analytic Number Theory under Prof. Pan (especially in the additive branch, where the central problem is the Goldbach conjecture), I was happy to have a talented student. I was surprised by Yitang's next request of working on the Jacobian conjecture as his thesis topic. I felt it was odd to select such a difficult task. After seeing his enthusiasm, I settled on it and did my best to guide him.

I thought Yitang as an ambitious, intelligent, and hard-working young man.

2 Education at Purdue

After we settled on his thesis topic, we met almost daily for a whole semester. Our discussions centered on my published paper

On the Jacobian conjecture and the configuration of roots. J. für die Reine und angewandte Mathematik, 340, 1983, pp. 140-212

Sometimes, they lasted into the dawn. That was the essential part of Yitang's first semester at Purdue. **All the time, Mr Zhang mentioned to me when nobody else is in my office that he will join his solution of Jacobian conjecture with me.** I always told him that I never joined with my students about Ph. D. theses, these were plain to anybody, just looked at my Ph. D. students records. I never published any joint work with my students. All discussions belong to my students. Certainly, I understood why some advisors joined with their students, because sometimes, the advisors had to write the Ph. D. theses for the students. On the other hand, students could be unfairly treated in the joint work. The above was my personal philosophy. Every few months, the our conversations would repeat. Until one day he claimed falsely the Conjecture.

The position of solving conjectures was controversy in mathematics. I believed that there were scientific part and logic part of mathematics. Conjectures belonged to the scientific part, and the proofs of conjectures belonged to logic part. For instance, in science, Newton's gravitation law could not be proved logically and was there as long as there was no important counter example. Scientific laws were the central parts.

In a letter I faxed to Prof Emilio R. Lluís (click here to see a copy), after he had heard that I claimed the proof with Prof S.S. Abhyankar of theorem of embedding lines, which is an open conjecture of Segre, he sent a letter to me claiming the priority of the pleasure of solving the open it, I showed that his proof is wrong also, and stated: "we are grateful to the first one who pointed out the mistake in Segre's article and you know that person is you (i.e., Prof Lluís)".

I believed that the ones who conjectured mathematical statements were the most important people, then someones kept the research lights on the statements, the third group of people were those who solved the conjectures.

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In general, science theories could not be proved, as the gravitational law of Newton, even if it was disproved by the phenomena of the bending of light by gravitation and the orbit of Mercury which was different from the prediction of Newton's law, Newton's law was still considered to be true within its scope and be treasured. For logic proof, it had been thought that AI could handle all logic problems in the near future, hence logic problems of solving conjectures might not be so interesting in the future.

For the next two semesters, I organized a seminar with five graduate students (including Yitang) on Prof. Hironaka's monumental papers on the theory of resolutions of singularities. I believed that we doubled the world population of those who had studied the papers after we finished two semesters. Prof. Grothendick once described those papers as among the most complicated theses in the human history.

Upon completing three semesters of intensive work, I felt that I might not have done the right thing (I behaved too much like a modern day "tiger mother"); I should give him the space to grow. I then laid back and did not do any pushing. He would come to me once a week to talk, and I listened carefully.

I told him my philosophy of Mathematics. I think that there are only good Mathematics and bad Mathematics. To find an original proof in a good Mathematics, one must use computations to do experiments, then you need an insight. It was not wise to totally believe in the partial results of the past. For instance, if you want to work on the grand unification theory of Physics and totally trust Einstein's thesis on unified fields, then you are a fool. You have to look at the new evidents. For instance, my work on the Jacobian Conjecture was beautiful while a generalization to a pair of degrees less than or equal to 1000 will require to handle a lot more exceptional cases, hence will not work by itself. We need more experiments and insight. What I want you to learn from that paper was the methods of computations and the methods of doing experiments.

Yitang spent all of his free time thinking of mathematics. After years, Yitang started to believe that he might have gotten a solution, one independent of my paper, to the Jacobian conjecture. **He told many people that he solved the Jacobian Conjecture without telling me a word. One day a senior Prof Yeh of Chemistry department told me that**

a student of math named Zhang solved a big problem. I called Yitang to my office to inquire the matter. Yitang told me that he solved the Jacobian Conjecture without my help. By that he meant he would not joined the work with me. I asked him to show me the proof. He told me his solution. I had read tens of wrong proofs, his was so strange and low level. It had nothing to do with what I taught him. I pointed out the errors and told him that his proof was wrong. Since then, he never mentioned again that he wished to join his result about Jacobian Conjecture with me. Clearly his words were fake. His work shows that he failed miserably. His talent should be somewhere else. It correlated with what he told Mr. Wilkinson, (New Yorker, Feb 2, 2015) that Zhang's professor insisted, though, that he change his major to algebraic geometry, his own field, "I studied it, but I didn't really like it.... He thought algebraic geometry was more important than number theory. He forced me. He was the university president. so he had this authority." (click here to see the original). Here the term "he" pointed to a grand old gentleman Prof Ding. I was acquainted with Prof Ding (click here to see my Chinese article about Prof Ding), in my opinion, it was impossible for Prof Ding to force Yitang to study algebraic geometry. On the other hand, in 1985, many Chinese students rushed to go aboard to study, some of them were capable to do strange things. It was possible that what happened was Yitang pretended to be interested in algebraic geometry and fooled Prof Ding to recommened him. Later on, Yitang denied that Prof Ding was his professor. For the last 33 years, Yitang never published any paper on algebraic geometry. The usual productivity was the Ph.D.thesis and a second paper in ten years for the top 10% new Ph.D.. Yitang published no paper in Algebraic Geometry. Yitang wasted 7 years of his own life and my time and an opportunity of a young Chinese Algebraic Geometor.

Years later, a young mathematician Su, Yucai, wrote three pieces of articles trying to solve Jacobian Conjecture, I reviewed two versions of his. I felt that he was a serious mathematician. I was afraid that he would be discouraged and even laughed at by some people, I published one of my reviews in Arxiv (click here to see a copy) which stated in part,
 "The problem of Jacobian Conjecture is very hard. Perhaps it will take human being another 100 years to solve it. Your attempt is

noble. Maybe the Gods of Olympus will smile on you one day. Donot be too disapointed.”

then I listed famous mathematicians who made mistakes about Jacobian Conjecture.

(After years, Yitang started to believe that he might have gotten a solution, one independent of my paper, to the Jacobian conjecture. As a gatekeeper of the palace of the Jacobian conjecture, I did my duty of examining every claim presented to me and denied the entrance of anybody (even if the claim has nothing to do with my work, especially Yitang’s false claim has nothing to do with my previous paper. Later in his PhD thesis he quoted extensively of my paper and never mention anything wrong with my paper.) if Yitang’s proof was invalid. “Maybe the Jacobian conjecture was a problem for the future”, I thought.)

In the year of 1991, his 7-th year at Purdue (which was the last year for Yitang at Purdue. according to school rules), I asked Yitang ito compose his Ph. D. thesis (click here to see a copy) *Jacobian Conjecture and the Degree of Field Extension* (Anybody’s thesis was in the public domain. This thesis in in the Purdue Library). He presented it to the committee which consisted of myself as the chair, L. de Brangles, J. Lipman and W. Heinzer. His thesis passed the committee with flying colors. All members of the committee agreed. It was a reasonable Ph. D. thesis (**rate B**), but clearly it was thousands miles away from a Field medal work. By the way, ”he attempted to prove something implied by the conjecture, rather than to prove the conjecture itself” (quoted from Wilkinson, New Yorker, Feb 2, 2015). **He falsely claimed that in his Ph. D. thesis, he solved the Jacobian Conjecture, and he should be awarded a ”Field medal”. However during a detail check, it was discovered that he used one of T.T.Moh’s theorems which was wrong, and his thesis could not be published.** His claim was a lie. Any rumor-spreading would not change the fact. Any one can see clearly from his thesis attached in this article.

3 Daily life

Some people believe that mathematicians are weird. I think differently. All the mathematicians I know are quite normal. Yitang was a normal person for those seven years at Purdue. He was elected the President of the Chinese Student Club at Purdue University, and he discharged his duty responsibly and served his community well.

Yitang had some background in classical Chinese literature. Sometimes

we talked about them. I mentioned to him a phrase of Confucius: "A person who knows a job is no match to a person who likes the job. A person who likes the job is no match to a person who enjoys the job. " I thought that applied to research very well. He agreed (**only superficially. He wanted to be famous all the time.**) Prof S.S.Chern said:"I think that the mathematics most of us are doing originated from the work of B.Riemann and H.Poincaré. They were Pu^2sa^4 (Bohdisattvas) in mathematics" and "no matter how hard we tried, at most became a Lou^2han^4 (Arhat). Maybe we all knew the name of a Pu^2sa^4 , but nobody could tell who's who for Lou^2han^4 s. We should not pay too much attention to one's name." (cf S.S.Chern:" Somethings about my mathematica career"(Chinese)

Sometimes I regreted not fixing him a job. But really, who could tell whether it was a good decision or not? **In anyway, his Purdue Ph.D. helped him on his long way.** Maybe it was his destiny to endure and turn out to be great **in number theory, while he showed clearly he could not do anything meaningful in algebraic geometry.** I indeed got a job for my first student at the end of 70's. Later when I told my stories to my colleagues, they all laughed and told me that it was only normal in the long-gone 60's. The times changed at the late 70's when there was a new term and trend. It was the "tenure track" and that the students should look for jobs on their own. So after Yitang graduated, I told him the normal way of seeking jobs. When I looked into his eyes, I found a disturbing soul, a burning bush, an explorer who wanted to reach the north pole, a mountaineer who determined to scale Mt. Everest, and a traveler who would brave thunders and lightnings to reach his destination. Yitang never came back to me requesting recommendation letters. Apparently, he did not seek a job. Even to the date Yitang announced his monumental result I did not know what was the best for him. Though I was sure of one thing, — he could not survive the life of "tenure-track," "tenure," and "promotions". It was not his type. I regarded him as a free spirit, and I should let him fly. Yitang flew away after he told me that he was going to Rutgers University to talk to Prof. Iwaniec. I bade him good luck. That was almost 22 years ago.

I inquired about Yitang's whereabouts from time to time. One student told me that he saw Yitang a few days ago in Library, and now he was gone. Then a friend of Yitang's family sent a letter to the department inquiring about Yitang for his father's funeral. They natureally asked me. I transmitted the questions to local Chinese students. After a few days, the answer came back that

he now knew the funeral and does not plan to go. Since it was an adult's privacy of keeping one's address secret, I do not plan to touch him with a five feet pole and just dropped the inquiry. The above fact might be seen from a letter from the secretary of the graduate committee to a family friend of Zhangs (the original was on file. The reply of the secretary was with the informations provided by me. All private informations as names and address were covered,) (click here to see).

4 Epilogue

here were some questions as to if my paper on the Jacobian Conjecture was correct. I will state reasons why that the paper was correct. In the year 1984, Prof. M. Miyanishi (the Provost of Osaka University) and Prof. S.S.S. Wang (Oakland University) visited Purdue University for one semester for the purpose of studying this paper on the Jacobian conjecture. After one semester seminar, they confirmed the paper. About 10 years after the publication of my paper, another mathematician published a paper that obtained the same results as mine using a completely different method (Prof. R. Heitmann (cf J. Pure Appl. Alg. 64(1990) 35-72) proved a general theory of the Jacobian Conjecture similar to mine (he thought that it might be identical to mine), and reduced his theory to pairs of polynomials that are up to degree 100, the results are identical to my theory that there are only 4 exceptional cases of pairs of degrees (64,48), (75,50), (84,56), (99,66). Our experiments showed that the number of the exceptions tended to infinity. What we are waiting for is either a theory with no exceptional case or only finitely many exceptional cases. The resolutions of a few isolated cases will not be very interesting at the present situations.) In fact, anybody who thinks that there was an error in my theory was welcomed to publish a paper to point it out. One should not overlook the tenure system in the USA. It might not be suitable for some participants, but it fits most situations.

The year 1985 was extremely difficult for the students from mainland China, they were unknown and without credits. I was the few professors who had personal contacts with them, I had taught the previous summer in the Peking University and had a vivid impression of them. The mission of education was to spread the knowledge everywhere. It was the noble American spirit. For some 10 years, I had recommended 100 mainland Chinese students to the department and all accepted by the department. I am always in debt to the trust of my judgements by the department. Only very few of

them misbehaved, bit the hands which fed them, none of them intended to murder their parents/friends, almost all of them performed well and became well-liked. I was happy that my recommendations brought fruits. Since then, due to the good office of Purdue administration, my friend Mr S. W. Lam donated a fund to a fellowship "T.T.Moh fellowship" in my name, I was greatly honored, for the students from mainland China. So far we only used the interests of it, and Mr S. W. Lam promised to continue donations until 2020. By the way, we offered 7 T.T.Moh fellowship⁸ to mainland Chinese students for 2018.