

## STANDARDS FOR EFFICIENT CRYPTOGRAPHY

# SEC 2: Recommended Elliptic Curve Domain Parameters

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# 1 Introduction

## 1.1 Overview

This document lists example elliptic curve domain parameters at commonly required security levels for use by implementers of SEC 1 [12] and other ECC standards like ANSI X9.62 [1], ANSI X9.63 [3], and IEEE P1363 [8].

It is strongly recommended that implementers select parameters from among the example parameters listed in this document when they deploy ECC-based products in order to encourage the deployment of interoperable ECC-based solutions.

## 1.2 Compliance

Implementations may claim compliance with the recommended parameters specified in this document provided some subset of the recommended parameters are used by the cryptographic schemes based on elliptic curve cryptography included in the implementation.

It is envisioned that implementations choosing to comply with this document will typically choose also to comply with its companion document, SEC 1 [12].

It is intended to make a validation system available so that implementors can check compliance with this document - see the SECG website, [www.secg.org](http://www.secg.org), for further information.

## 1.3 Document Evolution

This document will be reviewed every five years to ensure it remains up to date with cryptographic advances. The next scheduled review will therefore take place in September 2005.

Additional intermittent reviews may also be performed from time-to-time as deemed necessary by the Standards for Efficient Cryptography Group.

## 1.4 Intellectual Property

The reader's attention is called to the possibility that compliance with this document may require use of an invention covered by patent rights. By publication of this document, no position is taken with respect to the validity of this claim or of any patent rights in connection therewith. The patent holder(s) may have filed with the SECG a statement of willingness to grant a license under these rights on reasonable and nondiscriminatory terms and conditions to applicants desiring to obtain such a license. Additional details may be obtained from the patent holder and from the SECG website, [www.secg.org](http://www.secg.org).

## 1.5 Organization

This document is organized as follows.

The main body of the document focuses on the specification of recommended elliptic curve domain parameters. Section 2 describes recommended elliptic curve domain parameters over  $\mathbb{F}_p$ , and Section 3 describes recommended elliptic curve domain parameters over  $\mathbb{F}_{2^m}$ .

The appendices to the document provide additional relevant material. Appendix A provides reference ASN.1 syntax for implementations to use to identify the parameters. Appendix B lists the references cited in the document.

## 2 Recommended Elliptic Curve Domain Parameters over $\mathbb{F}_p$

This section specifies the elliptic curve domain parameters over  $\mathbb{F}_p$  recommended in this document.

The section is organized as follows. First Section 2.1 describes relevant properties of the recommended parameters over  $\mathbb{F}_p$ . Then Section 2.2 specifies recommended 112-bit elliptic curve domain parameters over  $\mathbb{F}_p$ , Section 2.3 specifies recommended 128-bit elliptic curve domain parameters over  $\mathbb{F}_p$ , Section 2.4 specifies recommended 160-bit elliptic curve domain parameters over  $\mathbb{F}_p$ , Section 2.5 specifies recommended 192-bit elliptic curve domain parameters over  $\mathbb{F}_p$ , Section 2.6 specifies recommended 224-bit elliptic curve domain parameters over  $\mathbb{F}_p$ , Section 2.7 specifies recommended 256-bit elliptic curve domain parameters over  $\mathbb{F}_p$ , Section 2.8 specifies recommended 384-bit elliptic curve domain parameters over  $\mathbb{F}_p$ , Section 2.9 specifies recommended 521-bit elliptic curve domain parameters over  $\mathbb{F}_p$ ,

### 2.1 Properties of Elliptic Curve Domain Parameters over $\mathbb{F}_p$

Following SEC 1 [12], elliptic curve domain parameters over  $\mathbb{F}_p$  are a sextuple:

$$T = (p, a, b, G, n, h)$$

consisting of an integer  $p$  specifying the finite field  $\mathbb{F}_p$ , two elements  $a, b \in \mathbb{F}_p$  specifying an elliptic curve  $E(\mathbb{F}_p)$  defined by the equation:

$$E : y^2 \equiv x^3 + a.x + b \pmod{p},$$

a base point  $G = (x_G, y_G)$  on  $E(\mathbb{F}_p)$ , a prime  $n$  which is the order of  $G$ , and an integer  $h$  which is the cofactor  $h = \#E(\mathbb{F}_p)/n$ .

When elliptic curve domain parameters are specified in this document, each component of this sextuple is represented as an octet string converted using the conventions specified in SEC 1 [12].

Again following SEC 1 [12], elliptic curve domain parameters over  $\mathbb{F}_p$  must have:

$$\lceil \log_2 p \rceil \in \{112, 128, 160, 192, 224, 256, 384, 521\}.$$

This restriction is designed to encourage interoperability while allowing implementers to supply commonly required security levels — recall that elliptic curve domain parameters over  $\mathbb{F}_p$  with  $\lceil \log_2 p \rceil = 2t$  supply approximately  $t$  bits of security — meaning that solving the logarithm problem on the associated elliptic curve is believed to take approximately  $2^t$  operations.

Here recommended elliptic curve domain parameters are supplied at each of the sizes allowed in SEC 1.

All the recommended elliptic curve domain parameters over  $\mathbb{F}_p$  use special form primes for their field order  $p$ . These special form primes facilitate especially efficient implementations like those described in [5]. Recommended elliptic curve domain parameters over  $\mathbb{F}_p$  which use random primes for their field order  $p$  may be added later if commercial demand for such parameters increases.

The elliptic curve domain parameters over  $\mathbb{F}_p$  supplied at each security level typically consist of examples of two different types of parameters — one type being parameters associated with a Koblitz curve and the

other type being parameters chosen verifiably at random — although only verifiably random parameters are supplied at export strength and at extremely high strength.

Parameters associated with a Koblitz curve admit especially efficient implementation. The name Koblitz curve is best-known when used to describe binary anomalous curves over  $\mathbb{F}_{2^m}$  which have  $a, b \in \{0, 1\}$  [9]. Here it is generalized to refer also to curves over  $\mathbb{F}_p$  which possess an efficiently computable endomorphism [7]. The recommended parameters associated with a Koblitz curve were chosen by repeatedly selecting parameters admitting an efficiently computable endomorphism until a prime order curve was found.

Verifiably random parameters offer some additional conservative features. These parameters are chosen from a seed using SHA-1 as specified in ANSI X9.62 [1]. This process ensures that the parameters cannot be predetermined. The parameters are therefore extremely unlikely to be susceptible to future special-purpose attacks, and no trapdoors can have been placed in the parameters during their generation. When elliptic curve domain parameters are chosen verifiably at random, the seed  $S$  used to generate the parameters may optionally be stored along with the parameters so that users can verify the parameters were chosen verifiably at random.

Here verifiably random parameters have been chosen either so that the associated elliptic curve has prime order, or so that scalar multiplication of points on the associated elliptic curve can be accelerated using Montgomery's method [10]. The recommended verifiably random parameters were chosen by repeatedly selecting a random seed and counting the number of points on the corresponding curve until appropriate parameters were found. Typically the parameters were chosen so that  $a = p - 3$  because such parameters admit efficient implementation. For a given  $p$ , approximately half the isomorphism classes of elliptic curves over  $\mathbb{F}_p$  contain a curve with  $a = p - 3$ .

See SEC 1 [12] for further guidance on the selection of elliptic curve domain parameters over  $\mathbb{F}_p$ .

The recommended elliptic curve domain parameters over  $\mathbb{F}_p$  have been given nicknames to enable them to be easily identified. The nicknames were chosen as follows. Each name begins with `sec` to denote ‘Standards for Efficient Cryptography’, followed by a `p` to denote parameters over  $\mathbb{F}_p$ , followed by a number denoting the length in bits of the field size  $p$ , followed by a `k` to denote parameters associated with a Koblitz curve or an `r` to denote verifiably random parameters, followed by a sequence number.

Table 1 summarizes salient properties of the recommended elliptic curve domain parameters over  $\mathbb{F}_p$ .

Information is represented in Table 1 as follows. The column labelled ‘parameters’ gives the nickname of the elliptic curve domain parameters. The column labelled ‘section’ refers to the section of this document where the parameters are specified. The column labelled ‘strength’ gives the approximate number of bits of security the parameters offer. The column labelled ‘size’ gives the length in bits of the field order. The column labelled ‘RSA/DSA’ gives the approximate size of an RSA or DSA modulus at comparable strength. (See SEC 1 [12] for precise technical guidance on the strength of elliptic curve domain parameters.) Finally the column labelled ‘Koblitz or random’ indicates whether the parameters are associated with a Koblitz curve — ‘`k`’ — or were chosen verifiably at random — ‘`r`’.

Table 2 summarizes the status of the recommended elliptic curve domain parameters over  $\mathbb{F}_p$  with respect to their alignment with other standards.

| Parameters | Section | Strength | Size | RSA/DSA | Koblitz or random |
|------------|---------|----------|------|---------|-------------------|
| secp112r1  | 2.2.1   | 56       | 112  | 512     | r                 |
| secp112r2  | 2.2.2   | 56       | 112  | 512     | r                 |
| secp128r1  | 2.3.1   | 64       | 128  | 704     | r                 |
| secp128r2  | 2.3.2   | 64       | 128  | 704     | r                 |
| secp160k1  | 2.4.1   | 80       | 160  | 1024    | k                 |
| secp160r1  | 2.4.2   | 80       | 160  | 1024    | r                 |
| secp160r2  | 2.4.3   | 80       | 160  | 1024    | r                 |
| secp192k1  | 2.5.1   | 96       | 192  | 1536    | k                 |
| secp192r1  | 2.5.2   | 96       | 192  | 1536    | r                 |
| secp224k1  | 2.6.1   | 112      | 224  | 2048    | k                 |
| secp224r1  | 2.6.2   | 112      | 224  | 2048    | r                 |
| secp256k1  | 2.7.1   | 128      | 256  | 3072    | k                 |
| secp256r1  | 2.7.2   | 128      | 256  | 3072    | r                 |
| secp384r1  | 2.8.1   | 192      | 384  | 7680    | r                 |
| secp521r1  | 2.9.1   | 256      | 521  | 15360   | r                 |

Table 1: Properties of Recommended Elliptic Curve Domain Parameters over  $\mathbb{F}_p$ 

Information is represented in Table 2 as follows. The column labelled ‘parameters’ gives the nickname of the elliptic curve domain parameters. The column labelled ‘section’ refers to the section of this document where the parameters are specified. The remaining columns give the status of the parameters with respect to various other standards which specify mechanisms based on elliptic curve cryptography: ‘ANSI X9.62’ refers to the ANSI X9.62 standard [1], ‘ANSI X9.63’ refers to the draft ANSI X9.63 standard [3], ‘echeck’ refers to the draft FSML standard [6], ‘IEEE P1363’ refers to the draft IEEE P1363 standard [8], ‘IPSec’ refers to the recent internet draft related to ECC [11] submitted to the IETF’s IPSec working group, ‘NIST’ refers to the list of recommended parameters recently released by the U.S. government [5], and ‘WAP’ refers to the Wireless Application Forum’s WTLS standard [13]. In these columns, a ‘-’ denotes parameters non-conformant with the standard, a ‘c’ denotes parameters conformant with the standard, and an ‘r’ denotes parameters explicitly recommended in the standard.

Note that ANSI X9.62 is currently being updated. The set of recommended parameters in the proposed ANSI X9.62-1 [2] is identical to the set of recommended parameters in this document.

| Parameters | Section | ANSI X9.62 | ANSI X9.63 | echeck | IEEE P1363 | IPSec | NIST | WAP |
|------------|---------|------------|------------|--------|------------|-------|------|-----|
| secp112r1  | 2.2.1   | -          | -          | -      | c          | c     | -    | r   |
| secp112r2  | 2.2.2   | -          | -          | -      | c          | c     | -    | c   |
| secp128r1  | 2.3.1   | -          | -          | -      | c          | c     | -    | c   |
| secp128r2  | 2.3.2   | -          | -          | -      | c          | c     | -    | c   |
| secp160k1  | 2.4.1   | c          | r          | c      | c          | c     | -    | c   |
| secp160r1  | 2.4.2   | c          | c          | c      | c          | c     | -    | r   |
| secp160r2  | 2.4.3   | c          | r          | c      | c          | c     | -    | c   |
| secp192k1  | 2.5.1   | c          | r          | c      | c          | c     | -    | c   |
| secp192r1  | 2.5.2   | r          | r          | c      | c          | c     | r    | c   |
| secp224k1  | 2.6.1   | c          | r          | c      | c          | c     | -    | c   |
| secp224r1  | 2.6.2   | c          | r          | c      | c          | c     | r    | c   |
| secp256k1  | 2.7.1   | c          | r          | c      | c          | c     | -    | c   |
| secp256r1  | 2.7.2   | r          | r          | c      | c          | c     | r    | c   |
| secp384r1  | 2.8.1   | c          | r          | c      | c          | c     | r    | c   |
| secp521r1  | 2.9.1   | c          | r          | c      | c          | c     | r    | c   |

Table 2: Status of Recommended Elliptic Curve Domain Parameters over  $\mathbb{F}_p$ 

## 2.2 Recommended 112-bit Elliptic Curve Domain Parameters over $\mathbb{F}_p$

This section specifies the two recommended 112-bit elliptic curve domain parameters over  $\mathbb{F}_p$  in this document: verifiably random parameters `secp112r1`, and verifiably random parameters `secp112r2`.

Section 2.2.1 specifies the elliptic curve domain parameters `secp112r1`, and Section 2.2.2 specifies the elliptic curve domain parameters `secp112r2`.

### 2.2.1 Recommended Parameters `secp112r1`

The verifiably random elliptic curve domain parameters over  $\mathbb{F}_p$  `secp112r1` are specified by the sextuple  $T = (p, a, b, G, n, h)$  where the finite field  $\mathbb{F}_p$  is defined by:

$$\begin{aligned} p &= \text{DB7C 2ABF62E3 5E668076 BEAD208B} \\ &= (2^{128} - 3)/76439 \end{aligned}$$

The curve  $E: y^2 = x^3 + ax + b$  over  $\mathbb{F}_p$  is defined by:

$$\begin{aligned} a &= \text{DB7C 2ABF62E3 5E668076 BEAD2088} \\ b &= \text{659E F8BA0439 16EEDE89 11702B22} \end{aligned}$$

$E$  was chosen verifiably at random as specified in ANSI X9.62 [1] from the seed:

$$S = \text{00F50B02 8E4D696E 67687561 51752904 72783FB1}$$

The base point  $G$  in compressed form is:

$$G = \text{020948 7239995A 5EE76B55 F9C2F098}$$

and in uncompressed form is:

$$\begin{aligned} G &= \text{04 09487239 995A5EE7 6B55F9C2 F098A89C E5AF8724 C0A23E0E} \\ &\quad \text{0FF77500} \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

$$\begin{aligned} n &= \text{DB7C 2ABF62E3 5E7628DF AC6561C5} \\ h &= \text{01} \end{aligned}$$

## 2.2.2 Recommended Parameters secp112r2

The verifiably random elliptic curve domain parameters over  $\mathbb{F}_p$  `secp112r2` are specified by the sextuple  $T = (p, a, b, G, n, h)$  where the finite field  $\mathbb{F}_p$  is defined by:

$$\begin{aligned} p &= \text{DB7C 2ABF62E3 5E668076 BEAD208B} \\ &= (2^{128} - 3)/76439 \end{aligned}$$

The curve  $E: y^2 = x^3 + ax + b$  over  $\mathbb{F}_p$  is defined by:

$$\begin{aligned} a &= \text{6127 C24C05F3 8A0AAAF6 5C0EF02C} \\ b &= \text{51DE F1815DB5 ED74FCC3 4C85D709} \end{aligned}$$

$E$  was chosen verifiably at random as specified in ANSI X9.62 [1] from the seed:

$$S = \text{002757A1 114D696E 67687561 51755316 C05E0BD4}$$

The base point  $G$  in compressed form is:

$$G = \text{034BA3 0AB5E892 B4E1649D D0928643}$$

and in uncompressed form is:

$$\begin{aligned} G &= \text{04 4BA30AB5 E892B4E1 649DD092 8643ADCD 46F5882E 3747DEF3} \\ &\quad \text{6E956E97} \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

$$\begin{aligned} n &= 36DF\ 0AAFD8B8\ D7597CA1\ 0520D04B \\ h &= 04 \end{aligned}$$

## 2.3 Recommended 128-bit Elliptic Curve Domain Parameters over $\mathbb{F}_p$

This section specifies the two recommended 128-bit elliptic curve domain parameters over  $\mathbb{F}_p$  in this document: verifiably random parameters `secp128r1`, and verifiably random parameters `secp128r2`.

Section 2.3.1 specifies the elliptic curve domain parameters `secp128r1`, and Section 2.3.2 specifies the elliptic curve domain parameters `secp128r2`.

### 2.3.1 Recommended Parameters `secp128r1`

The verifiably random elliptic curve domain parameters over  $\mathbb{F}_p$  `secp128r1` are specified by the sextuple  $T = (p, a, b, G, n, h)$  where the finite field  $\mathbb{F}_p$  is defined by:

$$\begin{aligned} p &= FFFFFFFD\ FFFFFFFF\ FFFFFFFF\ FFFFFFFF \\ &= 2^{128} - 2^{97} - 1 \end{aligned}$$

The curve  $E$ :  $y^2 = x^3 + ax + b$  over  $\mathbb{F}_p$  is defined by:

$$\begin{aligned} a &= FFFFFFFD\ FFFFFFFF\ FFFFFFFF\ FFFFFFFC \\ b &= E87579C1\ 1079F43D\ D824993C\ 2CEE5ED3 \end{aligned}$$

$E$  was chosen verifiably at random as specified in ANSI X9.62 [1] from the seed:

$$S = 000E0D4D\ 696E6768\ 75615175\ 0CC03A44\ 73D03679$$

The base point  $G$  in compressed form is:

$$G = 03\ 161FF752\ 8B899B2D\ 0C28607C\ A52C5B86$$

and in uncompressed form is:

$$\begin{aligned} G &= 04\ 161FF752\ 8B899B2D\ 0C28607C\ A52C5B86\ CF5AC839\ 5BAFEB13 \\ &\quad C02DA292\ DDED7A83 \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

$$\begin{aligned} n &= FFFFFFFE\ 00000000\ 75A30D1B\ 9038A115 \\ h &= 01 \end{aligned}$$

### 2.3.2 Recommended Parameters secp128r2

The verifiably random elliptic curve domain parameters over  $\mathbb{F}_p$  secp128r2 are specified by the sextuple  $T = (p, a, b, G, n, h)$  where the finite field  $\mathbb{F}_p$  is defined by:

$$\begin{aligned} p &= \text{FFFFFFFD FFFFFFFF FFFFFFFF FFFFFFFF} \\ &= 2^{128} - 2^{97} - 1 \end{aligned}$$

The curve  $E: y^2 = x^3 + ax + b$  over  $\mathbb{F}_p$  is defined by:

$$\begin{aligned} a &= \text{D6031998 D1B3BBFE BF59CC9B BFF9AEE1} \\ b &= \text{5EEEFCAs 80D02919 DC2C6558 BB6D8A5D} \end{aligned}$$

$E$  was chosen verifiably at random as specified in ANSI X9.62 [1] from the seed:

$$S = \text{004D696E 67687561 517512D8 F03431FC E63B88F4}$$

The base point  $G$  in compressed form is:

$$G = \text{02 7B6AA5D8 5E572983 E6FB32A7 CDEBC140}$$

and in uncompressed form is:

$$\begin{aligned} G = &\text{04 7B6AA5D8 5E572983 E6FB32A7 CDEBC140 27B6916A 894D3AEE} \\ &\text{7106FE80 5FC34B44} \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

$$\begin{aligned} n &= \text{3FFFFFFF 7FFFFFFF BE002472 0613B5A3} \\ h &= \text{04} \end{aligned}$$

## 2.4 Recommended 160-bit Elliptic Curve Domain Parameters over $\mathbb{F}_p$

This section specifies the three recommended 160-bit elliptic curve domain parameters over  $\mathbb{F}_p$  in this document: parameters secp160k1 associated with a Koblitz curve, verifiably random parameters secp160r1, and verifiably random parameters secp160r2.

Section 2.4.1 specifies the elliptic curve domain parameters secp160k1, Section 2.4.2 specifies the elliptic curve domain parameters secp160r1, and Section 2.4.3 specifies the elliptic curve domain parameters secp160r2.

### 2.4.1 Recommended Parameters secp160k1

The elliptic curve domain parameters over  $\mathbb{F}_p$  associated with a Koblitz curve secp160k1 are specified by the sextuple  $T = (p, a, b, G, n, h)$  where the finite field  $\mathbb{F}_p$  is defined by:

$$\begin{aligned} p &= \text{FFFFFFF FFFFFFFF FFFFFFFF FFFFFFFE FFFFAC73} \\ &= 2^{160} - 2^{32} - 2^{14} - 2^{12} - 2^9 - 2^8 - 2^7 - 2^3 - 2^2 - 1 \end{aligned}$$

The curve  $E: y^2 = x^3 + ax + b$  over  $\mathbb{F}_p$  is defined by:

$$\begin{aligned} a &= 00000000 00000000 00000000 00000000 00000000 \\ b &= 00000000 00000000 00000000 00000000 00000007 \end{aligned}$$

The base point  $G$  in compressed form is:

$$G = 02 3B4C382C E37AA192 A4019E76 3036F4F5 DD4D7EBB$$

and in uncompressed form is:

$$\begin{aligned} G &= 04 3B4C382C E37AA192 A4019E76 3036F4F5 DD4D7EBB 938CF935 \\ &\quad 318FDCE6 6BC28286 531733C3 F03C4FEE \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

$$\begin{aligned} n &= 01 00000000 00000000 0001B8FA 16DFAB9A CA16B6B3 \\ h &= 01 \end{aligned}$$

#### 2.4.2 Recommended Parameters secp160r1

The verifiably random elliptic curve domain parameters over  $\mathbb{F}_p$  `secp160r1` are specified by the sextuple  $T = (p, a, b, G, n, h)$  where the finite field  $\mathbb{F}_p$  is defined by:

$$\begin{aligned} p &= \text{FFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF 7FFFFFFF} \\ &= 2^{160} - 2^{31} - 1 \end{aligned}$$

The curve  $E: y^2 = x^3 + ax + b$  over  $\mathbb{F}_p$  is defined by:

$$\begin{aligned} a &= \text{FFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF 7FFFFFFC} \\ b &= 1C97BEFC 54BD7A8B 65ACF89F 81D4D4AD C565FA45 \end{aligned}$$

$E$  was chosen verifiably at random as specified in ANSI X9.62 [1] from the seed:

$$S = 1053CDE4 2C14D696 E6768756 1517533B F3F83345$$

The base point  $G$  in compressed form is:

$$G = 02 4A96B568 8EF57328 46646989 68C38BB9 13CBFC82$$

and in uncompressed form is:

$$\begin{aligned} G &= 04 4A96B568 8EF57328 46646989 68C38BB9 13CBFC82 23A62855 \\ &\quad 3168947D 59DCC912 04235137 7AC5FB32 \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

$$\begin{aligned} n &= 01\ 00000000\ 00000000\ 0001F4C8\ F927AED3\ CA752257 \\ h &= 01 \end{aligned}$$

### 2.4.3 Recommended Parameters secp160r2

The verifiably random elliptic curve domain parameters over  $\mathbb{F}_p$  `secp160r2` are specified by the sextuple  $T = (p, a, b, G, n, h)$  where the finite field  $\mathbb{F}_p$  is defined by:

$$\begin{aligned} p &= \text{FFFFFFF FFFFFFFF FFFFFFFF FFFFFFFE FFFFAC73} \\ &= 2^{160} - 2^{32} - 2^{14} - 2^{12} - 2^9 - 2^8 - 2^7 - 2^3 - 2^2 - 1 \end{aligned}$$

The curve  $E$ :  $y^2 = x^3 + ax + b$  over  $\mathbb{F}_p$  is defined by:

$$\begin{aligned} a &= \text{FFFFFFF FFFFFFFF FFFFFFFF FFFFFFFE FFFFAC70} \\ b &= \text{B4E134D3 FB59EB8B AB572749 04664D5A F50388BA} \end{aligned}$$

$E$  was chosen verifiably at random as specified in ANSI X9.62 [1] from the seed:

$$S = \text{B99B99B0 99B323E0 2709A4D6 96E67687 56151751}$$

The base point  $G$  in compressed form is:

$$G = \text{02 52DCB034 293A117E 1F4FF11B 30F7199D 3144CE6D}$$

and in uncompressed form is:

$$\begin{aligned} G &= \text{04 52DCB034 293A117E 1F4FF11B 30F7199D 3144CE6D FEAFFEF2} \\ &\quad \text{E331F296 E071FA0D F9982CFE A7D43F2E} \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

$$\begin{aligned} n &= 01\ 00000000\ 00000000\ 0000351E\ E786A818\ F3A1A16B \\ h &= 01 \end{aligned}$$

## 2.5 Recommended 192-bit Elliptic Curve Domain Parameters over $\mathbb{F}_p$

This section specifies the two recommended 192-bit elliptic curve domain parameters over  $\mathbb{F}_p$  in this document: parameters `secp192k1` associated with a Koblitz curve, and verifiably random parameters `secp192r1`.

Section 2.5.1 specifies the elliptic curve domain parameters `secp192k1`, and Section 2.5.2 specifies the elliptic curve domain parameters `secp192r1`.

### 2.5.1 Recommended Parameters secp192k1

The elliptic curve domain parameters over  $\mathbb{F}_p$  associated with a Koblitz curve `secp192k1` are specified by the sextuple  $T = (p, a, b, G, n, h)$  where the finite field  $\mathbb{F}_p$  is defined by:

$$\begin{aligned} p &= \text{FFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFE FFFFEE37} \\ &= 2^{192} - 2^{32} - 2^{12} - 2^8 - 2^7 - 2^6 - 2^3 - 1 \end{aligned}$$

The curve  $E: y^2 = x^3 + ax + b$  over  $\mathbb{F}_p$  is defined by:

$$\begin{aligned} a &= 00000000 00000000 00000000 00000000 00000000 00000000 \\ b &= 00000000 00000000 00000000 00000000 00000000 00000003 \end{aligned}$$

The base point  $G$  in compressed form is:

$$G = 03 \text{ DB4FF10E C057E9AE 26B07D02 80B7F434 1DA5D1B1 EAE06C7D}$$

and in uncompressed form is:

$$\begin{aligned} G = & 04 \text{ DB4FF10E C057E9AE 26B07D02 80B7F434 1DA5D1B1 EAE06C7D} \\ & 9B2F2F6D 9C5628A7 844163D0 15BE8634 4082AA88 D95E2F9D \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

$$\begin{aligned} n &= \text{FFFFFFF FFFFFFFF FFFFFFE 26F2FC17 0F69466A 74DEFD8D} \\ h &= 01 \end{aligned}$$

### 2.5.2 Recommended Parameters secp192r1

The verifiably random elliptic curve domain parameters over  $\mathbb{F}_p$  `secp192r1` are specified by the sextuple  $T = (p, a, b, G, n, h)$  where the finite field  $\mathbb{F}_p$  is defined by:

$$\begin{aligned} p &= \text{FFFFFFF FFFFFFFF FFFFFFFF FFFFFFE FFFFFFFF FFFFFFFF} \\ &= 2^{192} - 2^{64} - 1 \end{aligned}$$

The curve  $E: y^2 = x^3 + ax + b$  over  $\mathbb{F}_p$  is defined by:

$$\begin{aligned} a &= \text{FFFFFFF FFFFFFFF FFFFFFFF FFFFFFE FFFFFFFF FFFFFFFC} \\ b &= 64210519 E59C80E7 0FA7E9AB 72243049 FEB8DEEC C146B9B1 \end{aligned}$$

$E$  was chosen verifiably at random as specified in ANSI X9.62 [1] from the seed:

$$S = 3045AE6F C8422F64 ED579528 D38120EA E12196D5$$

The base point  $G$  in compressed form is:

$$G = 03 \text{ 188DA80E B03090F6 7CBF20EB 43A18800 F4FF0AFD 82FF1012}$$

and in uncompressed form is:

$$\begin{aligned} G = & \quad 04\ 188DA80E\ B03090F6\ 7CBF20EB\ 43A18800\ F4FF0AFD\ 82FF1012 \\ & 07192B95\ FFC8DA78\ 631011ED\ 6B24CDD5\ 73F977A1\ 1E794811 \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

$$\begin{aligned} n = & \text{ FFFFFFFF FFFFFFFF FFFFFFFF 99DEF836 146BC9B1 B4D22831} \\ h = & \text{ 01} \end{aligned}$$

## 2.6 Recommended 224-bit Elliptic Curve Domain Parameters over $\mathbb{F}_p$

This section specifies the two recommended 224-bit elliptic curve domain parameters over  $\mathbb{F}_p$  in this document: parameters `secp224k1` associated with a Koblitz curve, and verifiably random parameters `secp224r1`.

Section 2.6.1 specifies the elliptic curve domain parameters `secp224k1`, and Section 2.6.2 specifies the elliptic curve domain parameters `secp224r1`.

### 2.6.1 Recommended Parameters `secp224k1`

The elliptic curve domain parameters over  $\mathbb{F}_p$  associated with a Koblitz curve `secp224k1` are specified by the sextuple  $T = (p, a, b, G, n, h)$  where the finite field  $\mathbb{F}_p$  is defined by:

$$\begin{aligned} p = & \text{ FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFE FFFF56D} \\ = & 2^{224} - 2^{32} - 2^{12} - 2^{11} - 2^9 - 2^7 - 2^4 - 2 - 1 \end{aligned}$$

The curve  $E$ :  $y^2 = x^3 + ax + b$  over  $\mathbb{F}_p$  is defined by:

$$\begin{aligned} a = & \text{ 00000000 00000000 00000000 00000000 00000000 00000000 00000000} \\ b = & \text{ 00000000 00000000 00000000 00000000 00000000 00000000 00000005} \end{aligned}$$

The base point  $G$  in compressed form is:

$$\begin{aligned} G = & \quad 03\ A1455B33\ 4DF099DF\ 30FC28A1\ 69A467E9\ E47075A9\ 0F7E650E \\ & B6B7A45C \end{aligned}$$

and in uncompressed form is:

$$\begin{aligned} G = & \quad 04\ A1455B33\ 4DF099DF\ 30FC28A1\ 69A467E9\ E47075A9\ 0F7E650E \\ & B6B7A45C\ 7E089FED\ 7FBA3442\ 82CAFBD6\ F7E319F7\ C0B0BD59\ E2CA4BDB \\ & 556D61A5 \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

---

|     |  |
|-----|--|
| n = | 01 00000000 00000000 00000000 0001DCE8 D2EC6184 CAF0A971<br>769FB1F7 |
| h = | 01   |

### 2.6.2 Recommended Parameters secp224r1

The verifiably random elliptic curve domain parameters over  $\mathbb{F}_p$  secp224r1 are specified by the sextuple  $T = (p, a, b, G, n, h)$  where the finite field  $\mathbb{F}_p$  is defined by:

$$\begin{aligned} p &= \text{FFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF } 00000000 00000000 00000001 \\ &= 2^{224} - 2^{96} + 1 \end{aligned}$$

The curve  $E$ :  $y^2 = x^3 + ax + b$  over  $\mathbb{F}_p$  is defined by:

$$\begin{aligned} a &= \text{FFFFFFF FFFFFFFF FFFFFFFF FFFFFFFE FFFFFFFF FFFFFFFF FFFFFFFE} \\ b &= \text{B4050A85 0C04B3AB F5413256 5044B0B7 D7BFD8BA 270B3943 2355FFB4} \end{aligned}$$

$E$  was chosen verifiably at random as specified in ANSI X9.62 [1] from the seed:

$$S = \text{BD713447 99D5C7FC DC45B59F A3B9AB8F 6A948BC5}$$

The base point  $G$  in compressed form is:

$$\begin{aligned} G = \quad &02 \text{ B70E0CBD } 6BB4BF7F 321390B9 4A03C1D3 56C21122 343280D6 \\ &115C1D21 \end{aligned}$$

and in uncompressed form is:

$$\begin{aligned} G = \quad &04 \text{ B70E0CBD } 6BB4BF7F 321390B9 4A03C1D3 56C21122 343280D6 \\ &115C1D21 \text{ BD376388 B5F723FB } 4C22DFE6 \text{ CD4375A0 } 5A074764 44D58199 \\ &85007E34 \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

$$\begin{aligned} n &= \text{FFFFFFF FFFFFFFF FFFFFFFF FFFF16A2 E0B8F03E 13DD2945 5C5C2A3D} \\ h &= \quad 01 \end{aligned}$$

### 2.7 Recommended 256-bit Elliptic Curve Domain Parameters over $\mathbb{F}_p$

This section specifies the two recommended 256-bit elliptic curve domain parameters over  $\mathbb{F}_p$  in this document: parameters secp256k1 associated with a Koblitz curve, and verifiably random parameters secp256r1.

Section 2.7.1 specifies the elliptic curve domain parameters secp256k1, and Section 2.7.2 specifies the

elliptic curve domain parameters `secp256r1`.

### 2.7.1 Recommended Parameters `secp256k1`

The elliptic curve domain parameters over  $\mathbb{F}_p$  associated with a Koblitz curve `secp256k1` are specified by the sextuple  $T = (p, a, b, G, n, h)$  where the finite field  $\mathbb{F}_p$  is defined by:

$$\begin{aligned} p &= \text{FFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF} \\ &\quad \text{FFFFFFC2F} \\ &= 2^{256} - 2^{32} - 2^9 - 2^8 - 2^7 - 2^6 - 2^4 - 1 \end{aligned}$$

The curve  $E: y^2 = x^3 + ax + b$  over  $\mathbb{F}_p$  is defined by:

$$\begin{aligned} a &= 00000000 00000000 00000000 00000000 00000000 00000000 00000000 \\ &\quad 00000000 \\ b &= 00000000 00000000 00000000 00000000 00000000 00000000 00000000 \\ &\quad 00000007 \end{aligned}$$

The base point  $G$  in compressed form is:

$$\begin{aligned} G &= 02 \text{ 79BE667E F9DCBBAC 55A06295 CE870B07 029BFcdb 2DCE28D9} \\ &\quad 59F2815B 16F81798 \end{aligned}$$

and in uncompressed form is:

$$\begin{aligned} G &= 04 \text{ 79BE667E F9DCBBAC 55A06295 CE870B07 029BFcdb 2DCE28D9} \\ &\quad 59F2815B 16F81798 483ADA77 26A3C465 5DA4FBFC 0E1108A8 FD17B448 \\ &\quad A6855419 9C47D08F FB10D4B8 \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

$$\begin{aligned} n &= \text{FFFFFFF FFFFFFFF FFFFFFFF FFFFFFFE BAAEDCE6 AF48A03B BFD25E8C} \\ &\quad D0364141 \\ h &= 01 \end{aligned}$$

### 2.7.2 Recommended Parameters `secp256r1`

The verifiably random elliptic curve domain parameters over  $\mathbb{F}_p$  `secp256r1` are specified by the sextuple  $T = (p, a, b, G, n, h)$  where the finite field  $\mathbb{F}_p$  is defined by:

$$\begin{aligned}
 p &= \text{FFFFFFF } 00000001 \text{ 00000000 } 00000000 \text{ 00000000 FFFFFFFF FFFFFFFF} \\
 &\quad \text{FFFFFFF} \\
 &= 2^{224}(2^{32}-1) + 2^{192} + 2^{96} - 1
 \end{aligned}$$

The curve  $E: y^2 = x^3 + ax + b$  over  $\mathbb{F}_p$  is defined by:

$$\begin{aligned}
 a &= \text{FFFFFFF } 00000001 \text{ 00000000 } 00000000 \text{ 00000000 FFFFFFFF FFFFFFFF} \\
 &\quad \text{FFFFFFFC} \\
 b &= \text{5AC635D8 AA3A93E7 B3EBBD55 769886BC 651D06B0 CC53B0F6 3BCE3C3E} \\
 &\quad 27D2604B
 \end{aligned}$$

$E$  was chosen verifiably at random as specified in ANSI X9.62 [1] from the seed:

$$S = \text{C49D3608 86E70493 6A6678E1 139D26B7 819F7E90}$$

The base point  $G$  in compressed form is:

$$\begin{aligned}
 G = & \quad 03 \text{ 6B17D1F2 E12C4247 F8BCE6E5 63A440F2 77037D81 2DEB33A0} \\
 & \quad \text{F4A13945 D898C296}
 \end{aligned}$$

and in uncompressed form is:

$$\begin{aligned}
 G = & \quad 04 \text{ 6B17D1F2 E12C4247 F8BCE6E5 63A440F2 77037D81 2DEB33A0} \\
 & \quad \text{F4A13945 D898C296 4FE342E2 FE1A7F9B 8EE7EB4A 7C0F9E16 2BCE3357} \\
 & \quad 6B315ECE CBB64068 37BF51F5
 \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

$$\begin{aligned}
 n &= \text{FFFFFFF } 00000000 \text{ FFFFFFFF FFFFFFFF BCE6FAAD A7179E84 F3B9CAC2} \\
 &\quad \text{FC632551} \\
 h &= 01
 \end{aligned}$$

## 2.8 Recommended 384-bit Elliptic Curve Domain Parameters over $\mathbb{F}_p$

This section specifies the recommended 384-bit elliptic curve domain parameters over  $\mathbb{F}_p$  in this document: verifiably random parameters `secp384r1`.

Section 2.8.1 specifies the elliptic curve domain parameters `secp384r1`.

### 2.8.1 Recommended Parameters `secp384r1`

The verifiably random elliptic curve domain parameters over  $\mathbb{F}_p$  `secp384r1` are specified by the sextuple  $T = (p, a, b, G, n, h)$  where the finite field  $\mathbb{F}_p$  is defined by:

$$\begin{aligned}
 p &= \text{FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF} \\
 &\quad \text{FFFFFFFE FFFFFFFF 00000000 00000000 FFFFFFFF} \\
 &= 2^{384} - 2^{128} - 2^{96} + 2^{32} - 1
 \end{aligned}$$

The curve  $E: y^2 = x^3 + ax + b$  over  $\mathbb{F}_p$  is defined by:

$$\begin{aligned}
 a &= \text{FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF} \\
 &\quad \text{FFFFFFFE FFFFFFFF 00000000 00000000 FFFFFFFC} \\
 b &= \text{B3312FA7 E23EE7E4 988E056B E3F82D19 181D9C6E FE814112 0314088F} \\
 &\quad \text{5013875A C656398D 8A2ED19D 2A85C8ED D3EC2AEF}
 \end{aligned}$$

$E$  was chosen verifiably at random as specified in ANSI X9.62 [1] from the seed:

$$S = \text{A335926A A319A27A 1D00896A 6773A482 7ACDAC73}$$

The base point  $G$  in compressed form is:

$$\begin{aligned}
 G = & \quad \text{03 AA87CA22 BE8B0537 8EB1C71E F320AD74 6E1D3B62 8BA79B98} \\
 & \quad \text{59F741E0 82542A38 5502F25D BF55296C 3A545E38 72760AB7}
 \end{aligned}$$

and in uncompressed form is:

$$\begin{aligned}
 G = & \quad \text{04 AA87CA22 BE8B0537 8EB1C71E F320AD74 6E1D3B62 8BA79B98} \\
 & \quad \text{59F741E0 82542A38 5502F25D BF55296C 3A545E38 72760AB7 3617DE4A} \\
 & \quad \text{96262C6F 5D9E98BF 9292DC29 F8F41DBD 289A147C E9DA3113 B5F0B8C0} \\
 & \quad \text{0A60B1CE 1D7E819D 7A431D7C 90EA0E5F}
 \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

$$\begin{aligned}
 n &= \text{FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF C7634D81} \\
 &\quad \text{F4372DDF 581A0DB2 48B0A77A ECEC196A CCC52973} \\
 h &= \quad \text{01}
 \end{aligned}$$

## 2.9 Recommended 521-bit Elliptic Curve Domain Parameters over $\mathbb{F}_p$

This section specifies the recommended 521-bit elliptic curve domain parameters over  $\mathbb{F}_p$  in this document: verifiably random parameters `secp521r1`.

Section 2.9.1 specifies the elliptic curve domain parameters `secp521r1`.

### 2.9.1 Recommended Parameters secp521r1

The verifiably random elliptic curve domain parameters over  $\mathbb{F}_p$  secp521r1 are specified by the sextuple  $T = (p, a, b, G, n, h)$  where the finite field  $\mathbb{F}_p$  is defined by:

$$\begin{aligned} p &= 01FF\text{ FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF} \\ &\quad FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF \\ &\quad FFFFFFFF FFFFFFFF FFFFFFFF \\ &= 2^{521} - 1 \end{aligned}$$

The curve  $E: y^2 = x^3 + ax + b$  over  $\mathbb{F}_p$  is defined by:

$$\begin{aligned} a &= 01FF\text{ FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF} \\ &\quad FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF \\ &\quad FFFFFFFF FFFFFFFF FFFFFFFC \\ b &= 0051\text{ 953EB961 8E1C9A1F 929A21A0 B68540EE A2DA725B 99B315F3} \\ &\quad B8B48991 8EF109E1 56193951 EC7E937B 1652C0BD 3BB1BF07 3573DF88 \\ &\quad 3D2C34F1 EF451FD4 6B503F00 \end{aligned}$$

$E$  was chosen verifiably at random as specified in ANSI X9.62 [1] from the seed:

$$S = D09E8800\text{ 291CB853 96CC6717 393284AA A0DA64BA}$$

The base point  $G$  in compressed form is:

$$\begin{aligned} G &= 0200C6\text{ 858E06B7 0404E9CD 9E3ECB66 2395B442 9C648139 053FB521} \\ &\quad F828AF60 6B4D3DBA A14B5E77 EFE75928 FE1DC127 A2FFA8DE 3348B3C1 \\ &\quad 856A429B F97E7E31 C2E5BD66 \end{aligned}$$

and in uncompressed form is:

$$\begin{aligned} G &= 04\text{ 00C6858E 06B70404 E9CD9E3E CB662395 B4429C64 8139053F} \\ &\quad B521F828 AF606B4D 3DBAA14B 5E77EFE7 5928FE1D C127A2FF A8DE3348 \\ &\quad B3C1856A 429BF97E 7E31C2E5 BD660118 39296A78 9A3BC004 5C8A5FB4 \\ &\quad 2C7D1BD9 98F54449 579B4468 17AFBD17 273E662C 97EE7299 5EF42640 \\ &\quad C550B901 3FAD0761 353C7086 A272C240 88BE9476 9FD16650 \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

$n = 01FF\text{ FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF}$   
 $\text{ FFFFFFFF FFFFFFA 51868783 BF2F966B 7FCC0148 F709A5D0 3BB5C9B8}$   
 $\text{ 899C47AE BB6FB71E 91386409}$

$h = 01$

### 3 Recommended Elliptic Curve Domain Parameters over $\mathbb{F}_{2^m}$

This section specifies the elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  recommended in this document.

The section is organized as follows. First Section 3.1 describes relevant properties of the recommended parameters over  $\mathbb{F}_{2^m}$ . Then Section 3.2 specifies recommended 113-bit elliptic curve domain parameters over  $\mathbb{F}_{2^m}$ , Section 3.3 specifies recommended 131-bit elliptic curve domain parameters over  $\mathbb{F}_{2^m}$ , Section 3.4 specifies recommended 163-bit elliptic curve domain parameters over  $\mathbb{F}_{2^m}$ , Section 3.5 specifies recommended 193-bit elliptic curve domain parameters over  $\mathbb{F}_{2^m}$ , Section 3.6 specifies recommended 233-bit elliptic curve domain parameters over  $\mathbb{F}_{2^m}$ , Section 3.7 specifies recommended 239-bit elliptic curve domain parameters over  $\mathbb{F}_{2^m}$ , Section 3.8 specifies recommended 283-bit elliptic curve domain parameters over  $\mathbb{F}_{2^m}$ , Section 3.9 specifies recommended 409-bit elliptic curve domain parameters over  $\mathbb{F}_{2^m}$ , and Section 3.10 specifies recommended 571-bit elliptic curve domain parameters over  $\mathbb{F}_{2^m}$ .

#### 3.1 Properties of Elliptic Curve Domain Parameters over $\mathbb{F}_{2^m}$

Following SEC 1 [12], elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  are a septuple:

$$T = (m, f(x), a, b, G, n, h)$$

consisting of an integer  $m$  specifying the finite field  $\mathbb{F}_{2^m}$ , an irreducible binary polynomial  $f(x)$  of degree  $m$  specifying the polynomial basis representation of  $\mathbb{F}_{2^m}$ , two elements  $a, b \in \mathbb{F}_{2^m}$  specifying an elliptic curve  $E(\mathbb{F}_{2^m})$  defined by the equation:

$$E : y^2 + x.y = x^3 + a.x^2 + b \text{ in } \mathbb{F}_{2^m},$$

a base point  $G = (x_G, y_G)$  on  $E(\mathbb{F}_{2^m})$ , a prime  $n$  which is the order of  $G$ , and an integer  $h$  which is the cofactor  $h = \#E(\mathbb{F}_{2^m})/n$ .

When elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  are specified in this document,  $m$  is represented directly as an integer,  $f(x)$  is represented directly as a polynomial, and the remaining components are represented as octet strings converted using the conventions specified in SEC 1 [12].

Again following SEC 1 [12], elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  must have:

$$m \in \{113, 131, 163, 193, 233, 239, 283, 409, 571\}.$$

Furthermore elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  must use the reduction polynomials listed in Table 3 below.

This restriction is designed to encourage interoperability while allowing implementers to supply efficient implementations at commonly required security levels.

Here recommended elliptic curve domain parameters are supplied at each of the sizes allowed by SEC 1. The elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  supplied at each security level typically consist of examples of two different types of parameters — one type being parameters associated with a Koblitz curve

| Field                  | Reduction Polynomial(s)                                  |
|------------------------|--|
| $\mathbb{F}_{2^{113}}$ | $f(x) = x^{113} + x^9 + 1$                               |
| $\mathbb{F}_{2^{131}}$ | $f(x) = x^{131} + x^8 + x^3 + x^2 + 1$                   |
| $\mathbb{F}_{2^{163}}$ | $f(x) = x^{163} + x^7 + x^6 + x^3 + 1$                   |
| $\mathbb{F}_{2^{193}}$ | $f(x) = x^{193} + x^{15} + 1$                            |
| $\mathbb{F}_{2^{233}}$ | $f(x) = x^{233} + x^{74} + 1$                            |
| $\mathbb{F}_{2^{239}}$ | $f(x) = x^{239} + x^{36} + 1$ or $x^{239} + x^{158} + 1$ |
| $\mathbb{F}_{2^{283}}$ | $f(x) = x^{283} + x^{12} + x^7 + x^5 + 1$                |
| $\mathbb{F}_{2^{409}}$ | $f(x) = x^{409} + x^{87} + 1$                            |
| $\mathbb{F}_{2^{571}}$ | $f(x) = x^{571} + x^{10} + x^5 + x^2 + 1$                |

Table 3: Representations of  $\mathbb{F}_{2^m}$ 

and the other type being parameters chosen verifiably at random — although only verifiably random parameters are supplied at export strength.

Parameters associated with a Koblitz curve admit especially efficient implementation. Koblitz curves over  $\mathbb{F}_{2^m}$  are binary anomalous curves which have  $a, b \in \{0, 1\}$  [9].

Verifiably random parameters offer some additional conservative features. These parameters are chosen from a seed using SHA-1 as specified in ANSI X9.62 [1]. This process ensures that the parameters cannot be predetermined. The parameters are therefore extremely unlikely to be susceptible to future special-purpose attacks, and no trapdoors can have been placed in the parameters during their generation. When elliptic curve domain parameters are chosen verifiably at random, the seed  $S$  used to generate the parameters may optionally be stored along with the parameters so that users can verify the parameters were chosen verifiably at random.

The recommended verifiably random parameters were chosen by repeatedly selecting a random seed and counting the points on the corresponding curve using Schoof's algorithm until appropriate parameters were found. The parameters were chosen so that either  $a$  is random or  $a = 1$ . For a given  $m$ , approximately half the isomorphism classes of elliptic curves over  $\mathbb{F}_{2^m}$  contain a curve with  $a = 1$ .

See SEC 1 [12] for further guidance on the selection of elliptic curve domain parameters over  $\mathbb{F}_{2^m}$ .

The example elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  have been given nicknames to enable them to be easily identified. The nicknames were chosen as follows. Each name begins with `sec` to denote ‘Standards for Efficient Cryptography’, followed by a `t` to denote parameters over  $\mathbb{F}_{2^m}$ , followed by a number denoting the field size  $m$ , followed by a `k` to denote parameters associated with a Koblitz curve or an `r` to denote verifiably random parameters, followed by a sequence number.

Table 4 summarizes salient properties of the recommended elliptic curve domain parameters over  $\mathbb{F}_{2^m}$ .

| Parameters | Section | Strength | Size | RSA/DSA | Koblitz or random |
|------------|---------|----------|------|---------|-------------------|
| sect113r1  | 3.2.1   | 56       | 113  | 512     | r                 |
| sect113r2  | 3.2.2   | 56       | 113  | 512     | r                 |
| sect131r1  | 3.3.1   | 64       | 131  | 704     | r                 |
| sect131r2  | 3.3.2   | 64       | 131  | 704     | r                 |
| sect163k1  | 3.4.1   | 80       | 163  | 1024    | k                 |
| sect163r1  | 3.4.2   | 80       | 163  | 1024    | r                 |
| sect163r2  | 3.4.3   | 80       | 163  | 1024    | r                 |
| sect193r1  | 3.5.1   | 96       | 193  | 1536    | r                 |
| sect193r2  | 3.5.2   | 96       | 193  | 1536    | r                 |
| sect233k1  | 3.6.1   | 112      | 233  | 2240    | k                 |
| sect233r1  | 3.6.2   | 112      | 233  | 2240    | r                 |
| sect239k1  | 3.7.1   | 115      | 239  | 2304    | k                 |
| sect283k1  | 3.8.1   | 128      | 283  | 3456    | k                 |
| sect283r1  | 3.8.2   | 128      | 283  | 3456    | r                 |
| sect409k1  | 3.9.1   | 192      | 409  | 7680    | k                 |
| sect409r1  | 3.9.2   | 192      | 409  | 7680    | r                 |
| sect571k1  | 3.10.1  | 256      | 571  | 15360   | k                 |
| sect571r1  | 3.10.2  | 256      | 571  | 15360   | r                 |

Table 4: Properties of Recommended Elliptic Curve Domain Parameters over  $\mathbb{F}_{2^m}$

Information is represented in Table 4 as follows. The column labelled ‘parameters’ gives the nickname of the elliptic curve domain parameters. The column labelled ‘section’ refers to the section of this document where the parameters are specified. The column labelled ‘strength’ gives the approximate number of bits of security the parameters offer. The column labelled ‘size’ gives the field size  $m$ . The column labelled ‘RSA/DSA’ gives the approximate size of an RSA or DSA modulus at comparable strength. (See SEC 1 [12] for precise technical guidance on the strength of elliptic curve domain parameters.) Finally the column labelled ‘Koblitz or random’ indicates whether the parameters are associated with a Koblitz curve — ‘k’ — or were chosen verifiably at random — ‘r’.

| Parameters | Section | ANSI X9.62 | ANSI X9.63 | echeck | IEEE P1363 | IPSec | NIST | WAP |
|------------|---------|------------|------------|--------|------------|-------|------|-----|
| sect113r1  | 3.2.1   | -          | -          | -      | c          | c     | -    | r   |
| sect113r2  | 3.2.2   | -          | -          | -      | c          | c     | -    | c   |
| sect131r1  | 3.3.1   | -          | -          | -      | c          | c     | -    | c   |
| sect131r2  | 3.3.2   | -          | -          | -      | c          | c     | -    | c   |
| sect163k1  | 3.4.1   | c          | r          | r      | c          | r     | r    | r   |
| sect163r1  | 3.4.2   | c          | c          | r      | c          | r     | -    | c   |
| sect163r2  | 3.4.3   | c          | r          | r      | c          | c     | r    | c   |
| sect193r1  | 3.5.1   | c          | r          | c      | c          | c     | -    | c   |
| sect193r2  | 3.5.2   | c          | r          | c      | c          | c     | -    | c   |
| sect233k1  | 3.6.1   | c          | r          | c      | c          | c     | r    | c   |
| sect233r1  | 3.6.2   | c          | r          | c      | c          | c     | r    | c   |
| sect239k1  | 3.7.1   | c          | c          | c      | c          | c     | -    | c   |
| sect283k1  | 3.8.1   | c          | r          | r      | c          | r     | r    | c   |
| sect283r1  | 3.8.2   | c          | r          | r      | c          | r     | r    | c   |
| sect409k1  | 3.9.1   | c          | r          | c      | c          | c     | r    | c   |
| sect409r1  | 3.9.2   | c          | r          | c      | c          | c     | r    | c   |
| sect571k1  | 3.10.1  | c          | r          | c      | c          | c     | r    | c   |
| sect571r1  | 3.10.2  | c          | r          | c      | c          | c     | r    | c   |

Table 5: Status of Recommended Elliptic Curve Domain Parameters over  $\mathbb{F}_{2^m}$

Table 5 summarizes the status of the recommended elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  with respect to their alignment with other standards.

Information is represented in Table 5 as follows. The column labelled ‘parameters’ gives the nickname

of the elliptic curve domain parameters. The column labelled ‘section’ refers to the section of this document where the parameters are specified. The remaining columns give the status of the parameters with respect to various other standards which specify mechanisms based on elliptic curve cryptography: ‘ANSI X9.62’ refers to the ANSI X9.62 standard [1], ‘ANSI X9.63’ refers to the draft ANSI X9.63 standard [3], ‘echeck’ refers to the draft FSML standard [6], ‘IEEE P1363’ refers to the draft IEEE P1363 standard [8], ‘IPSec’ refers to the recent internet draft related to ECC [11] submitted to the IETF’s IPSec working group, ‘NIST’ refers to the list of recommended parameters recently released by the U.S. government [5], and ‘WAP’ refers to the Wireless Application Forum’s WTLS standard [13]. In these columns, a ‘-’ denotes parameters non-conformant with the standard, a ‘c’ denotes parameters conformant with the standard, and an ‘r’ denotes parameters explicitly recommended in the standard.

Note that ANSI X9.62 is currently being updated. The set of recommended parameters in the proposed ANSI X9.62-1 [2] is identical to the set of recommended parameters in this document.

## 3.2 Recommended 113-bit Elliptic Curve Domain Parameters over $\mathbb{F}_{2^m}$

This section specifies the two recommended 113-bit elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  in this document: verifiably random parameters `sect113r1`, and verifiably random parameters `sect113r2`.

Section 3.2.1 specifies the elliptic curve domain parameters `sect113r1`, and Section 3.2.2 specifies the elliptic curve domain parameters `sect113r2`.

### 3.2.1 Recommended Parameters `sect113r1`

The verifiably random elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  `sect113r1` are specified by the septuple  $T = (m, f(x), a, b, G, n, h)$  where  $m = 113$  and the representation of  $\mathbb{F}_{2^{113}}$  is defined by:

$$f(x) = x^{113} + x^9 + 1$$

The curve  $E$ :  $y^2 + xy = x^3 + ax^2 + b$  over  $\mathbb{F}_{2^m}$  is defined by:

$$a = 003088\ 250CA6E7\ C7FE649C\ E85820F7$$

$$b = 00E8BE\ E4D3E226\ 0744188B\ E0E9C723$$

$E$  was chosen verifiably at random as specified in ANSI X9.62 [1] from the seed:

$$S = 10E723AB\ 14D696E6\ 76875615\ 1756FEBF\ 8FCB49A9$$

The base point  $G$  in compressed form is:

$$G = 03009D73\ 616F35F4\ AB1407D7\ 3562C10F$$

and in uncompressed form is:

$$G = \begin{aligned} & 04009D\ 73616F35\ F4AB1407\ D73562C1\ 0F00A528\ 30277958\ EE84D131 \\ & 5ED31886 \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

$$\begin{aligned} n &= 010000\ 00000000\ 00D9CCEC\ 8A39E56F \\ h &= 02 \end{aligned}$$

### 3.2.2 Recommended Parameters sect113r2

The verifiably random elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  `sect113r2` are specified by the septuple  $T = (m, f(x), a, b, G, n, h)$  where  $m = 113$  and the representation of  $\mathbb{F}_{2^{113}}$  is defined by:

$$f(x) = x^{113} + x^9 + 1$$

The curve  $E$ :  $y^2 + xy = x^3 + ax^2 + b$  over  $\mathbb{F}_{2^m}$  is defined by:

$$\begin{aligned} a &= 006899\ 18DBEC7E\ 5A0DD6DF\ C0AA55C7 \\ b &= 0095E9\ A9EC9B29\ 7BD4BF36\ E059184F \end{aligned}$$

$E$  was chosen verifiably at random as specified in ANSI X9.62 [1] from the seed:

$$S = 10C0FB15\ 760860DE\ F1EEF4D6\ 96E67687\ 5615175D$$

The base point  $G$  in compressed form is:

$$G = 0301A57A\ 6A7B26CA\ 5EF52FCD\ B8164797$$

and in uncompressed form is:

$$\begin{aligned} G &= 0401A5\ 7A6A7B26\ CA5EF52F\ CDB81647\ 9700B3AD\ C94ED1FE\ 674C06E6 \\ &\quad 95BABA1D \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

$$\begin{aligned} n &= 010000\ 00000000\ 0108789B\ 2496AF93 \\ h &= 02 \end{aligned}$$

## 3.3 Recommended 131-bit Elliptic Curve Domain Parameters over $\mathbb{F}_{2^m}$

This section specifies the two recommended 131-bit elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  in this document: verifiably random parameters `sect131r1`, and verifiably random parameters `sect131r2`.

Section 3.3.1 specifies the elliptic curve domain parameters `sect131r1`, and Section 3.3.2 specifies the elliptic curve domain parameters `sect131r2`.

### 3.3.1 Recommended Parameters sect131r1

The verifiably random elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  sect131r1 are specified by the septuple  $T = (m, f(x), a, b, G, n, h)$  where  $m = 131$  and the representation of  $\mathbb{F}_{2^{131}}$  is defined by:

$$f(x) = x^{131} + x^8 + x^3 + x^2 + 1$$

The curve  $E$ :  $y^2 + xy = x^3 + ax^2 + b$  over  $\mathbb{F}_{2^m}$  is defined by:

$$a = 07\ A11B09A7\ 6B562144\ 418FF3FF\ 8C2570B8$$

$$b = 02\ 17C05610\ 884B63B9\ C6C72916\ 78F9D341$$

$E$  was chosen verifiably at random as specified in ANSI X9.62 [1] from the seed:

$$S = 4D696E67\ 68756151\ 75985BD3\ ADBADA21\ B43A97E2$$

The base point  $G$  in compressed form is:

$$G = 0300\ 81BAF91F\ DF9833C4\ 0F9C1813\ 43638399$$

and in uncompressed form is:

$$\begin{aligned} G = & \quad 040081\ BAF91FDF\ 9833C40F\ 9C181343\ 63839907\ 8C6E7EA3\ 8C001F73 \\ & \quad C8134B1B\ 4EF9E150 \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

$$n = 04\ 00000000\ 00000002\ 3123953A\ 9464B54D$$

$$h = 02$$

### 3.3.2 Recommended Parameters sect131r2

The verifiably random elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  sect131r2 are specified by the septuple  $T = (m, f(x), a, b, G, n, h)$  where  $m = 131$  and the representation of  $\mathbb{F}_{2^{131}}$  is defined by:

$$f(x) = x^{131} + x^8 + x^3 + x^2 + 1$$

The curve  $E$ :  $y^2 + xy = x^3 + ax^2 + b$  over  $\mathbb{F}_{2^m}$  is defined by:

$$a = 03\ E5A88919\ D7CAFBCF\ 415F07C2\ 176573B2$$

$$b = 04\ B8266A46\ C55657AC\ 734CE38F\ 018F2192$$

$E$  was chosen verifiably at random as specified in ANSI X9.62 [1] from the seed:

$$S = 985BD3AD\ BAD4D696\ E6768756\ 15175A21\ B43A97E3$$

The base point  $G$  in compressed form is:

$$G = 0303\ 56DCD8F2\ F95031AD\ 652D2395\ 1BB366A8$$

and in uncompressed form is:

$$\begin{aligned} G = & \quad 040356\text{ DCD8F2F9 5031AD65 2D23951B B366A806 48F06D86 7940A536} \\ & \quad 6D9E265D E9EB240F \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

$$\begin{aligned} n = & \quad 04\text{ 00000000 00000001 6954A233 049BA98F} \\ h = & \quad 02 \end{aligned}$$

### 3.4 Recommended 163-bit Elliptic Curve Domain Parameters over $\mathbb{F}_{2^m}$

This section specifies the three recommended 163-bit elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  in this document: parameters `sect163k1` associated with a Koblitz curve, verifiably random parameters `sect163r1`, and verifiably random parameters `sect163r2`.

Section 3.4.1 specifies the elliptic curve domain parameters `sect163k1`, Section 3.4.2 specifies the elliptic curve domain parameters `sect163r1`, and Section 3.4.3 specifies the elliptic curve domain parameters `sect163r2`.

#### 3.4.1 Recommended Parameters `sect163k1`

The elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  associated with a Koblitz curve `sect163k1` are specified by the septuple  $T = (m, f(x), a, b, G, n, h)$  where  $m = 163$  and the representation of  $\mathbb{F}_{2^{163}}$  is defined by:

$$f(x) = x^{163} + x^7 + x^6 + x^3 + 1$$

The curve  $E$ :  $y^2 + xy = x^3 + ax^2 + b$  over  $\mathbb{F}_{2^m}$  is defined by:

$$\begin{aligned} a = & \quad 00\text{ 00000000 00000000 00000000 00000000 00000001} \\ b = & \quad 00\text{ 00000000 00000000 00000000 00000000 00000001} \end{aligned}$$

The base point  $G$  in compressed form is:

$$G = 0302\text{ FE13C053 7BBC11AC AA07D793 DE4E6D5E 5C94EEE8}$$

and in uncompressed form is:

$$\begin{aligned} G = & \quad 0402FE\text{ 13C0537B BC11ACAA 07D793DE 4E6D5E5C 94EEE802 89070FB0} \\ & \quad 5D38FF58 321F2E80 0536D538 CCDA3D9 \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

$$\begin{aligned} n = & \quad 04\text{ 00000000 00000000 00020108 A2E0CC0D 99F8A5EF} \\ h = & \quad 02 \end{aligned}$$

### 3.4.2 Recommended Parameters sect163r1

The verifiably random elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  sect163r1 are specified by the septuple  $T = (m, f(x), a, b, G, n, h)$  where  $m = 163$  and the representation of  $\mathbb{F}_{2^{163}}$  is defined by:

$$f(x) = x^{163} + x^7 + x^6 + x^3 + 1$$

The curve  $E$ :  $y^2 + xy = x^3 + ax^2 + b$  over  $\mathbb{F}_{2^m}$  is defined by:

$$a = 07\ B6882CAA\ EFA84F95\ 54FF8428\ BD88E246\ D2782AE2$$

$$b = 07\ 13612DCD\ DCB40AAB\ 946BDA29\ CA91F73A\ F958AFD9$$

$E$  was chosen verifiably at random from the seed:

$$S = 24B7B137\ C8A14D69\ 6E676875\ 6151756F\ D0DA2E5C$$

However for historical reasons the method used to generate  $E$  from  $S$  differs slightly from the method described in ANSI X9.62 [1]. Specifically the coefficient  $b$  produced from  $S$  is the reverse of the coefficient that would have been produced by the method described in ANSI X9.62.

The base point  $G$  in compressed form is:

$$G = 0303\ 69979697\ AB438977\ 89566789\ 567F787A\ 7876A654$$

and in uncompressed form is:

$$G = 040369\ 979697AB\ 43897789\ 56678956\ 7F787A78\ 76A65400\ 435EDB42$$

$$\quad \quad \quad EFAFB298\ 9D51FEFC\ E3C80988\ F41FF883$$

Finally the order  $n$  of  $G$  and the cofactor are:

$$n = 03\ FFFFFFFF\ FFFFFFFF\ FFFF48AA\ B689C29C\ A710279B$$

$$h = 02$$

### 3.4.3 Recommended Parameters sect163r2

The verifiably random elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  sect163r2 are specified by the septuple  $T = (m, f(x), a, b, G, n, h)$  where  $m = 163$  and the representation of  $\mathbb{F}_{2^{163}}$  is defined by:

$$f(x) = x^{163} + x^7 + x^6 + x^3 + 1$$

The curve  $E$ :  $y^2 + xy = x^3 + ax^2 + b$  over  $\mathbb{F}_{2^m}$  is defined by:

$$a = 00\ 0000000\ 0000000\ 0000000\ 0000000\ 0000001$$

$$b = 02\ 0A601907\ B8C953CA\ 1481EB10\ 512F7874\ 4A3205FD$$

$E$  was chosen verifiably at random from the seed:

$$S = 85E25BFE\ 5C86226C\ DB12016F\ 7553F9D0\ E693A268$$

$E$  was selected from  $S$  as specified in ANSI X9.62 [1] in normal basis representation and converted into polynomial basis representation.

The base point  $G$  in compressed form is:

$$G = 0303\text{ F0EBA162 86A2D57E A0991168 D4994637 E8343E36}$$

and in uncompressed form is:

$$\begin{aligned} G = & \quad 0403\text{F0 EBA16286 A2D57EA0 991168D4 994637E8 343E3600 D51FBC6C} \\ & \quad 71A0094F A2CDD545 B11C5C0C 797324F1 \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

$$\begin{aligned} n = & \quad 04\ 00000000\ 00000000\ 000292\text{FE}\ 77E70C12\ A4234C33 \\ h = & \quad 02 \end{aligned}$$

### 3.5 Recommended 193-bit Elliptic Curve Domain Parameters over $\mathbb{F}_{2^m}$

This section specifies the two recommended 193-bit elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  in this document: verifiably random parameters `sect193r1`, and verifiably random parameters `sect193r1`. Section 3.5.1 specifies the elliptic curve domain parameters `sect193r1`, and Section 3.5.2 specifies the elliptic curve domain parameters `sect193r2`.

#### 3.5.1 Recommended Parameters `sect193r1`

The verifiably random elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  `sect193r1` are specified by the septuple  $T = (m, f(x), a, b, G, n, h)$  where  $m = 193$  and the representation of  $\mathbb{F}_{2^{193}}$  is defined by:

$$f(x) = x^{193} + x^{15} + 1$$

The curve  $E$ :  $y^2 + xy = x^3 + ax^2 + b$  over  $\mathbb{F}_{2^m}$  is defined by:

$$\begin{aligned} a = & \quad 00\ 17858\text{FEB}\ 7A989751\ 69E171F7\ 7B4087DE\ 098AC8A9\ 11DF7B01 \\ b = & \quad 00\ FDFB49BF\ E6C3A89F\ ACADA7A\ 1E5BBC7C\ C1C2E5D8\ 31478814 \end{aligned}$$

$E$  was chosen verifiably at random as specified in ANSI X9.62 [1] from the seed:

$$S = 103FAEC7\ 4D696E67\ 68756151\ 75777FC5\ B191EF30$$

The base point  $G$  in compressed form is:

$$G = 0301\text{ F481BC5F\ 0FF84A74\ AD6CDF6F\ DEF4BF61\ 79625372\ D8C0C5E1}$$

and in uncompressed form is:

$$\begin{aligned} G = & \quad 0401F4\ 81BC5F0F\ F84A74AD\ 6CDF6FDE\ F4BF6179\ 625372D8\ C0C5E100 \\ & 25E399F2\ 903712CC\ F3EA9E3A\ 1AD17FB0\ B3201B6A\ F7CE1B05 \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

$$\begin{aligned} n = & \quad 01\ 00000000\ 00000000\ 00000000\ C7F34A77\ 8F443ACC\ 920EBA49 \\ h = & \quad 02 \end{aligned}$$

### 3.5.2 Recommended Parameters sect193r2

The verifiably random elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  `sect193r2` are specified by the septuple  $T = (m, f(x), a, b, G, n, h)$  where  $m = 193$  and the representation of  $\mathbb{F}_{2^{193}}$  is defined by:

$$f(x) = x^{193} + x^{15} + 1$$

The curve  $E$ :  $y^2 + xy = x^3 + ax^2 + b$  over  $\mathbb{F}_{2^m}$  is defined by:

$$\begin{aligned} a = & \quad 01\ 63F35A51\ 37C2CE3E\ A6ED8667\ 190B0BC4\ 3ECD6997\ 7702709B \\ b = & \quad 00\ C9BB9E89\ 27D4D64C\ 377E2AB2\ 856A5B16\ E3EFB7F6\ 1D4316AE \end{aligned}$$

$E$  was chosen verifiably at random as specified in ANSI X9.62 [1] from the seed:

$$S = 10B7B4D6\ 96E67687\ 56151751\ 37C8A16F\ D0DA2211$$

The base point  $G$  in compressed form is:

$$G = 0300\ D9B67D19\ 2E0367C8\ 03F39E1A\ 7E82CA14\ A651350A\ AE617E8F$$

and in uncompressed form is:

$$\begin{aligned} G = & \quad 0400D9\ B67D192E\ 0367C803\ F39E1A7E\ 82CA14A6\ 51350AAE\ 617E8F01 \\ & CE943356\ 07C304AC\ 29E7DEFB\ D9CA01F5\ 96F92722\ 4CDECF6C \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

$$\begin{aligned} n = & \quad 01\ 00000000\ 00000000\ 00000001\ 5AAB561B\ 005413CC\ D4EE99D5 \\ h = & \quad 02 \end{aligned}$$

## 3.6 Recommended 233-bit Elliptic Curve Domain Parameters over $\mathbb{F}_{2^m}$

This section specifies the two recommended 233-bit elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  in this document: parameters `sect233k1` associated with a Koblitz curve, and verifiably random parameters `sect233r1`.

Section 3.6.1 specifies the elliptic curve domain parameters `sect233k1`, and Section 3.6.2 specifies the elliptic curve domain parameters `sect233r1`.

### 3.6.1 Recommended Parameters sect233k1

The elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  associated with a Koblitz curve `sect233k1` are specified by the septuple  $T = (m, f(x), a, b, G, n, h)$  where  $m = 233$  and the representation of  $\mathbb{F}_{2^{233}}$  is defined by:

$$f(x) = x^{233} + x^{74} + 1$$

The curve  $E$ :  $y^2 + xy = x^3 + ax^2 + b$  over  $\mathbb{F}_{2^m}$  is defined by:

$$\begin{aligned} a &= 0000\ 0000000\ 0000000\ 0000000\ 0000000\ 0000000\ 0000000 \\ &\quad 0000000 \\ b &= 0000\ 0000000\ 0000000\ 0000000\ 0000000\ 0000000\ 0000000 \\ &\quad 00000001 \end{aligned}$$

The base point  $G$  in compressed form is:

$$\begin{aligned} G &= 020172\ 32BA853A\ 7E731AF1\ 29F22FF4\ 149563A4\ 19C26BF5\ 0A4C9D6E \\ &\quad EFAD6126 \end{aligned}$$

and in uncompressed form is:

$$\begin{aligned} G &= 04\ 017232BA\ 853A7E73\ 1AF129F2\ 2FF41495\ 63A419C2\ 6BF50A4C \\ &\quad 9D6EEFAD\ 612601DB\ 537DECE8\ 19B7F70F\ 555A67C4\ 27A8CD9B\ F18AEB9B \\ &\quad 56E0C11056FAE6A3 \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

$$\begin{aligned} n &= 80\ 0000000\ 0000000\ 0000000\ 00069D5B\ B915BCD4\ 6EFB1AD5 \\ &\quad F173ABDF \\ h &= 04 \end{aligned}$$

### 3.6.2 Recommended Parameters sect233r1

The verifiably random elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  `sect233r1` are specified by the septuple  $T = (m, f(x), a, b, G, n, h)$  where  $m = 233$  and the representation of  $\mathbb{F}_{2^{233}}$  is defined by:

$$f(x) = x^{233} + x^{74} + 1$$

The curve  $E$ :  $y^2 + xy = x^3 + ax^2 + b$  over  $\mathbb{F}_{2^m}$  is defined by:

$$\begin{aligned} a &= 0000\ 0000000\ 0000000\ 0000000\ 0000000\ 0000000\ 0000000 \\ &\quad 00000001 \\ b &= 0066\ 647EDE6C\ 332C7F8C\ 0923BB58\ 213B333B\ 20E9CE42\ 81FE115F \\ &\quad 7D8F90AD \end{aligned}$$

$E$  was chosen verifiably at random from the seed:

$$S = 74D59FF0 7F6B413D 0EA14B34 4B20A2DB 049B50C3$$

$E$  was selected from  $S$  as specified in ANSI X9.62 [1] in normal basis representation and converted into polynomial basis representation.

The base point  $G$  in compressed form is:

$$\begin{aligned} G = & \quad 0300FA C9DFCBAC 8313BB21 39F1BB75 5FEF65BC 391F8B36 F8F8EB73 \\ & 71FD558B \end{aligned}$$

and in uncompressed form is:

$$\begin{aligned} G = & \quad 04 00FAC9DF CBAC8313 BB2139F1 BB755FEF 65BC391F 8B36F8F8 \\ & EB7371FD 558B0100 6A08A419 03350678 E58528BE BF8A0BEF F867A7CA \\ & 36716F7E 01F81052 \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

$$\begin{aligned} n = & \quad 0100 0000000 0000000 0000000 0013E974 E72F8A69 22031D26 \\ & 03CFE0D7 \\ h = & \quad 02 \end{aligned}$$

## 3.7 Recommended 239-bit Elliptic Curve Domain Parameters over $\mathbb{F}_{2^m}$

This section specifies the recommended 239-bit elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  in this document: parameters `sect239k1` associated with a Koblitz curve.

Section 3.7.1 specifies the elliptic curve domain parameters `sect239k1`.

### 3.7.1 Recommended Parameters `sect239k1`

The elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  associated with a Koblitz curve `sect239k1` are specified by the septuple  $T = (m, f(x), a, b, G, n, h)$  where  $m = 239$  and the representation of  $\mathbb{F}_{2^{239}}$  is defined by:

$$f(x) = x^{239} + x^{158} + 1$$

The curve  $E$ :  $y^2 + xy = x^3 + ax^2 + b$  over  $\mathbb{F}_{2^m}$  is defined by:

$$\begin{aligned} a = & \quad 0000 0000000 00000000 00000000 00000000 00000000 00000000 \\ & 00000000 \\ b = & \quad 0000 0000000 00000000 00000000 00000000 00000000 00000000 \\ & 00000001 \end{aligned}$$

The base point  $G$  in compressed form is:

$$\begin{aligned} G = & \quad 0329A0\ B6A887A9\ 83E97309\ 88A68727\ A8B2D126\ C44CC2CC\ 7B2A6555 \\ & 193035DC \end{aligned}$$

and in uncompressed form is:

$$\begin{aligned} G = & \quad 04\ 29A0B6A8\ 87A983E9\ 730988A6\ 8727A8B2\ D126C44C\ C2CC7B2A \\ & 65551930\ 35DC7631\ 0804F12E\ 549BDB01\ 1C103089\ E73510AC\ B275FC31 \\ & 2A5DC6B7\ 6553FOCA \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

$$\begin{aligned} n = & \quad 2000\ 0000000\ 0000000\ 0000000\ 005A79FE\ C67CB6E9\ 1F1C1DA8 \\ & 00E478A5 \\ h = & \quad 04 \end{aligned}$$

### 3.8 Recommended 283-bit Elliptic Curve Domain Parameters over $\mathbb{F}_{2^m}$

This section specifies the two recommended 283-bit elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  in this document: parameters `sect283k1` associated with a Koblitz curve, and verifiably random parameters `sect283r1`.

Section 3.8.1 specifies the elliptic curve domain parameters `sect283k1`, and Section 3.8.2 specifies the elliptic curve domain parameters `sect283r1`.

#### 3.8.1 Recommended Parameters `sect283k1`

The elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  associated with a Koblitz curve `sect283k1` are specified by the septuple  $T = (m, f(x), a, b, G, n, h)$  where  $m = 283$  and the representation of  $\mathbb{F}_{2^{283}}$  is defined by:

$$f(x) = x^{283} + x^{12} + x^7 + x^5 + 1$$

The curve  $E$ :  $y^2 + xy = x^3 + ax^2 + b$  over  $\mathbb{F}_{2^m}$  is defined by:

$$\begin{aligned} a = & \quad 0000000\ 0000000\ 0000000\ 0000000\ 0000000\ 0000000\ 0000000 \\ & 0000000\ 0000000 \\ b = & \quad 0000000\ 0000000\ 0000000\ 0000000\ 0000000\ 0000000\ 0000000 \\ & 0000000\ 0000001 \end{aligned}$$

The base point  $G$  in compressed form is:

$$\begin{aligned} G = & \quad 02\ 0503213F\ 78CA4488\ 3F1A3B81\ 62F188E5\ 53CD265F\ 23C1567A \\ & 16876913\ B0C2AC24\ 58492836 \end{aligned}$$

and in uncompressed form is:

$$\begin{aligned} G = & \quad 04\ 0503213F\ 78CA4488\ 3F1A3B81\ 62F188E5\ 53CD265F\ 23C1567A \\ & 16876913\ B0C2AC24\ 58492836\ 01CCDA38\ 0F1C9E31\ 8D90F95D\ 07E5426F \\ & E87E45C0\ E8184698\ E4596236\ 4E341161\ 77DD2259 \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

$$\begin{aligned} n = & \quad 01FFFFFF\ FFFFFFFF\ FFFFFFFF\ FFFFFF9AE\ 2ED07577\ 265DFF7F \\ & 94451E06\ 1E163C61 \\ h = & \quad 04 \end{aligned}$$

### 3.8.2 Recommended Parameters sect283r1

The verifiably random elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  sect283r1 are specified by the septuple  $T = (m, f(x), a, b, G, n, h)$  where  $m = 283$  and the representation of  $\mathbb{F}_{2^{283}}$  is defined by:

$$f(x) = x^{283} + x^{12} + x^7 + x^5 + 1$$

The curve  $E$ :  $y^2 + xy = x^3 + ax^2 + b$  over  $\mathbb{F}_{2^m}$  is defined by:

$$\begin{aligned} a = & \quad 00000000\ 00000000\ 00000000\ 00000000\ 00000000\ 00000000\ 00000000 \\ & 00000000\ 00000001 \\ b = & \quad 027B680A\ C8B8596D\ A5A4AF8A\ 19A0303F\ CA97FD76\ 45309FA2\ A581485A \\ & F6263E31\ 3B79A2F5 \end{aligned}$$

$E$  was chosen verifiably at random from the seed:

$$S = 77E2B073\ 70EB0F83\ 2A6DD5B6\ 2DFC88CD\ 06BB84BE$$

$E$  was selected from  $S$  as specified in ANSI X9.62 [1] in normal basis representation and converted into polynomial basis representation.

The base point  $G$  in compressed form is:

$$\begin{aligned} G = & \quad 03\ 05F93925\ 8DB7DD90\ E1934F8C\ 70B0DFEC\ 2EED25B8\ 557EAC9C \\ & 80E2E198\ F8CDBECD\ 86B12053 \end{aligned}$$

and in uncompressed form is:

$$\begin{aligned} G = & \quad 04\ 05F93925\ 8DB7DD90\ E1934F8C\ 70B0DFEC\ 2EED25B8\ 557EAC9C \\ & 80E2E198\ F8CDBECD\ 86B12053\ 03676854\ FE24141C\ B98FE6D4\ B20D02B4 \\ & 516FF702\ 350EDDB0\ 826779C8\ 13F0DF45\ BE8112F4 \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

$n = 03FFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFF90 399660FC 938A9016$   
 $5B042A7C EFADB307$   
 $h = 02$

### 3.9 Recommended 409-bit Elliptic Curve Domain Parameters over $\mathbb{F}_{2^m}$

This section specifies the two recommended 409-bit elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  in this document: parameters `sect409k1` associated with a Koblitz curve, and verifiably random parameters `sect409r1`.

Section 3.9.1 specifies the elliptic curve domain parameters `sect409k1`, and Section 3.9.2 specifies the elliptic curve domain parameters `sect409r1`.

#### 3.9.1 Recommended Parameters `sect409k1`

The elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  associated with a Koblitz curve `sect409k1` are specified by the septuple  $T = (m, f(x), a, b, G, n, h)$  where  $m = 409$  and the representation of  $\mathbb{F}_{2^{409}}$  is defined by:

$$f(x) = x^{409} + x^{87} + 1$$

The curve  $E: y^2 + xy = x^3 + ax^2 + b$  over  $\mathbb{F}_{2^m}$  is defined by:

$$\begin{aligned} a &= 00000000 00000000 00000000 00000000 00000000 00000000 00000000 \\ &\quad 00000000 00000000 00000000 00000000 00000000 00000000 00000000 \\ b &= 00000000 00000000 00000000 00000000 00000000 00000000 00000000 \\ &\quad 00000000 00000000 00000000 00000000 00000000 00000000 00000001 \end{aligned}$$

The base point  $G$  in compressed form is:

$$\begin{aligned} G &= 03 0060F05F 658F49C1 AD3AB189 0F718421 0EFD0987 E307C84C \\ &\quad 27ACCFB8 F9F67CC2 C460189E B5AAAA62 EE222EB1 B35540CF E9023746 \end{aligned}$$

and in uncompressed form is:

$$\begin{aligned} G &= 04 0060F05F 658F49C1 AD3AB189 0F718421 0EFD0987 E307C84C \\ &\quad 27ACCFB8 F9F67CC2 C460189E B5AAAA62 EE222EB1 B35540CF E9023746 \\ &\quad 01E36905 0B7C4E42 ACBA1DAC BF04299C 3460782F 918EA427 E6325165 \\ &\quad E9EA10E3 DA5F6C42 E9C55215 AA9CA27A 5863EC48 D8E0286B \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

---

$n = \text{7FFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFE5F}$   
 $\quad \quad \quad \text{83B2D4EA 20400EC4 557D5ED3 E3E7CA5B 4B5C83B8 E01E5FCF}$   
 $h = \text{04}$

### 3.9.2 Recommended Parameters sect409r1

The verifiably random elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  sect409r1 are specified by the septuple  $T = (m, f(x), a, b, G, n, h)$  where  $m = 409$  and the representation of  $\mathbb{F}_{2^{409}}$  is defined by:

$$f(x) = x^{409} + x^{87} + 1$$

The curve  $E: y^2 + xy = x^3 + ax^2 + b$  over  $\mathbb{F}_{2^m}$  is defined by:

$a = \text{00000000 00000000 00000000 00000000 00000000 00000000 00000000}$   
 $\quad \quad \quad \text{00000000 00000000 00000000 00000000 00000000 00000001}$   
 $b = \text{0021A5C2 C8EE9FEB 5C4B9A75 3B7B476B 7FD6422E F1F3DD67 4761FA99}$   
 $\quad \quad \quad \text{D6AC27C8 A9A197B2 72822F6C D57A55AA 4F50AE31 7B13545F}$

$E$  was chosen verifiably at random from the seed:

$$S = \text{4099B5A4 57F9D69F 79213D09 4C4BCD4D 4262210B}$$

$E$  was selected from  $S$  as specified in ANSI X9.62 [1] in normal basis representation and converted into polynomial basis representation.

The base point  $G$  in compressed form is:

$$G = \text{03 015D4860 D088DDB3 496B0C60 64756260 441CDE4A F1771D4D}$$
  
 $\quad \quad \quad \text{B01FFE5B 34E59703 DC255A86 8A118051 5603AEAB 60794E54 BB7996A7}$

and in uncompressed form is:

$$G = \text{04 015D4860 D088DDB3 496B0C60 64756260 441CDE4A F1771D4D}$$
  
 $\quad \quad \quad \text{B01FFE5B 34E59703 DC255A86 8A118051 5603AEAB 60794E54 BB7996A7}$   
 $\quad \quad \quad \text{0061B1CF AB6BE5F3 2BBFA783 24ED106A 7636B9C5 A7BD198D 0158AA4F}$   
 $\quad \quad \quad \text{5488D08F 38514F1F DF4B4F40 D2181B36 81C364BA 0273C706}$

Finally the order  $n$  of  $G$  and the cofactor are:

$n = \text{01000000 00000000 00000000 00000000 00000000 00000000 000001E2}$   
 $\quad \quad \quad \text{AAD6A612 F33307BE 5FA47C3C 9E052F83 8164CD37 D9A21173}$   
 $h = \text{02}$

### 3.10 Recommended 571-bit Elliptic Curve Domain Parameters over $\mathbb{F}_{2^m}$

This section specifies the two recommended 571-bit elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  in this document: parameters `sect571k1` associated with a Koblitz curve, and verifiably random parameters `sect571r1`.

Section 3.10.1 specifies the elliptic curve domain parameters `sect571k1`, and Section 3.10.2 specifies the elliptic curve domain parameters `sect571r1`.

#### 3.10.1 Recommended Parameters `sect571k1`

The elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  associated with a Koblitz curve `sect571k1` are specified by the septuple  $T = (m, f(x), a, b, G, n, h)$  where  $m = 571$  and the representation of  $\mathbb{F}_{2^{571}}$  is defined by:

$$f(x) = x^{571} + x^{10} + x^5 + x^2 + 1$$

The curve  $E$ :  $y^2 + xy = x^3 + ax^2 + b$  over  $\mathbb{F}_{2^m}$  is defined by:

$$\begin{aligned} a &= 00000000\ 00000000\ 00000000\ 00000000\ 00000000\ 00000000\ 00000000 \\ &\quad 00000000\ 00000000\ 00000000\ 00000000\ 00000000\ 00000000\ 00000000 \\ &\quad 00000000\ 00000000\ 00000000\ 00000000 \\ b &= 00000000\ 00000000\ 00000000\ 00000000\ 00000000\ 00000000\ 00000000 \\ &\quad 00000000\ 00000000\ 00000000\ 00000000\ 00000000\ 00000000\ 00000000 \\ &\quad 00000000\ 00000000\ 00000000\ 00000001 \end{aligned}$$

The base point  $G$  in compressed form is:

$$\begin{aligned} G &= 02\ 026EB7A8\ 59923FBC\ 82189631\ F8103FE4\ AC9CA297\ 0012D5D4 \\ &\quad 60248048\ 01841CA4\ 43709584\ 93B205E6\ 47DA304D\ B4CEB08C\ BBD1BA39 \\ &\quad 494776FB\ 988B4717\ 4DCA88C7\ E2945283\ A01C8972 \end{aligned}$$

and in uncompressed form is:

$$\begin{aligned} G &= 04\ 026EB7A8\ 59923FBC\ 82189631\ F8103FE4\ AC9CA297\ 0012D5D4 \\ &\quad 60248048\ 01841CA4\ 43709584\ 93B205E6\ 47DA304D\ B4CEB08C\ BBD1BA39 \\ &\quad 494776FB\ 988B4717\ 4DCA88C7\ E2945283\ A01C8972\ 0349DC80\ 7F4FBF37 \\ &\quad 4F4AEADE\ 3BCA9531\ 4DD58CEC\ 9F307A54\ FFC61EFC\ 006D8A2C\ 9D4979C0 \\ &\quad AC44AEAT\ 4FBEBBB9\ F772AEDC\ B620B01A\ 7BA7AF1B\ 320430C8\ 591984F6 \\ &\quad 01CD4C14\ 3EF1C7A3 \end{aligned}$$

Finally the order  $n$  of  $G$  and the cofactor are:

---

$n = 02000000\ 00000000\ 00000000\ 00000000\ 00000000\ 00000000\ 00000000$   
 $00000000\ 00000000\ 131850E1\ F19A63E4\ B391A8DB\ 917F4138\ B630D84B$   
 $E5D63938\ 1E91DEB4\ 5CFE778F\ 637C1001$

$h = 04$

### 3.10.2 Recommended Parameters sect571r1

The verifiably random elliptic curve domain parameters over  $\mathbb{F}_{2^m}$  sect571r1 are specified by the septuple  $T = (m, f(x), a, b, G, n, h)$  where  $m = 571$  and the representation of  $\mathbb{F}_{2^{571}}$  is defined by:

$$f(x) = x^{571} + x^{10} + x^5 + x^2 + 1$$

The curve  $E: y^2 + xy = x^3 + ax^2 + b$  over  $\mathbb{F}_{2^m}$  is defined by:

$a = 00000000\ 00000000\ 00000000\ 00000000\ 00000000\ 00000000\ 00000000$   
 $00000000\ 00000000\ 00000000\ 00000000\ 00000000\ 00000000\ 00000000$   
 $00000000\ 00000000\ 00000000\ 00000000\ 00000001$

$b = 02F40E7E\ 2221F295\ DE297117\ B7F3D62F\ 5C6A97FF\ CB8CEFF1\ CD6BA8CE$   
 $4A9A18AD\ 84FFABBD\ 8EFA5933\ 2BE7AD67\ 56A66E29\ 4AFD185A\ 78FF12AA$   
 $520E4DE7\ 39BACA0C\ 7FFEFFF7F\ 2955727A$

$E$  was chosen verifiably at random from the seed:

$$S = 2AA058F7\ 3A0E33AB\ 486B0F61\ 0410C53A\ 7F132310$$

$E$  was selected from  $S$  as specified in ANSI X9.62 [1] in normal basis representation and converted into polynomial basis representation.

The base point  $G$  in compressed form is:

$G = 03\ 0303001D\ 34B85629\ 6C16C0D4\ 0D3CD775\ 0A93D1D2\ 955FA80A$   
 $A5F40FC8\ DB7B2ABD\ BDE53950\ F4C0D293\ CDD711A3\ 5B67FB14\ 99AE6003$   
 $8614F139\ 4ABFA3B4\ C850D927\ E1E7769C\ 8EEC2D19$

and in uncompressed form is:

$G = 04\ 0303001D\ 34B85629\ 6C16C0D4\ 0D3CD775\ 0A93D1D2\ 955FA80A$   
 $A5F40FC8\ DB7B2ABD\ BDE53950\ F4C0D293\ CDD711A3\ 5B67FB14\ 99AE6003$   
 $8614F139\ 4ABFA3B4\ C850D927\ E1E7769C\ 8EEC2D19\ 037BF273\ 42DA639B$   
 $6DCCFFFE\ B73D69D7\ 8C6C27A6\ 009CBBBCA\ 1980F853\ 3921E8A6\ 84423E43$   
 $BAB08A57\ 6291AF8F\ 461BB2A8\ B3531D2F\ 0485C19B\ 16E2F151\ 6E23DD3C$   
 $1A4827AF\ 1B8AC15B$

Finally the order  $n$  of  $G$  and the cofactor are:

$$\begin{aligned} n &= 03FFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF \\ &\quad FFFFFFFF FFFFFFFF E661CE18 FF559873 08059B18 6823851E C7DD9CA1 \\ &\quad 161DE93D 5174D66E 8382E9BB 2FE84E47 \\ h &= 02 \end{aligned}$$

## A ASN.1 Syntax

This section discusses the representation of elliptic curve domain parameters using ASN.1 syntax and specifies ASN.1 object identifiers for the elliptic curve domain parameters recommended in this document.

### A.1 Syntax for Elliptic Curve Domain Parameters

There are a number of ways of representing elliptic curve domain parameters using ASN.1 syntax. The following syntax is recommended in SEC 1 [12] for use in X.509 certificates and elsewhere (following [4]).

```
Parameters{CURVES:IOSet} ::= CHOICE {
    ecParameters ECParameters,
    namedCurve    CURVES.&id({IOSet}),
    implicitCA   NULL
}
```

where

- `ecParameters` of type `ECParameters` indicates that the full elliptic curve domain parameters are given,
- `namedCurve` of type `CURVES` indicates that a named curve from the set delimited by `CurveNames` is to be used, and
- `implicitCA` of type `NULL` indicates that the curve is known implicitly, that is, the actual curve is known to both parties by other means.

The following syntax is then used to describe explicit representations of elliptic curve domain parameters, if need be.

```
ECParameters ::= SEQUENCE {
    version  INTEGER { ecpVer1(1) } (ecpVer1),
    fieldID  FieldID {{FieldTypes}},
    curve    Curve,
    base     ECPoint,
    order    INTEGER,
    cofactor INTEGER OPTIONAL,
    ...
}
```

See SEC 1 [12] for more details on the explicit representation of elliptic curve domain parameters.

## A.2 Object Identifiers for Recommended Parameters

This section specifies object identifiers for the elliptic curve domain parameters recommended in this document. These object identifiers may be used, for example, to represent parameters using the named-Curve syntax described in the previous section.

Parameters that have not previously been assigned object identifiers appear in the tree whose root is designated by the object identifier `certicom-arc`. It has the following value.

```
certicom-arc OBJECT IDENTIFIER ::= {
    iso(1) identified-organization(3) certicom(132)
}
```

Parameters that are given as examples in ANSI X9.62 [1] appear in the tree whose root is designated by the object identifier `ansi-X9-62`. It has the following value.

```
ansi-X9-62 OBJECT IDENTIFIER ::= {
    iso(1) member-body(2) us(840) 10045
}
```

The values of the object identifiers of parameters given in ANSI X9.62 are duplicated here for convenience.

To reduce the encoded lengths, the parameters under `certicom-arc` appear just below the main node. The object identifier `ellipticCurve` represents the root of the tree containing all such parameters in this document and has the following value.

```
ellipticCurve OBJECT IDENTIFIER ::= { certicom-arc curve(0) }
```

The actual parameters appear immediately below this; their object identifiers may be found in the following sections. Section A.2.1 specifies object identifiers for the parameters over  $\mathbb{F}_p$ , and Section A.2.2 specifies object identifiers for the parameters over  $\mathbb{F}_{2^m}$ .

### A.2.1 OIDs for Recommended Parameters over $\mathbb{F}_p$

The object identifiers for the recommended parameters over  $\mathbb{F}_p$  have the following values. The names of the identifiers agree with the nicknames given to the parameters in this document. In ANSI X9.62 [1], the curve `secp192r1` is designated `prime192v1`, and the curve `secp256r1` is designated `prime256v1`.

```
--  
-- Curves over prime-order fields:
```

```
--  

secp112r1 OBJECT IDENTIFIER ::= { ellipticCurve 6 }  

secp112r2 OBJECT IDENTIFIER ::= { ellipticCurve 7 }  

  

secp128r1 OBJECT IDENTIFIER ::= { ellipticCurve 28 }  

secp128r2 OBJECT IDENTIFIER ::= { ellipticCurve 29 }  

  

secp160k1 OBJECT IDENTIFIER ::= { ellipticCurve 9 }  

secp160r1 OBJECT IDENTIFIER ::= { ellipticCurve 8 }  

secp160r2 OBJECT IDENTIFIER ::= { ellipticCurve 30 }  

  

secp192k1 OBJECT IDENTIFIER ::= { ellipticCurve 31 }  

secp192r1 OBJECT IDENTIFIER ::= { ansi-X9-62 curves(3) prime(1) 1 }  

  

secp224k1 OBJECT IDENTIFIER ::= { ellipticCurve 32 }  

secp224r1 OBJECT IDENTIFIER ::= { ellipticCurve 33 }  

  

secp256k1 OBJECT IDENTIFIER ::= { ellipticCurve 10 }  

secp256r1 OBJECT IDENTIFIER ::= { ansi-X9-62 curves(3) prime(1) 7 }  

  

secp384r1 OBJECT IDENTIFIER ::= { ellipticCurve 34 }  

  

secp521r1 OBJECT IDENTIFIER ::= { ellipticCurve 35 }
```

### A.2.2 OIDs for Recommended Parameters over $\mathbb{F}_{2^m}$

The object identifiers for the recommended parameters over  $\mathbb{F}_{2^m}$  have the following values. The names of the identifiers agree with the nicknames given to the parameters in this document.

```
--  

-- Curves over characteristic 2 fields.  

--  

sect113r1 OBJECT IDENTIFIER ::= { ellipticCurve 4 }  

sect113r2 OBJECT IDENTIFIER ::= { ellipticCurve 5 }  

  

sect131r1 OBJECT IDENTIFIER ::= { ellipticCurve 22 }  

sect131r2 OBJECT IDENTIFIER ::= { ellipticCurve 23 }  

  

sect163k1 OBJECT IDENTIFIER ::= { ellipticCurve 1 }  

sect163r1 OBJECT IDENTIFIER ::= { ellipticCurve 2 }  

sect163r2 OBJECT IDENTIFIER ::= { ellipticCurve 15 }
```

```

sect193r1 OBJECT IDENTIFIER ::= { ellipticCurve 24 }
sect193r2 OBJECT IDENTIFIER ::= { ellipticCurve 25 }

sect233k1 OBJECT IDENTIFIER ::= { ellipticCurve 26 }
sect233r1 OBJECT IDENTIFIER ::= { ellipticCurve 27 }

sect239k1 OBJECT IDENTIFIER ::= { ellipticCurve 3 }

sect283k1 OBJECT IDENTIFIER ::= { ellipticCurve 16 }
sect283r1 OBJECT IDENTIFIER ::= { ellipticCurve 17 }

sect409k1 OBJECT IDENTIFIER ::= { ellipticCurve 36 }
sect409r1 OBJECT IDENTIFIER ::= { ellipticCurve 37 }

sect571k1 OBJECT IDENTIFIER ::= { ellipticCurve 38 }
sect571r1 OBJECT IDENTIFIER ::= { ellipticCurve 39 }

```

### A.2.3 The Information Object Set SECGCurveNames

The following information object set SECGCurveNames of class CURVES may be used to delineate the use of a curve recommended in this document. When it is used to govern the component namedCurve of Parameters (defined in section A.1), the value of namedCurve must be one of the values of the set.

```

SECGCurveNames CURVES ::= {
    -- Curves over prime-order fields:
    { ID secp112r1 } |
    { ID secp112r2 } |
    { ID secp128r1 } |
    { ID secp128r2 } |
    { ID secp160k1 } |
    { ID secp160r1 } |
    { ID secp160r2 } |
    { ID secp192k1 } |
    { ID secp192r1 } |
    { ID secp224k1 } |
    { ID secp224r1 } |
    { ID secp256k1 } |
    { ID secp256r1 } |
    { ID secp384r1 } |
    { ID secp521r1 } |
    -- Curves over characteristic 2 fields:

```

```
{ ID sect113r1 } |
{ ID sect113r2 } |
{ ID sect131r1 } |
{ ID sect131r2 } |
{ ID sect163k1 } |
{ ID sect163r1 } |
{ ID sect163r2 } |
{ ID sect193r1 } |
{ ID sect193r2 } |
{ ID sect233k1 } |
{ ID sect233r1 } |
{ ID sect239k1 } |
{ ID sect283k1 } |
{ ID sect283r1 } |
{ ID sect409k1 } |
{ ID sect409r1 } |
{ ID sect571k1 } |
{ ID sect571r1 } ,
    ...
}
```

The type CURVES used above is defined below.

```
CURVES ::= CLASS {
    &curve-id OBJECT IDENTIFIER UNIQUE
} WITH SYNTAX { ID &curve-id }
```

## B References

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