MODELS FOR CONFINEMENT [*]

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Abstract

The dynamical mechanisms causing quarks to be absolutely confined within the boundaries of a hadronic particle are discussed from various perspectives.

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1. Introduction: The Sakata Model

According to the model proposed by Sakata[1], there could be a relation between baryonic particles, then thought to be represented by the proton (P), the neutron (N) and the lambda (Λ) on the one hand, and the leptons, ν, e^- and μ^- on the other:

$$\begin{aligned} P &= \langle B^+\,\nu \rangle\,, \\ N &= \langle B^+\,e^- \rangle\,, \\ \Lambda &= \langle B^+\,\mu^- \rangle\,. \end{aligned} \qquad (1.1)$$

The distinction between ν_e and ν_μ was of course not yet known. B^+ must have been thought of as being some boson with lepton number -1 and baryon number 1 (a "leptobaryon"), and clearly it must have been thought to be sufficiently tightly bound to the leptons to escape detection at that time. What kinds of forces could have been responsible for that?

The modern versions of our models for baryons and mesons assume them to be composed of quarks. In the 1960s, the arguments used were of the same algebraical nature as those that led Sakata to his model. Then too, the question of the nature of the binding force was posed. When it was assumed that the constituents were strongly charged, and that the binding was caused by the attraction between the charges[2][3], this could only be part of an answer. In contrast with the electromagnetic case, it seems to be impossible to "ionize" hadronic particles: individual quarks are never produced. Why should this be so?

2. Absolute quark confinement in lattice QCD

One of the first indications that quark confinement really might be absolute, so that ionization leading to isolated quarks is fundamentally impossible, came from the lattice formulation of Quantum Chromodynamics[4]. In the lattice theory, the continuum of space and time is replaced by a lattice of individual *points* x ,of which the nearest neighbors, x and $x \pm a\,e_\mu$ are connected by *links* $[x,\, x \pm a\,e_\mu]$, where the index μ is a Lorentz index, and a is the size of the meshes of the lattice. The quark fields $q^i(x)$ are defined on the points x. The index i here is the color index (the Dirac spin index is suppressed here). The gauge field $A_\mu(x)$ is replaced by a connection operator $U(x,\mu)$ defined on the link $[x,\, x + a\,e_\mu]$:

$$U(x,\mu) \stackrel{\text{def}}{=} \exp\left(ig \int_x^{x+e_\mu} A_\mu \mathrm{d}x^\mu \right)\,. \qquad (2.1)$$

The same link in the opposite direction, by definition, corresponds to the inverse of the group element U. In the limit of small lattice length a, we consider the links at the boundary of an *elementary plaquette* in the 1-2 direction:

$$U_1 U_2 U_{-1} U_{-2} \rightarrow \exp\left(igF_{12}\delta x^1 \wedge \delta x^2 \right) = \mathbb{I} + igF_{12}a^2 - \tfrac{1}{2}g^2 F_{12}^2 a^4 + \cdots\,. \qquad (2.2)$$

This enables one to approximate the QCD action, $-\frac{1}{4}\mathrm{Tr}\,(F_{\mu\nu}F_{\mu\nu})$ by

$$\frac{1}{2g^2}\sum_{x,\mu,\nu}\mathrm{Tr}\,(UUUU)_{\mu\nu} - C \ , \tag{2.3}$$

where we also sum over the *orientations* of the plaquettes (giving six terms), such that in Eq. (2.2) the terms linear in $F_{\mu\nu}$ cancel out. The constant C is chosen so as to cancel $\mathrm{Tr}\,(\mathbb{I})$ but in general it has no effect.

The QCD amplitudes are now described in terms of the functional integrals over the field variables, with the exponent of this action as integrand. Now, we consider the $1/g^2$ expansion in the Euclidean case. It allows us to expand the exponent of the action, so that integrals of the type

$$\int \mathcal{D}\bar{q}\mathcal{D}q\mathcal{D}U\ \bar{q}_i(x_1)\ U\,U\,\cdots U\,q(x_2)\ , \tag{2.4}$$

are encountered. By merely applying group symmetry, one derives the general outcome of such integrals:

$$\int \mathrm{d}U\ U_i^p U_j^q \cdots U_r^{\dagger k}U_s^{\dagger \ell}\cdots = \sum_{\mathrm{Perm}} C_{\mathrm{Perm}}\ \delta_s^k \delta_j^\ell \cdots \delta_r^p \delta_s^q \cdots \ , \tag{2.5}$$

where the sum is over various allowed permutations between the upper indices only. The coefficients C_{Perm} can readily be determined. Local gauge-invariance requires indices to match pairwise at every lattice point separately. If at the l.h.s. of Eq. (2.5), the indices cannot be paired up point by point, the integral vanishes. Now, of we deal with a quark field $q^i(x_1)$ and an antiquark field $\bar{q}_j(x_2)$, then the only expressions where the indices can be paired, are the ones where all U fields occur, on all links that connect x_1 with x_2.

In the functional integral, this implies that, if we have a quark propagator running from $x_1(0)$ to $x_1(t)$ and an antiquark propagator running from $x_2(0)$ to $x_2(t)$, the only non-vanishing contribution to the functional integral comes from configurations where a sequence of plaquettes contributes in such a way that the plaquettes form a closed surface with the quark lines at its edges. Since each plaquette gives a suppression factor $1/g^2$ (which were assumed to be small in the $1/g^2$ expansion), we obtain a total suppression factor equal to $1/g^2$ raised to a power which is equal to the number of plaquettes needed to produce the surface in between, see Fig. 1. This factor can be written as

$$(1/g^2)^{L\,t} = e^{-2t\,L\log(1/g)} = e^{-tV(x_1,x_2)}\ , \tag{2.6}$$

where L is the distance between the quark lines, and t the interval in the Euclidean time direction. In Euclidean space, this generates a matrix element of the operator e^{-tH}. Apparently, all these matrix elements of H contain a potential term V increasing linearly with the distance L between the quark lines. Thus, one obtains a confining potential.

Note that this argument explaining absolute confinement could be misleading. Indeed, we could apply the same argument to the lattice version of Maxwell's theory of electromagnetism. Would one not also conclude that the electromagnetic force gives absolute
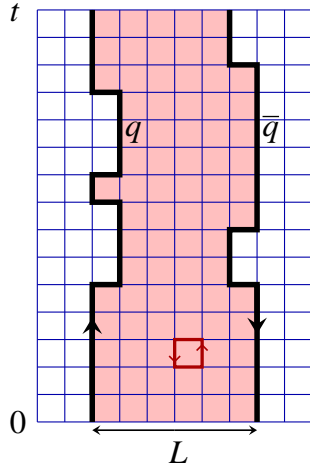
Figure 1: Lattice plaquettes contributing to the functional integral for an amplitude containing a quark antiquark pair at Euclidean time $\tau = 0$ and $\tau = t$. Solid lines are the quark trajectories. In this picture, all plaquettes in between the quark lines contribute, but in four-dimensional space-time, the contributing plaquettes may form any closed surface connecting the two lines. The surface area is bounded by $t\,L$, where L is the average distance between the quarks.

confinement? That is obviously false. Where does the argument then fail? The answer to this question is that one would be replacing the Maxwell vector potential A_μ by the connection matrices $U(x, \mu)$. Maxwell's action would be replaced by a non-quadratic action (2.3), and this would allow for new solutions: *magnetic monopoles*. Thus, the lattice action allows for unphysical particles. For large values of the fine structure constant α, these could indeed lead to absolute confinement, as is further explained in the next section. For small values of α, the $1/\alpha$ expansion as the one used here diverges too strongly to be reliable.

3. Absolute quark confinement as a topological phenomenon

We conclude from the above that the lattice approximation shows how absolute quark confinement *may* take place, but that the argument is not conclusive. It would be helpful to see how a similar phenomenon could be described in a strictly continuous theory. An indication as to what might happen can be obtained from the familiar Higgs theory[5]. Consider first the Abelian Higgs model. The Lagrangian is chosen to be

$$\mathcal{L}(A, \varphi) = -\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} - D_\mu\varphi^* D_\mu\varphi - V(\varphi) , \qquad (3.1)$$

where, as usual,

$$F_{\mu\nu} - \partial_\mu A_\nu - \partial_\nu A_\mu , \qquad D_\mu\varphi = (\partial_\mu + iqA_\mu)\varphi ;$$

$$V(\varphi) = \tfrac{1}{2}(\varphi^*\varphi - F^2)^2 \ . \tag{3.2}$$

q is the unit of electric charge, and F is the vacuum expectation value of the Higgs field φ. This Lagrangian allows for classical solutions where the Higgs field φ makes a full twist:

$$\varphi(x,y,z,t) \rightarrow \varrho(x^2 + y^2)\frac{x+iy}{|x+iy|} \ , \tag{3.3}$$

where the continuous function ϱ has to vanish when its argument vanishes, but the Higgs field quickly approaches the vacuum value $\varrho \rightarrow \langle\varphi\rangle_0 = F$ when its argument is large. This configuration is a vortex, and, assuming that the scalar field must obey some continuity conditions, one can easily convince oneself that this vortex is stable. The energy of the vortex can easily be estimated, and is found to be proportional to its length in the z-direction.[6]

Now imagine this theory to possess magnetic monopoles.[7] A magnetic monopole with magnetic charge $2\pi/q$ is defined to be an object that is attached to a *Dirac string*.[8] The Dirac string is essentially just a gauge artifact; putting such a string along the z axis, a gauge transformation of the form $\Lambda(x,y,z,t) \rightarrow \frac{x+iy}{|x+iy|}$ will remove the string. We now notice that this gauge transformation would also replace the φ field of a vortex by a field that can be dissolved in the vacuum, which has $\varphi \rightarrow F$ everywhere. Apparently, magnetic monopoles also form end points of our vortex. Indeed, the vortex can be seen to carry a magnetic flux equalling exactly $2\pi/q$. We end up with a picture where magnetic monopoles are tied together by vortex lines.

Indeed, in the Higgs model, photons are massive so that all fields are short-range. Long-range magnetic fields should also not be allowed, but that would clash with the law of magnetic flux conservation: div $\vec{B} = 0$. We see that, when monopoles are present, their magnetic flux is carried away by vortices.

Since a vortex carries energy proportional to its length, one finds that in a Higgs theory, magnetic monopoles are *absolutely confined*, exactly in the way one expects quarks to be confined in QCD. This idea was first demonstrated by Nielsen and Olesen.[6]

Quarks, however, do not carry large amounts of magnetic charge. Since QCD is asymptotically free,[9] one actually expects the electric coupling constant q for quarks to be small in the small-distance regime. Since the unit of magnetic charge is $2\pi/q$, quarks would be very strongly interacting at small distances, if they would indeed be 'color magnetic' monopoles.

There is of course a better theory for the confinement of quarks. What we have to do is replace all magnetic charges and fields by electric charges and *vice versa*.[10] It is the 'Higgs field' that we should assume to be magnetically charged, whereas the quarks carry only color-electric charges. Such a theory would work if we could show that scalar fields with pure color-magnetic monopole charges exist in QCD, and that their self interaction is driven by a potential similar to the one of Eq. (3.2).

The first, we can do. Non-Abelian gauge theories naturally carry field configurations that have color-magnetic monopole charges. In the case where the gauge group is $SU(2)$,
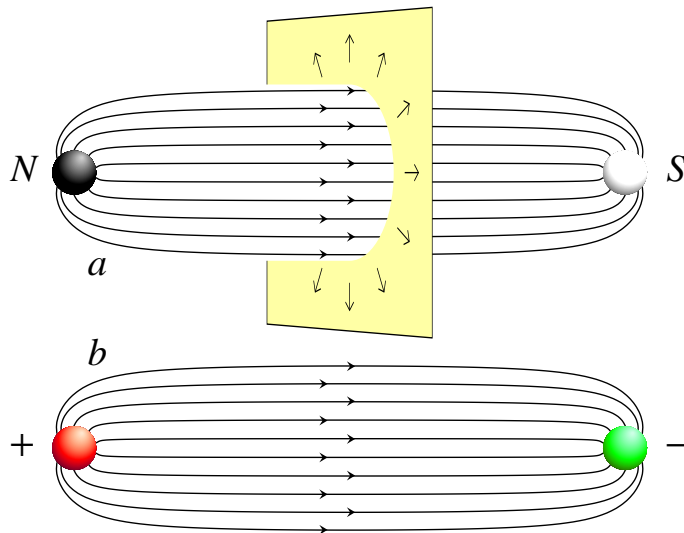
Figure 2: *a*)Confinement of magnetic monopoles in a superconductor. The Higgs field makes a full rotation if we follow a line around the vortex. *b*) Confinement of (color-) electric xharges when magnetic monopoles condense to produce magnetic superconductivity.

one may assume the presence of some scalar field configuration that transforms as the adjoint representation $I = 1$ of the gauge group. If it is not explicitly present, it might be formed by combining gluon fields, which are also in the adjoint representation. If it is explicitly present then one can describe magnetic monopoles in perturbation theory and their properties can be calculated. In QCD, we do not expect scalar fields in the adjoint representation to be present explicitly, but if we formulate the theory in the *Abelian projection* gauge,[11] we find artificial singularities that play the role of magnetic monopoles. It is these singularities that we may assume to condense just like Higgs particles, and then the electric-magnetic dual of the magnetic vortex confinement mechanism is established. It is this mechanism by which quarks can be absolutely confined.

A disadvantage of the Abelian projection procedure to describe hadrons in QCD is that it does not seem to be very accessible to systematic perturbative calculations. One has to *assume* that the singularities carrying color-magnetic charge will Bose-condense, but first-principle derivations are hard. In the lattice theory, first principle derivations are possible, but require the $1/g^2$ expansion, while g^2 is not really large enough. Furthermore, the lattice breaks several of the good symmetries of our particle models, such as rotation invariance, Lorentz invariance and the chiral part of the flavor group $SU(N_F)_L \otimes SU(N_F)_R$. We still want something better.

4.   A chain of gluons

We will claim that permanent confinement does not really conflict with the standard perturbation expansion. First, let us see how far we can push a very intuitive picture. Confining gluonic forces can be imagined to be structured as follows. Consider a spectator quark and spectator antiquark. We call a quark spectator quark if pair production of this quark species in the vacuum can be ignored, such as is the case when we endow these quarks with a large algebraical quark mass m_q.

Imagine that we slowly pull these quarks apart. Will the energy of the gluonic fields in their environment tend to infinity when the interquark distance goes to infinity? There is actually no very strong reason why this should *not* be the case. It happens to be so that the energy remains finite in perturbative Abelian gauge theories, but in the non-Abelian case the infrared structure of the theory is far too complicated to tell for sure. Greensite and Thorn[12] conjectured that more and more gluons will be created in the region between the quarks. They make an ansatz for their wave functions. Let the quarks be fixed on the positions x_0 and x_N. We take gluons to be at $x_1, x_2, \cdots, x_{N-1}$. Their ansatz wave function is

$$\psi(\vec{x}_1, \vec{x}_2, \cdots, \vec{x}_{N-1}) = A \prod_{i=1}^{N} \varphi(\vec{u}_i) \ ,$$
$$\vec{u}_i = \vec{x}_i - \vec{x}_{i-1} \ , \tag{4.1}$$

where $\varphi(\vec{u})$ is a universal wave function whose value follows from the extremal principle. The Hamiltonian contains a kinetic part $T^{(\mathrm{kin})}$ and a potential part $V^{(\mathrm{Coulomb})}$. One minimizes the energy $\mathcal{E} = \sum_N T^{(\mathrm{kin})} + \sum_N V^{(\mathrm{Coulomb})}$, both with respect to the form of the fnction φ and the number of gluons $N-1$. Then, one systematically tries to improve the ansatz.

There will be problems with this program. It is hard to see why N has to tend to infinity when the separation is large, and why the omission of color non-singlets would not violate unitarity. Could we do this more systematically? A very important question in this respect is what to choose as our "Coulomb potential" and why. We claim that this will have to be a confining potential from the start, but can this be justified and quantified?

There may well be more systematic ways to do this. The point we wish to stress here is that the expected infrared behavior can indeed be incorporated in a perturbative scheme such that better convergence may be expected. What we would advocate is to be called *infrared renormalization.*[13]

Compare what is usually done in UV renormalization procedures: we replace the Lagrangian $\mathcal{L}$ by

$$\mathcal{L}(g, A) = \mathcal{L}^{\mathrm{bare}} + \Delta\mathcal{L} - \Delta\mathcal{L} \ . \tag{4.2}$$

Here $\mathcal{L}^{\mathrm{bare}}$ is the bare Lagrangian, which may actually be quite different from the effective Lagrangian that we expect to see in the lowest order expressions of the amplitudes. We

take $\mathcal{L}^{\text{bare}} + \Delta\mathcal{L}$ as out lowest order Lagrangian. We then find that infinities cancel provided that we choose a counter term $-\Delta\mathcal{L}$. So the effective Lagrangian $\mathcal{L}^{\text{bare}} + \Delta\mathcal{L}$ is taken as our lowest order Lagrangian. This is just standard renormalization theory. The *renormalization group* is obtained when it is found that $\Delta\mathcal{L}$ has to be chosen differently at different scales.

This procedure, adding and subsequently subtracting the same term $\Delta\mathcal{L}$ to our Lagrangian, can also be done for infrared divergences. We write

$$\mathcal{L}(A) = -\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} + \mathcal{L}^{\text{gaugefix}} + \cdots \overset{\text{def}}{=} \mathcal{L}^0 - \Delta\mathcal{L} , \qquad (4.3)$$

where $\mathcal{L}^0$ is expected to be the appropriate Lagrangian for the lower order expressions, which will require $-\Delta\mathcal{L}$ as a counter term later. Choose

$$\mathcal{L}^0(A, x) = -\tfrac{1}{4}\int dx'\, F_{\mu\nu}(x)G(x - x')F_{\mu\nu}(x') + \cdots . \qquad (4.4)$$

Pick the radiation gauge:

$$\mathcal{L}^{\text{gaugefix}} = \lambda\partial_i A_i + \cdots , \qquad (4.5)$$

where λ is a Lagrange multiplier. So,

$$\mathcal{L}^0 = -\tfrac{1}{2}(\partial_\mu A_i)G(\partial_\mu A_i) + \tfrac{1}{2}(\partial_i A_0)G(\partial_i A_0) + \lambda\partial_i A_i + \cdots . \qquad (4.6)$$

A_0 now generates a potential V between charges, which obeys

$$\int dx'\, G(\vec{x} - \vec{x}')\partial_i^2 V(\vec{x}' - \vec{y}) = \delta^3(\vec{x} - \vec{y}) . \qquad (4.7)$$

We now assume that V is a confining potential, typically:

$$e^2 V(\vec{x}) = \frac{-\alpha}{r} + \varrho r + C^{\text{st}} ; \quad r = |\vec{x}| , \qquad (4.8)$$

where ϱ is the effective string constant, to be determined later by imposing consistency. In $\vec{k}$ space:

$$4\pi\alpha\, V(\vec{k}) = -\frac{4\pi\alpha}{\vec{k}^2} - \frac{8\pi\varrho}{(\vec{k}^2)^2} . \qquad (4.9)$$

The required Green function G now follows from solving Eq. (4.7), first in $\vec{k}$ space:

$$G(\vec{k}) = -\left(\vec{k}^2 V(\vec{k})\right)^{-1} = \frac{\vec{k}^2}{\vec{k}^2 + 2\varrho/\alpha} = 1 - \frac{2\varrho}{\alpha\vec{k}^2 + 2\varrho} , \qquad (4.10)$$

so that

$$G(\vec{x} - \vec{x}') = \delta^3(\vec{x} - \vec{x}') - \frac{8\pi\varrho}{\alpha r}e^{-\sqrt{\frac{2\varrho}{\alpha}}r} ; \quad r = |\vec{x} - \vec{x}'| , \qquad (4.11)$$

the latter part being, of course, just a Yukawa potential.

This way, we find that one has to try the following $\Delta\mathcal{L}$:

$$\Delta\mathcal{L}(A) = -\tfrac{1}{4} \int \mathrm{d}^3\vec{y}\, F_{\mu\nu}(x) \frac{8\pi\varrho}{\alpha r} e^{-\sqrt{2\varrho/\alpha}\, r} F_{\mu\nu}(x + \vec{y})\,, \quad r = \sqrt{\vec{y}^2}\,. \tag{4.12}$$

We see that this, innocent-looking, extra term $\Delta\mathcal{L}$ has to be treated exactly like a renormalization counter term. If all is well, one will find closed loop diagrams to generate infrared divergences that require just such a term to be cancelled out.

$\Delta\mathcal{L}$, as derived in Eq. (4.12) looks fairly local and unobtruding, but it does have drastic consequences for the infrared behavior of the color electric forces, because, in the zero momentum limit ($\vec{k} \to 0$), it cancels out completely the naive $-\tfrac{1}{4}FF$ term of the gauge theory.

The procedure has not yet been tested. We do not know whether this infrared renormalization would suffice to produce meaningful hadronic wave functions in perturbation expansion, but it appears to be worth trying.

5.   Renormalized gauge-invariant effective actions

In our final chapter, we consider effective theories all by themselves, in the classical (zero-loop) approximation. The Lagrangians to be produced now should be regarded in the context of the previous discussion. Again, although the real theory is expected to be a non-Abelian one, the considerations that follow now would apply to Abelian Lagrangians as well, so, for simplicity, we restrict ourselves to the Abelian case.

Take a vector potential field A_μ and a neutral scalar φ. Let the Lagrangian be

$$\mathcal{L}(A, \varphi) = -\tfrac{1}{4} Z(\varphi)\, F_{\mu\nu} F_{\mu\nu} - V(\varphi) + J_\mu A_\mu\,. \tag{5.1}$$

We omit the kinetic term for φ. As one may verified later, a kinetic term would not play any important role in the solutions that follow, other than making them a bit more complicated to derive. J_μ is an external current — one could think of the charge density $(\rho, \vec{0})$ of a stationary separated quark and antiquark pair. We are looking for a stationary solution:

$$\partial_i D_i = \rho(x)\,; \qquad D_i = Z(\varphi) E_i\,. \tag{5.2}$$

The energy density is

$$\mathcal{H} = \tfrac{1}{2} \frac{D^2}{Z(\varphi)} + V(\varphi)\,, \tag{5.3}$$

and since φ has no kinetic term, the energy density of the $\vec{D}$ field becomes

$$U(D) = \min_\varphi \left(\tfrac{1}{2} \frac{D^2}{Z(\varphi)} + V(\varphi) \right)\,. \tag{5.4}$$

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By choosing the functions Z and V (relative to one another), we can obtain any kind of monotonically increasing function of D.

Now, consider a flux tube of the D-field. The conservation law (5.2) ensures that the field lines are conserved, so the total flux $Q = D\Sigma$ (where Σ is the surface area over which D is spread) equals the total charge of a quark or antiquark at the end point of a flux tube, and it is fixed. The vortex with given total flux Q will spread over a surface Σ in such a way that the total energy is minimized. Thus, the energy per unit of length of this vortex will be

$$\varrho^{\text{string}} = \min(\Sigma\, U(D)) = \min_D \left(Q \frac{U(D)}{D} \right) . \tag{5.5}$$

$U(D)/D$ has a non-trivial minimum if, at low values for D, the energy will increase not faster than linearly in D, rather than quadratically in ordinary electrodyunamics:

$$U(D) \to \varrho^{\text{string}} D . \tag{5.6}$$

Which functions Z and V will do the job? This is an exercise in Legendre transformations. From (5.4):

$$\mathrm{d}U = 0 \quad \to \quad \tfrac{1}{2} D^2 = -\frac{\mathrm{d}V}{\mathrm{d}(1/Z)} = Z^2 \frac{\mathrm{d}V}{\mathrm{d}Z} . \tag{5.7}$$

I we assume some power law relation between $V(\varphi)$ and $Z(\varphi)$:

$$V(Z) \approx Z^\alpha , \quad \alpha > 0 , \tag{5.8}$$

then we find

$$D \approx Z^{\frac{1}{2}(\alpha+1)} ; \quad U \approx Z^\alpha ; \quad U \approx D^{\frac{2\alpha}{\alpha+1}} , \tag{5.9}$$

So, Eq. (5.6) requires $\alpha = 1$, that is, a linear dependence between V and Z. The proportionality constant can then easily be related to the one in (5.6):

$$V \approx \tfrac{1}{2} \varrho^2\, Z ; \quad U \approx \varrho^2 Z \approx \varrho D . \tag{5.10}$$

If we keep $\alpha < 3$ the the energy density of the D field surrounding a charge decreases slower than $1/r^3$ so that the total energy is infinite: such theories give confinement, though flux lines only develop if $\alpha \le 1$. The Maxwell case has $\alpha = \infty$, that is, Z is fixed. In general, however, Eqs. (5.8)–(5.10) are too crude to be used in detailed models.

Now that we know which relationship we want between Z and V, we can reconsider the Lagrangian (5.1), and try to eliminate the field φ altogether, which might be possible because there is no kinetic term for φ. Writing $\tfrac{1}{4} F_{\mu\nu} F_{\mu\nu} \stackrel{\text{def}}{=} x$, and realizing that, so far, everything we did was classical, we find

$$\mathcal{L} = f(x) = \operatorname{extr}_\varphi(Zx - V) , \tag{5.11}$$

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so that we have to solve φ out of

$$x = \frac{\mathrm{d}V}{\mathrm{d}Z} \ . \tag{5.12}$$

Thus, the Lagrangian will be a function of x, and Eq. (5.12) will force V and Z to be functions of x as well. So, everything depends on only one parameter. Suppose now that we wish the flux energy ϱ of a unit of flux (see Eq. (5.6)) to be bounded from below, so as to obtain vortex formation:

$$U(D) \stackrel{\mathrm{def}}{=} \varrho(D)D \ , \qquad \varrho \geq \varrho_0 \ . \tag{5.13}$$

We then ask what kind of function $\mathcal{L}(x)$ can be, given the function $\varrho(D)$. It is then convenient, temporarily, to use D as the independent variable. We have

$$V = \varrho D - \frac{\frac{1}{2}D^2}{Z} \ ; \qquad \frac{\partial V}{\partial Z} = \frac{\frac{1}{2}D^2}{Z^2} \ , \tag{5.14}$$

and by differentiating:

$$\mathrm{d}V = \mathrm{d}(\varrho D) - \frac{D\mathrm{d}D}{Z} + \frac{\frac{1}{2}D^2\mathrm{d}Z}{Z^2} \quad \rightarrow \quad \frac{\mathrm{d}(\varrho D)}{D\mathrm{d}D} = \frac{1}{Z} \ . \tag{5.15}$$

Combining Eqs. (5.11), (5.12) and (5.14), we find

$$\mathcal{L}(x) = \frac{D^2}{Z} - \varrho D \ = \ D^2\frac{\partial \varrho}{\partial D} \ ; \qquad \sqrt{2x} = \frac{\partial(\varrho D)}{\partial D} \ . \tag{5.16}$$

Furthermore, using (5.12), we derive

$$\mathrm{d}\mathcal{L} = Z\mathrm{d}x + x\mathrm{d}Z - \mathrm{d}V \quad \rightarrow \quad Z = \frac{\mathrm{d}\mathcal{L}}{\mathrm{d}x} \ . \tag{5.17}$$

One then discovers that, insisting that $\varrho \geq \varrho_0$ and also that $\varrho D = U(D) \rightarrow 0$ if $D \rightarrow 0$, the equation for $x(D)$ has no unambiguous inverse, so we cannot write the Lagrangian as a single-valued function of $x = \frac{1}{2}E^2$. Thus, to describe a theory with confinement, we must keep the functions V and Z in the Lagrangian[1].

6. Conclusions

Quantum Chromodynamica is an extremely arrurately defined theory. At short distance scales, due to asymptotic freedom, the forces become weak, so that perturbative treatment is possible, and calculating the QCD contributions to high-energy scattering processes has become routine. Its salient property of permanent quark confinement is usually regarded as a fundamentally non-perturbative feature of the theory; indeed, usual perturbative

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[1]During the meeting, Ref[13], was cited, where it was stated (Eq. (8.9) and Fig. 9) that $\partial\mathcal{L}/\partial x = 0$ at $x = \frac{1}{2}\varrho^2$; this, unfortunately, was an error.

methods do not allow us to understand it. But perturbation expansion can be performed in a modified fashion, such that confinement is assumed to be realized from the start. There are several ways to perform such unusual expansions; they are less systematic, and there is no guarantee for convergence better than the usual procedure, but one may suspect improvement since one uses the right spectrum of physical states from the start. We think that such approaches deserve further study. In any case, one obtains models that provide us with a good qualitative understanding of how permanent confinement of quarks can take place. It is not a very mysterious feature of the theory, but rather a consequence of the behavior of the effective actions at large distances and relatively weak field strengths.

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