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Possible Advanced Topics Course

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ABSTRACT: Potential List of Topics for an Advanced Topics version of Physics 695, Fall 2015 June 14, 2015

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Contents

1. Basic Plan

The basic plan will be to cover several different topics in Physical Mathematics. There is much more material here than can be covered in one semester so some choices need to be made. I welcome student input in making the decision as to what topics to cover.

I will assume knowledge of basic manifold, cohomology and bundle theory. I am quite happy to go back and review any of this material.

The references are to various notes I have used in the past. For any topics we do agree upon I will try to provide more extensive sources, including lecture notes.

2. Quantum symmetries and phases of gapped quantum systems

This is the topic I covered in Fall 2013. Lecture notes are available at
http://www.physics.rutgers.edu/∼gmoore/695Fall2013/Physics695Fall2013.html
http://www.physics.rutgers.edu/∼gmoore/ViennaLecturesF.pdf

2.1 Quantum Symmetry

2.2 Group Extensions

a little crystallography

2.3 Wigner's theorem and quantum symmetry

2.4 Corepresentations

2.5 Real, complex, and quaternionic vector spaces and representations

2.6 Dyson's 3-fold way

Division algebras; Schur's lemma; Dyson's problem; Random matrices

2.7 Symmetries and time reversal

2.8 Gapped Hamiltonians and topological phases of matter

2.9 The 10 CT groups

2.10 Clifford algebras and the 10 super division algebras

2.11 Spinors and spin representations

2.12 Finite dimensional free fermions with symmetry

2.13 The Altland-Zirnbauer classification

2.14 Bott periodicity and Cartan symmetric spaces

Other references:

1. Twisted equivariant matter, Daniel S. Freed and Gregory W. Moore, e-Print: arXiv:1208.5055

2. http://www.physics.rutgers.edu/ gmoore/QuantumSymmetryKTheory-Part1.pdf

3. http://www.physics.rutgers.edu/ gmoore/SCGP-TWISTEDMATTER-2013D.pdf

3. K-theory in string theory and condensed matter physics

3.1 Bundle basics, homotopy classification, BG, Serre-Swan theorem

3.2 Grothendieck construction and K-theory

3.3 D-branes in K-theory

Just a little bit. Really belongs later on.

3.4 Virtual representations

3.5 Twisted and equivariant

3.6 Topological phases, again

References:

1. GMP notes
2. Freed-Moore above.
3. Husemoller et. al. Basic Bundle Theory and K-Cohomological Invariants
4. Notes from course at Aspen 2000.

Expected 3-4 lectures. Try to be brief here.

4. Supersymmetric Quantum Mechanics and Morse Theory

4.1 The idea of topological field theory

4.2 Some math background: Thom class and Thom isomorphism. Poincaré dual class.

4.3 Topological Field Theory path integrals and Cohomological Field Theory.

4.4 Supersymmetric Quantum mechanics

Witten index, and localization. Simple examples. Hilbert space and Hodge theory.

4.5 Supersymmetry and index theorems.

4.6 Morse theory, according to Morse

4.7 Morse theory, according to Witten

SQM and Morse theory: Morse-Smale-Witten complex

Nicely ties into the discussion of the symmetry classes via Milnor's discussion of Bott periodicity.

4.8 Morse theory in infinite dimensions

1. LG models in 1+1 dimensions with (2,2) supersymmetry.
2. Floer theory and its cousins.

References:

1. Witten 82
2. Les Houches lectures
3. GMP notes
4. Recent paper of Gaiotto, Moore, and Witten.

5. Generalized Maxwell theory, Dirac quantization, Chern-Simons, and Self-Dual Theories

5.1 Generalized Maxwell Theory and (abelian) Electric-Magnetic Duality

5.2 Dirac quantization

5.3 Role of differential cohomology

5.4 Partition functions

5.5 Hilbert space and Pontryagin-Poincare Duality

5.6 Electric and magnetic sources

5.7 Self-duality

5.8 Chern-Simons theory

5.9 The abelian gauge field of M-theory

5.10 RR fields and differential cohomology

5.11 A general theory of self-dual fields

References:

1. Chapter 5, Felix Klein lectures
2. http://www.physics.rutgers.edu/ gmoore/SCGP-Minicourse.pdf

Expected 6-8 lectures

6. Conformal Theories

6.1 Conformal groups, deSitter, and anti-deSitter geometry

6.2 A tiny bit about conformally invariant QFT

References:

1. Group Theory Notes.

7. Super Lie algebras

7.1 Super Lie algebras

7.2 Superconformal Lie algebras: Nahm's theorem

7.3 The Poincaré super algebras

7.4 Representations of the Poincaré and superconformal algebras

7.5 Superconformal indices

7.6 Two-dimensional superconformal geometry

7.7 Comments on the six-dimensional (2,0) theories

References:

1. Group Theory notes.
2. Felix Klein lectures, chapters 2,3,6

8. Magnetic monopoles and Instantons

8.1 Topological sectors

Yang-Mills on a 4-manifold. Yang-Mills-Higgs on $\mathbb{R}^3$.

8.2 't Hooft-Polyakov monopole

8.3 Monopole moduli space and collective coordinates

8.4 ADHM construction

Expected 4-5 lectures

9. Donaldson theory and Seiberg-Witten invariants

9.1 Four-manifold topology

9.2 Twisting N=2 SYM and Donaldson invariants

9.3 Low energy Seiberg-Witten solution

9.4 Seiberg-Witten equations

9.5 Relating the Donaldson and Seiberg-Witten equations

References:

2. Lectures at http://scgp.stonybrook.edu/archives/1964
3. Lecture notes from SCGP March school lecture.

Expected: 6-8 lectures.

10. BPS wall-crossing and Hitchin systems

PiTP lectures.

Felix Klein lectures: Chapter 4, Chapters 11-20.

About 10 - 15 lectures

11. Automorphic forms in conformal field theory and string theory

Basics of modular forms.

Modular forms and functions in two dimensional CFT.

Automorphic forms and target space string dualities

Black hole counting functions

Rademacher expansion.

Elliptic genus

Borcherds products and theta lifts. K3 sigma model, mock modular forms, Mathieu Moonshine.

References:

1. Trieste lectures 2008. (Home page, talk 17)

Expected: About 6 lectures