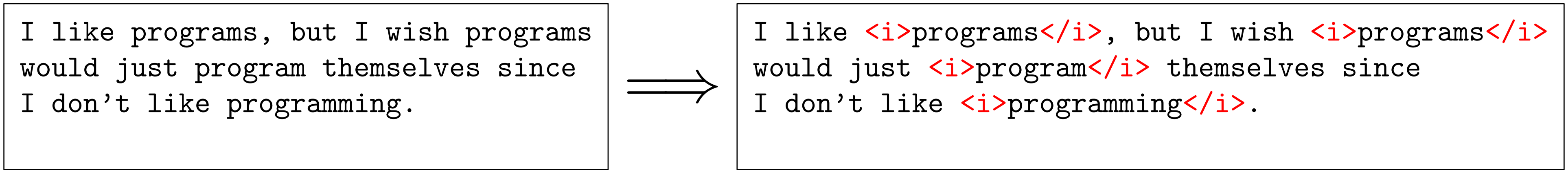
Learning Programs:
A Hierarchical Bayesian Approach

ICML - Haifa, Israel June 24, 2010

Percy Liang Michael I. Jordan Dan Klein

Motivating Application: Repetitive Text Editing



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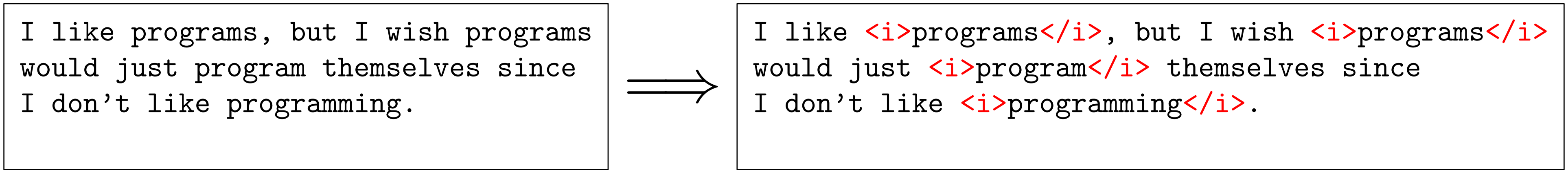

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Challenge: learn from very few examples

General Setup

Goal:

$$\begin{gathered}(X_1, Y_1)\\ \cdots \\ (X_n, Y_n)\end{gathered}$$

Training data

General Setup

Goal:

$$\begin{array}{c} (X_1, Y_1) \\ \cdots \\ (X_n, Y_n) \end{array} \quad \Longrightarrow \quad Z \text{ such that } (Z\ X_j) = Y_j$$

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When n small, many programs consistent with training data.

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Move to beginning of third word, ...
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Move to word with prefix `program`, ...
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Which program to choose?

Key Intuition

One task:

Want to choose a program which is simple (Occam's razor).

Examples $\Longrightarrow$ Z

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$$\text{Examples} \implies Z$$

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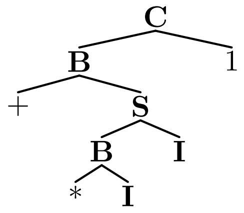
Find programs that share common subprograms.

- Programs do tend to share common components.
- Penalize joint complexity of all K programs.

Outline of Proposed Solution

Program representation: What are subprograms?

Combinatory logic



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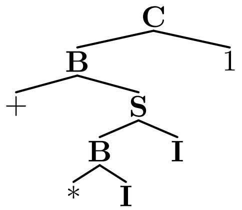
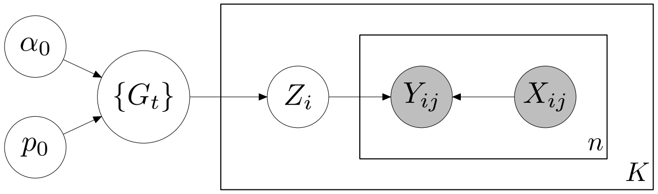


Probabilistic model: Which programs are favorable?

Nonparametric Bayes



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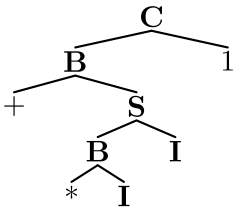

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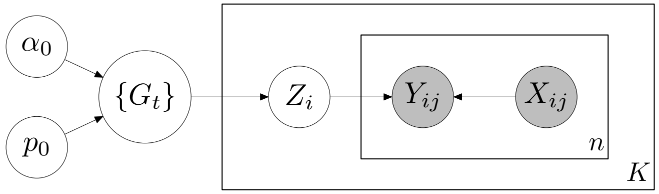
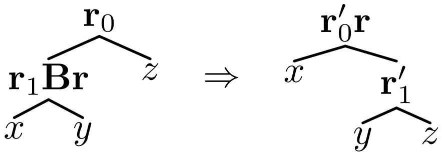


Statistical inference: How do we search for good programs?

MCMC



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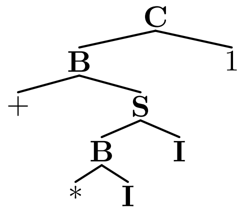

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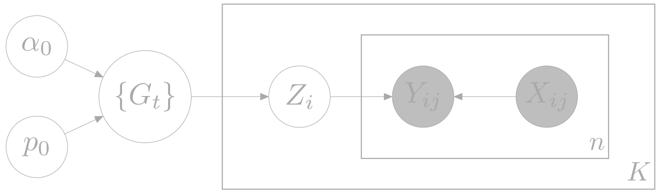

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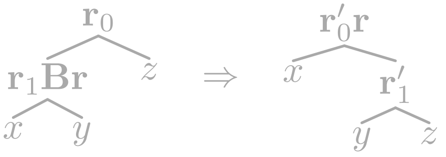

Representation: What Language?

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Our language:

Combinatory logic [Schönfinkel, 1924]

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Properties: no mutation, no variables $\Rightarrow$ simple semantics

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Result:

- Programs are trees
- Subprograms are subtrees

Programs with No Arguments

Example: compute $\min(3,4)$

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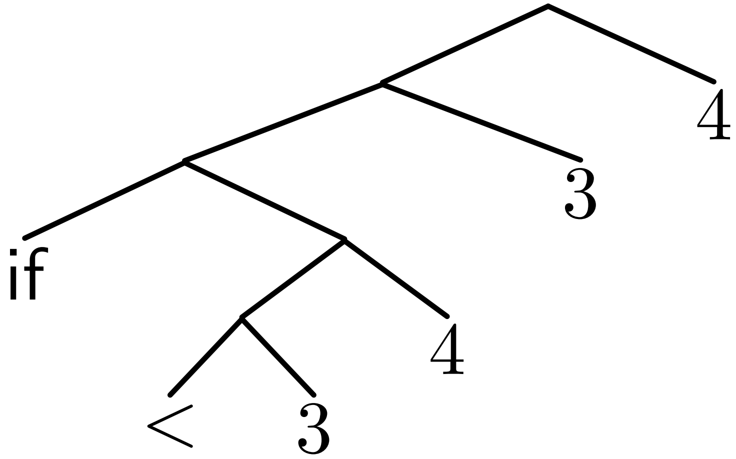

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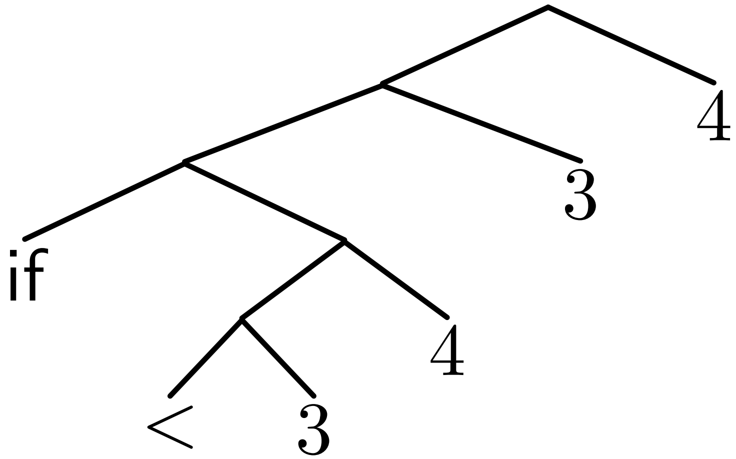
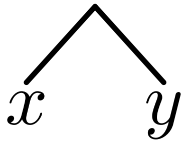


General:



$\Rightarrow$ result of applying function x to argument y

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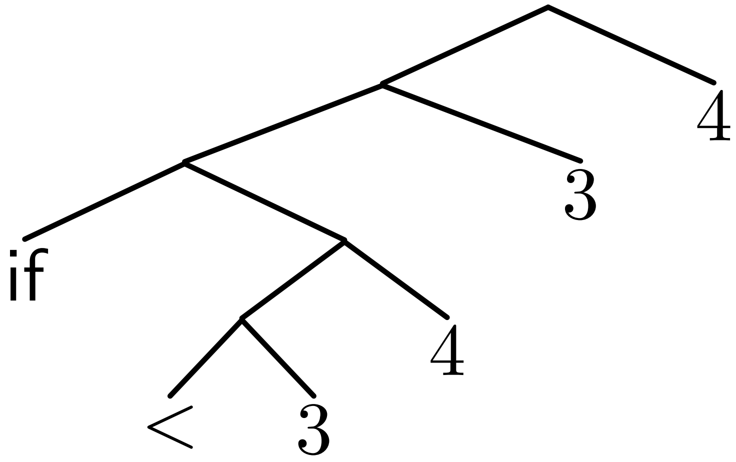

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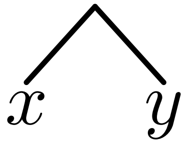

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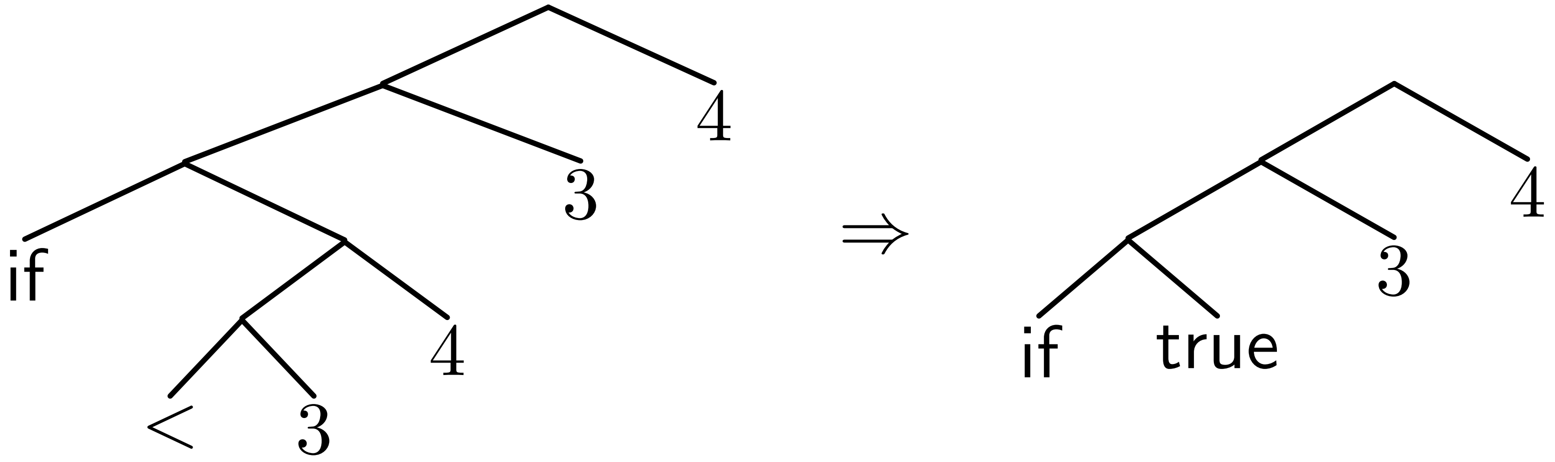

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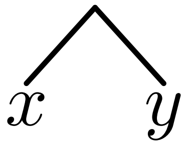

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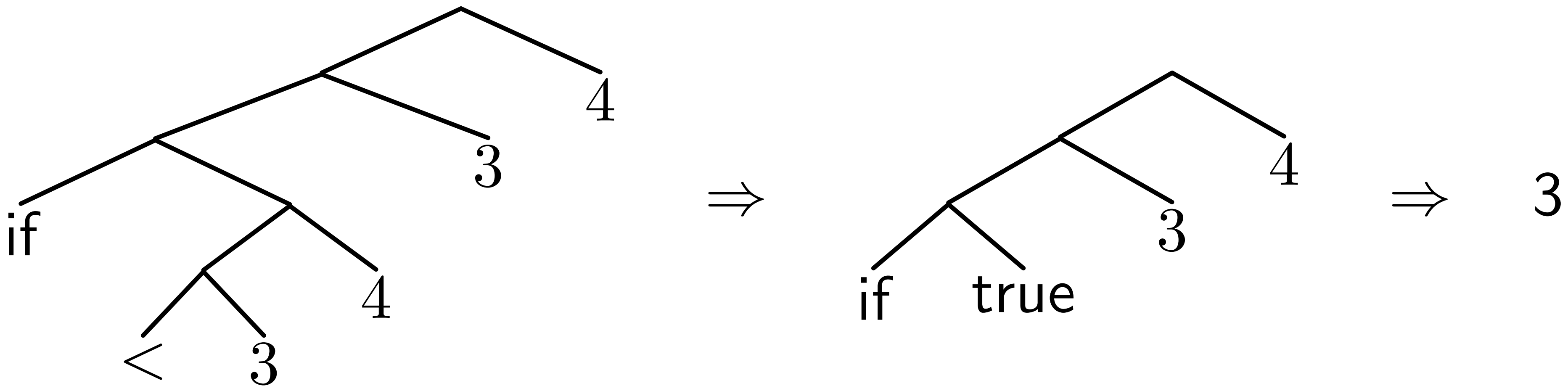

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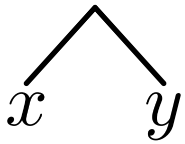 $\Rightarrow$ result of applying function x to argument y

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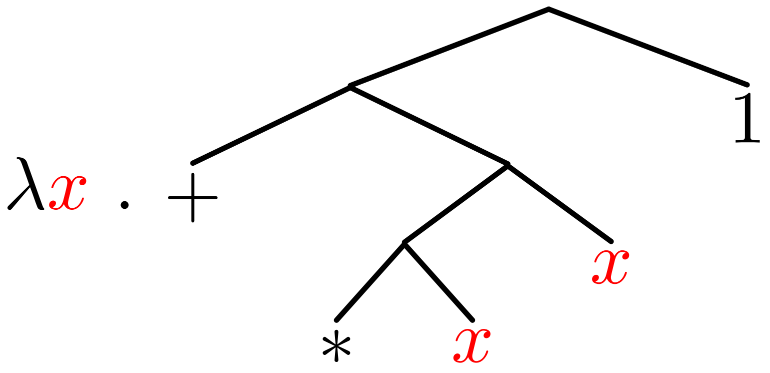

Lambda calculus

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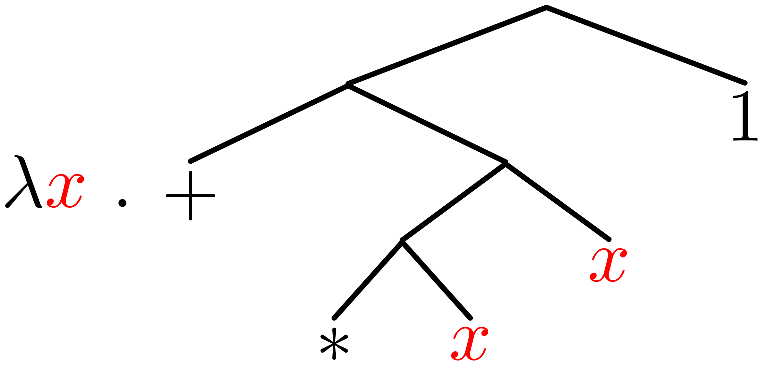

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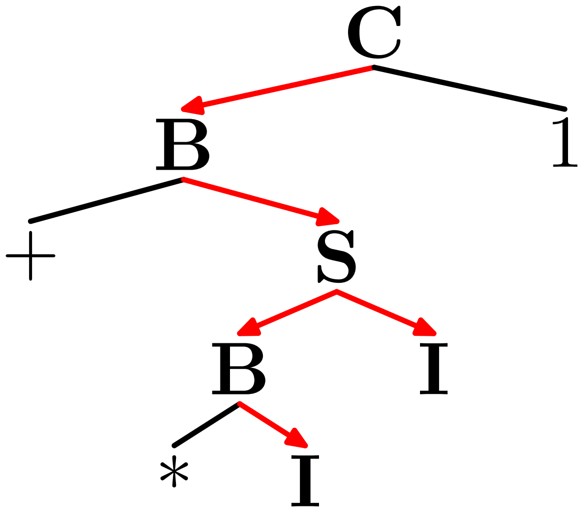

Combinatory logic

Programs with One Argument

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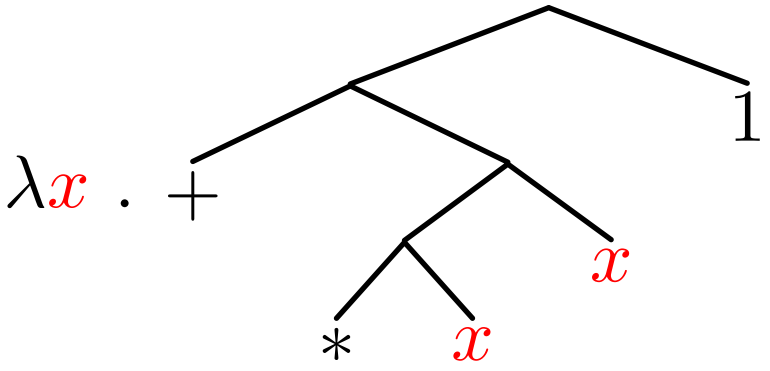

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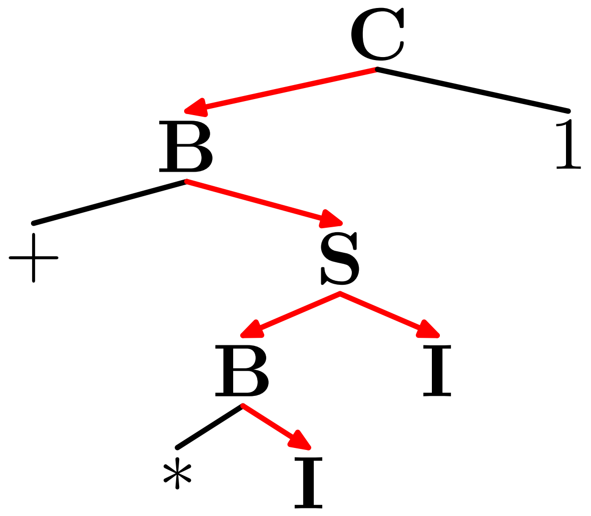

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Combinators $\{\mathbf{B}, \mathbf{C}, \mathbf{S}, \mathbf{I}\}$ encode placement of arguments

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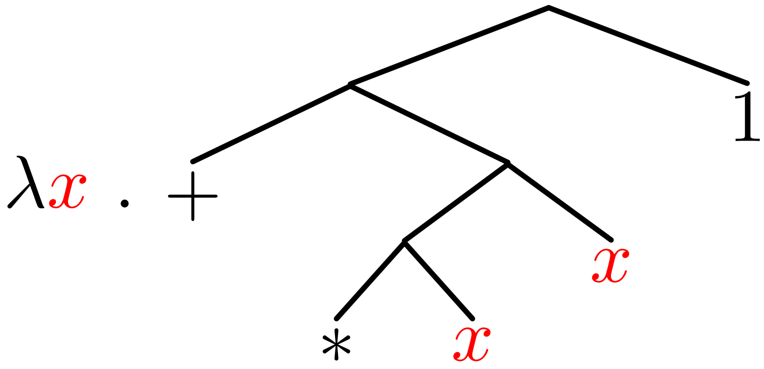

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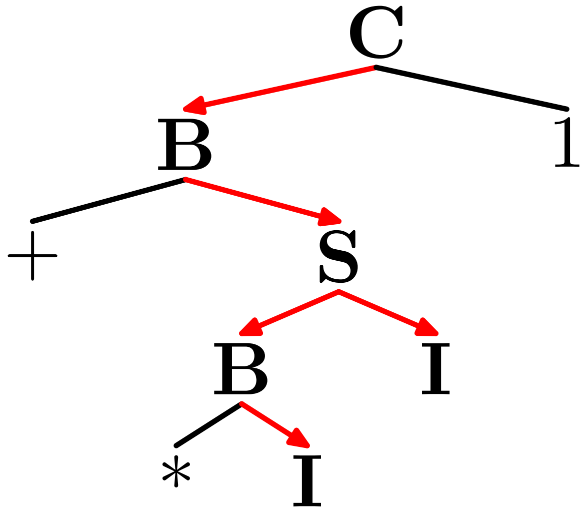

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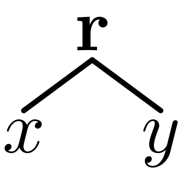

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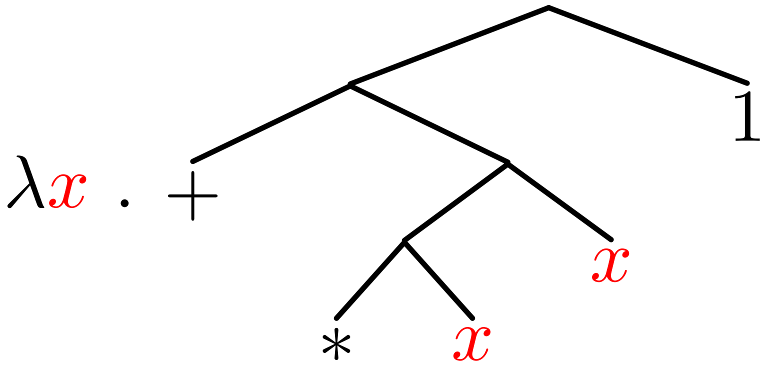

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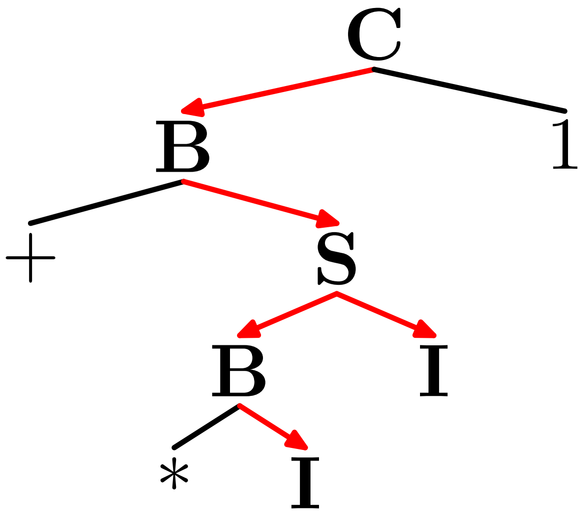

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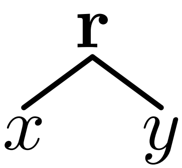

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Rules:

$(\mathbf{B}\ f\ g\ x) = (f\ (g\ x))$

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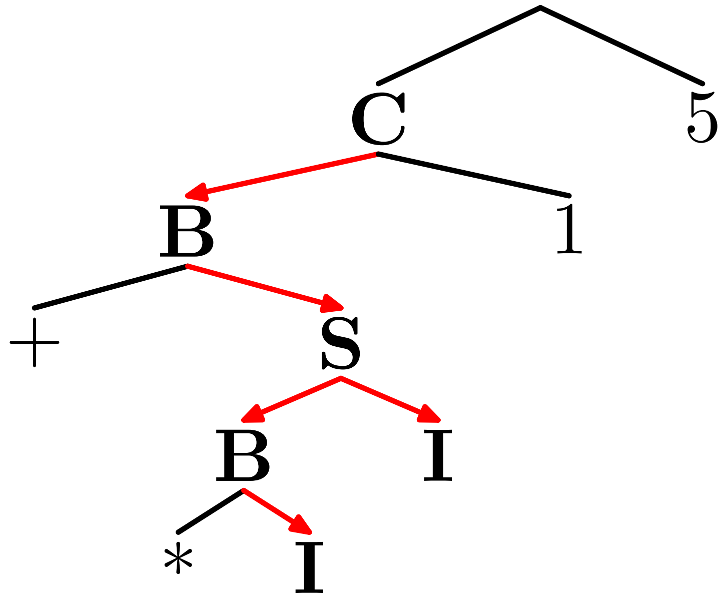

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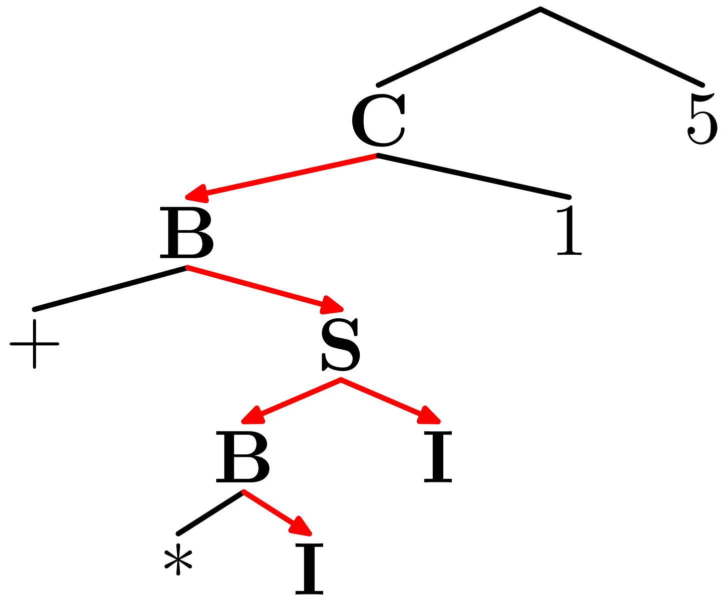

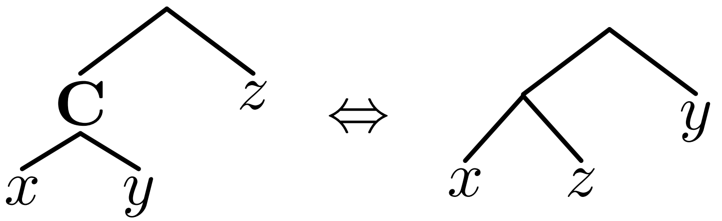

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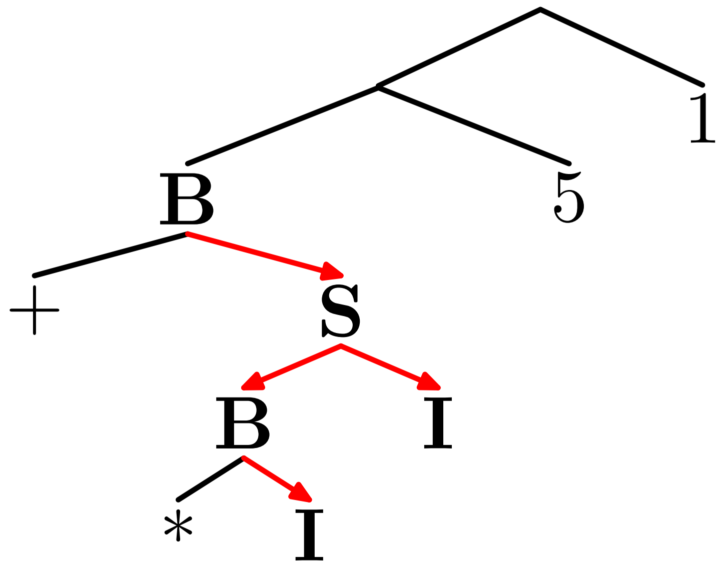

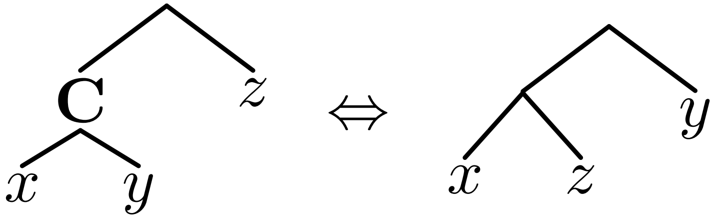

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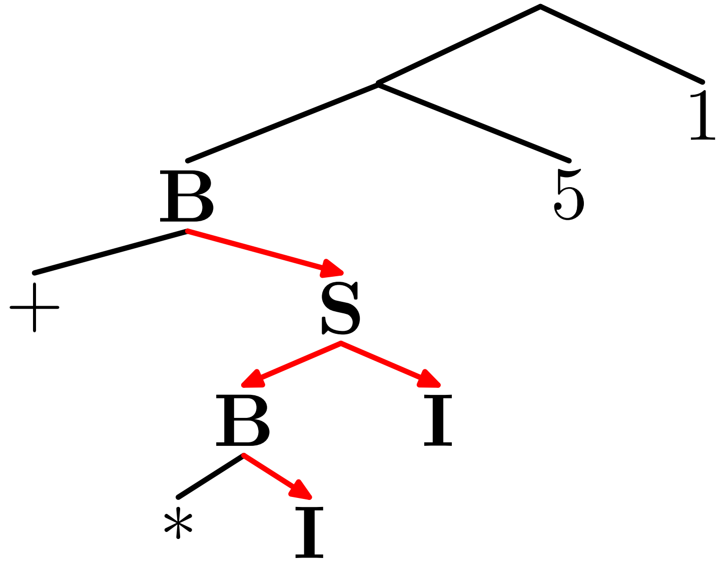

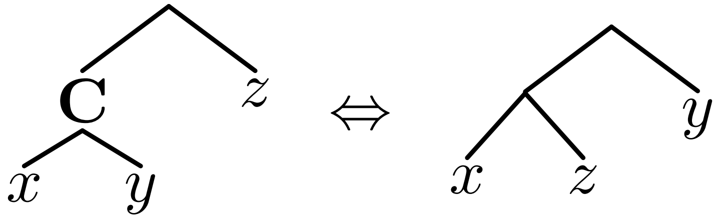

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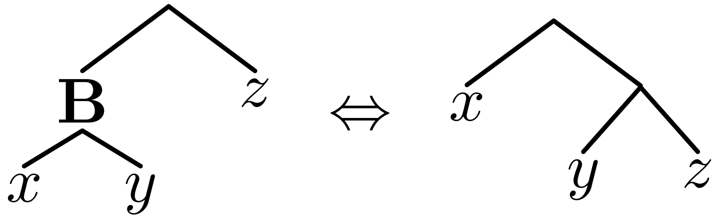

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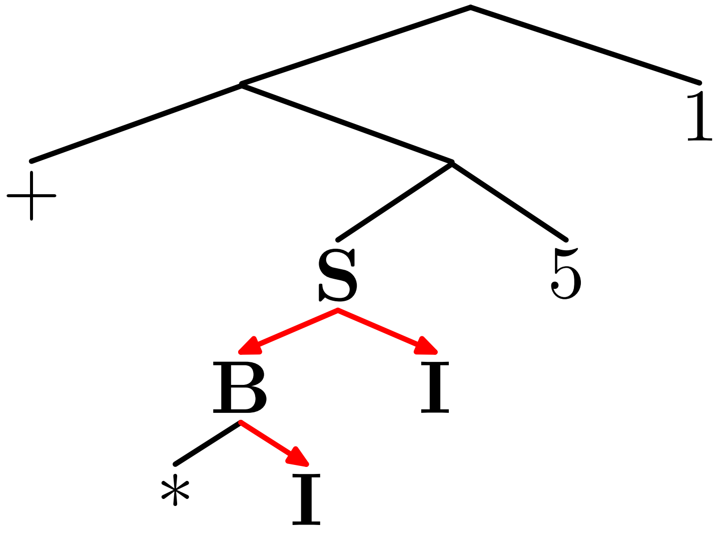

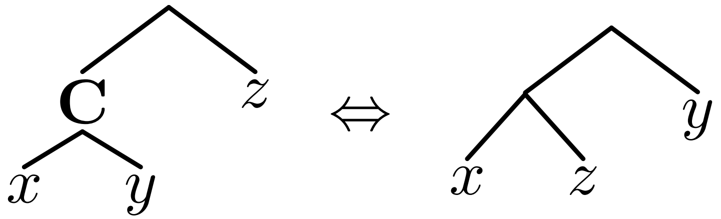

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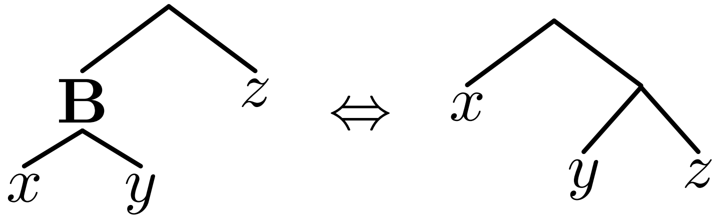

route right

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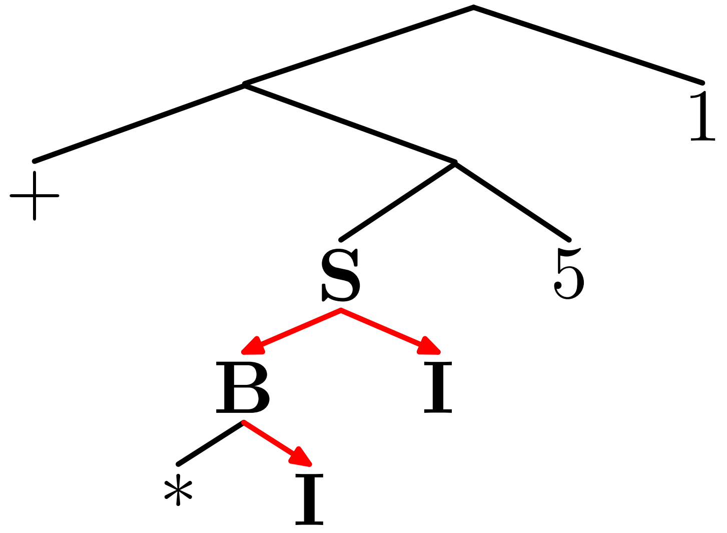

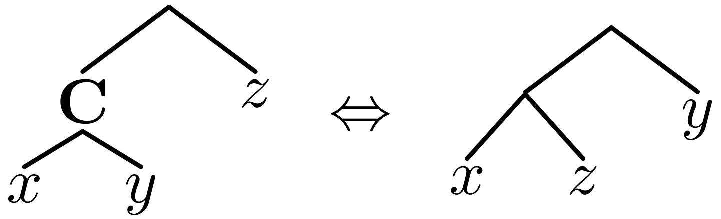

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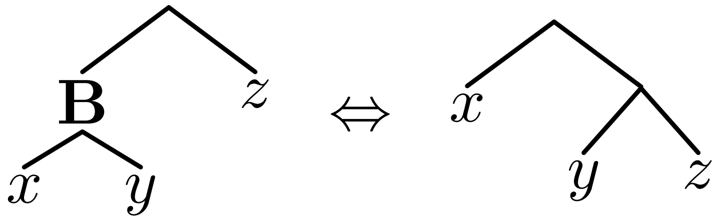

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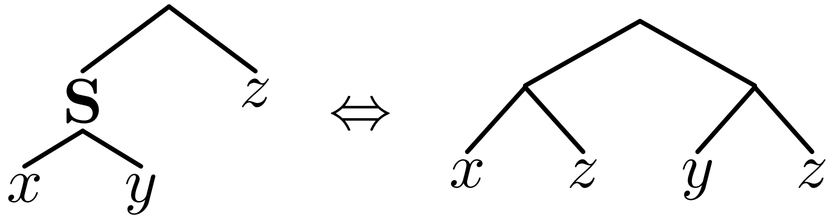

route left and right

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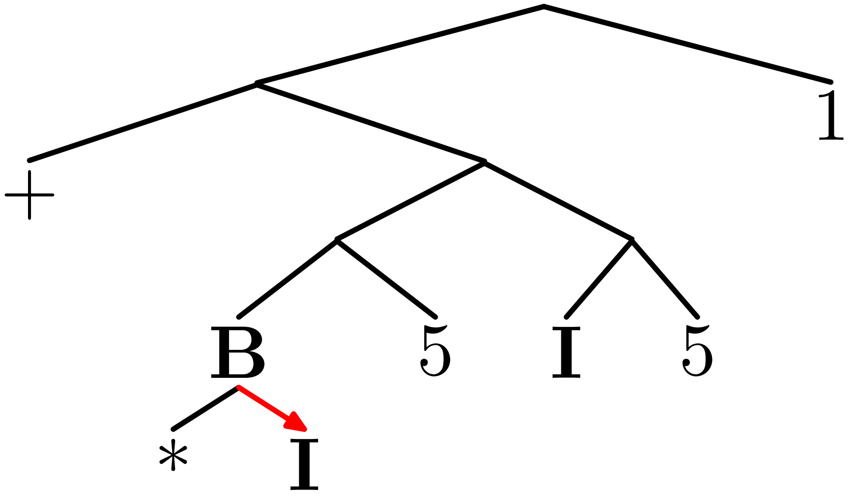
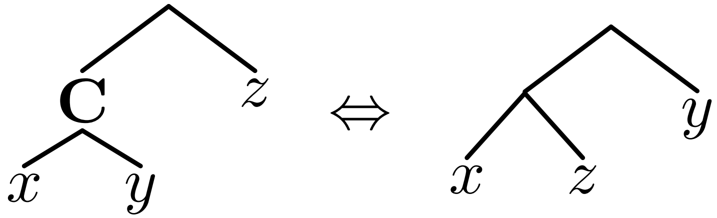
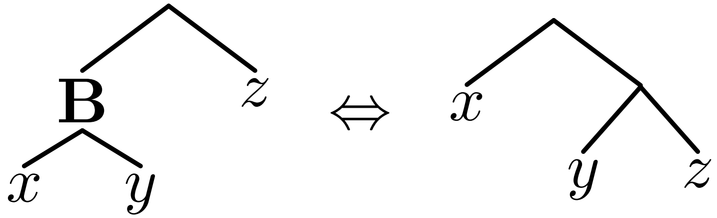




route left



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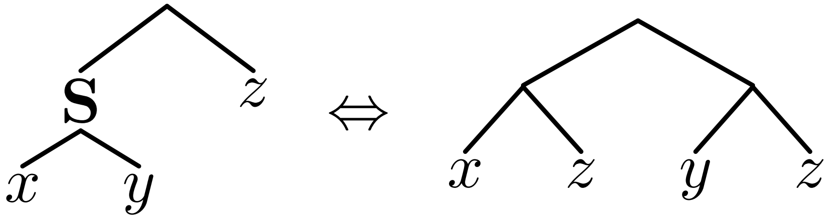

route left and right

Programs with One Argument

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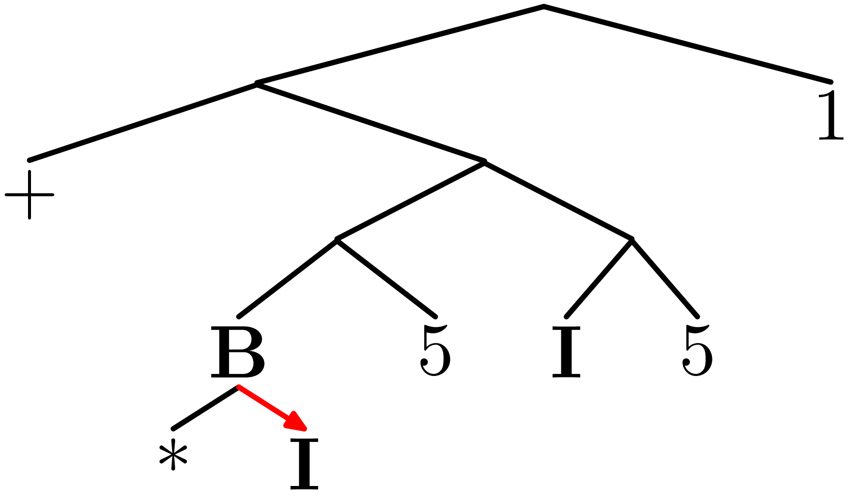

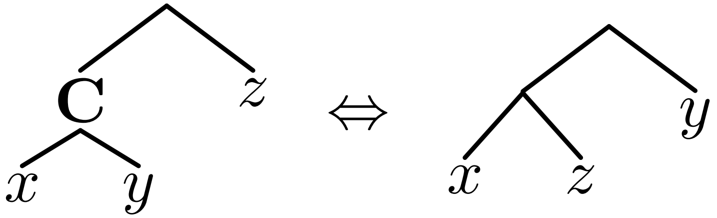

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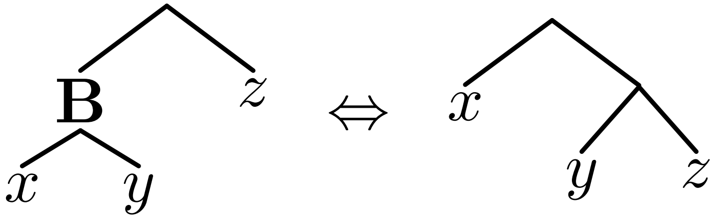

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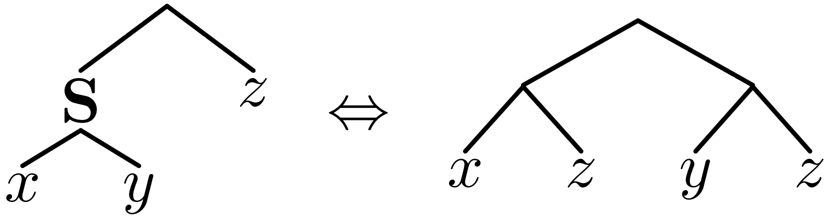

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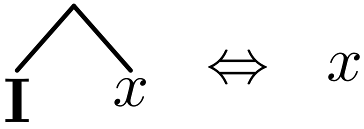

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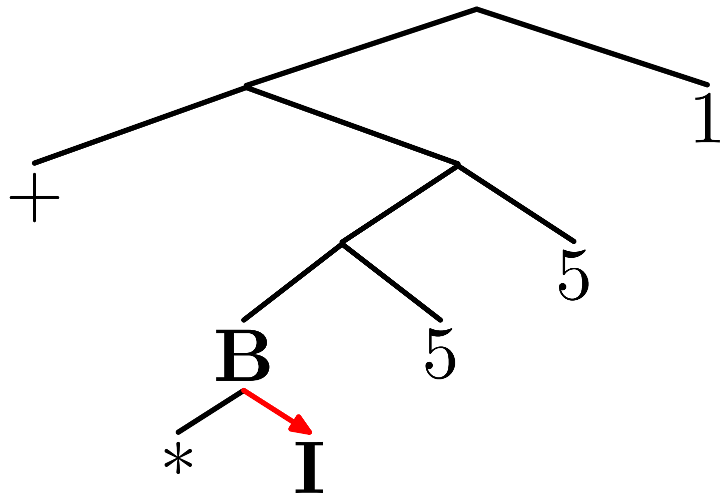

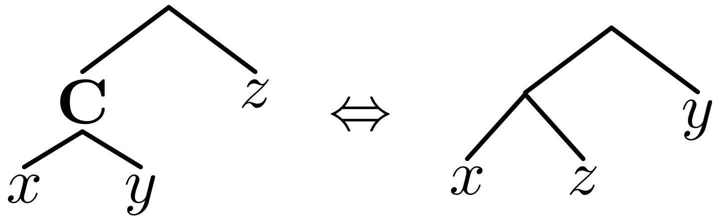

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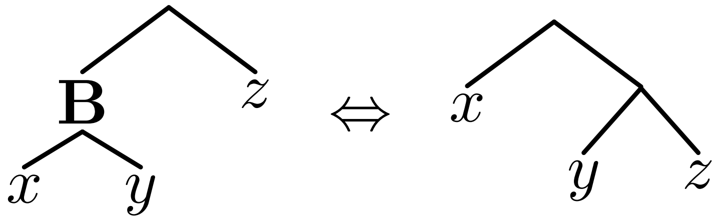

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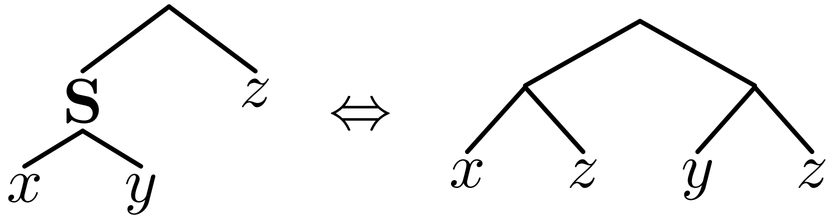

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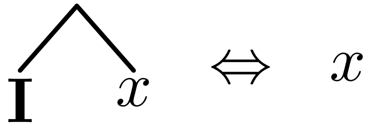

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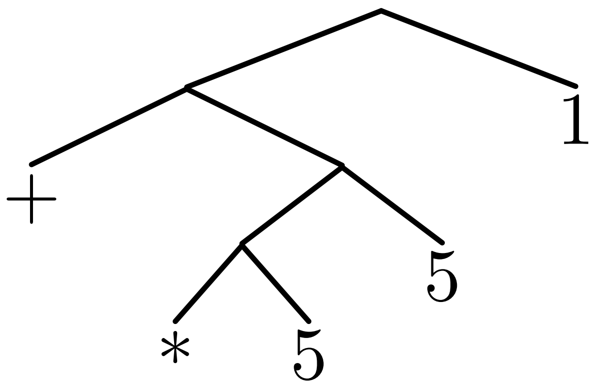

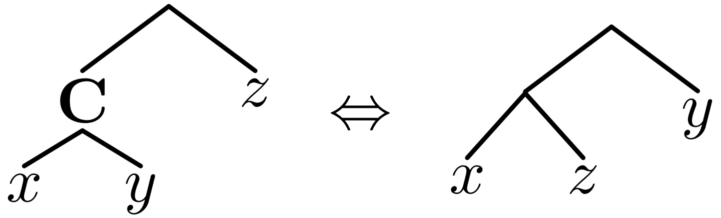

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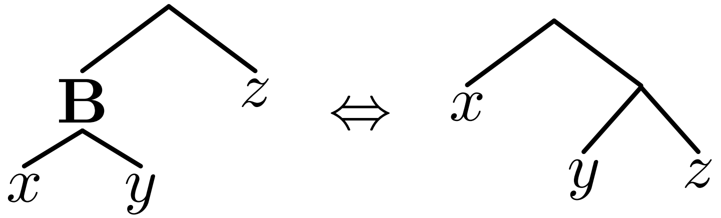

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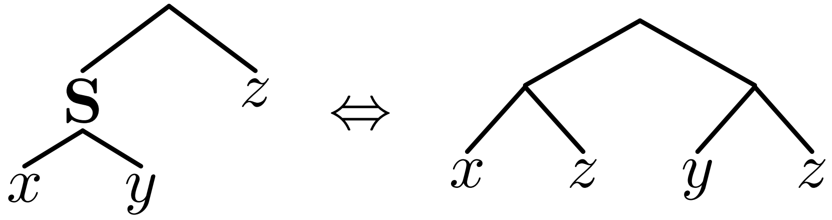

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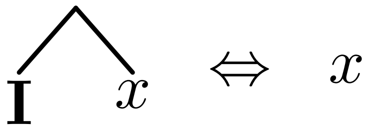

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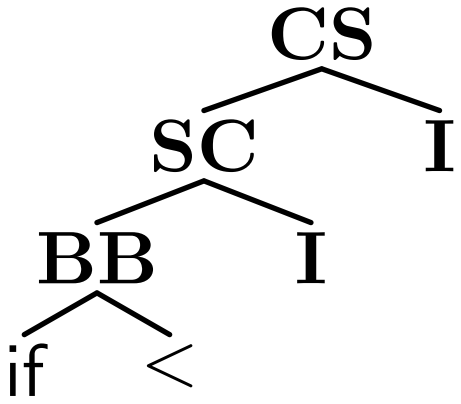

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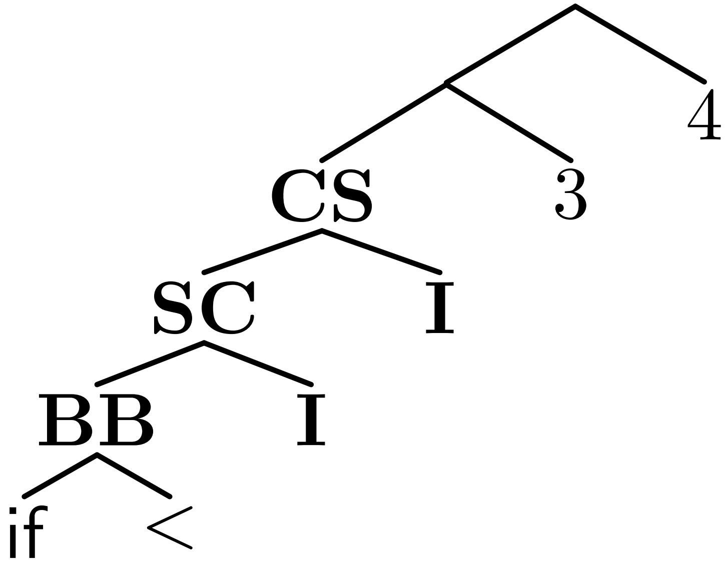

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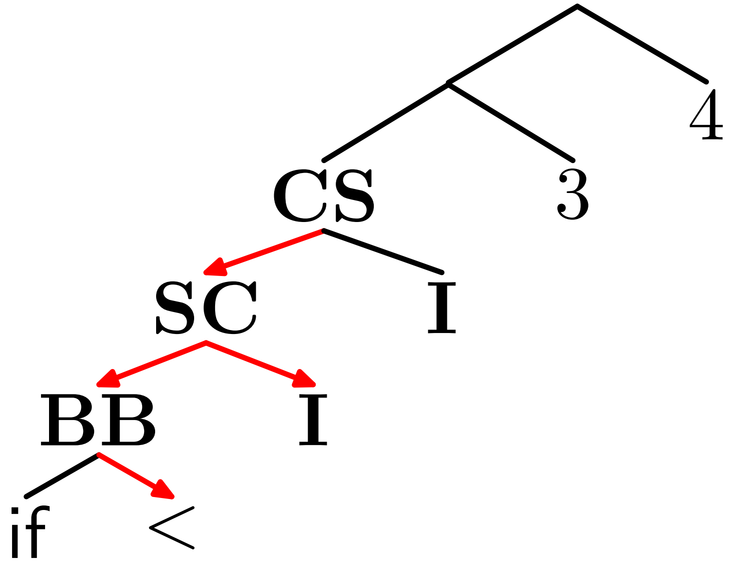

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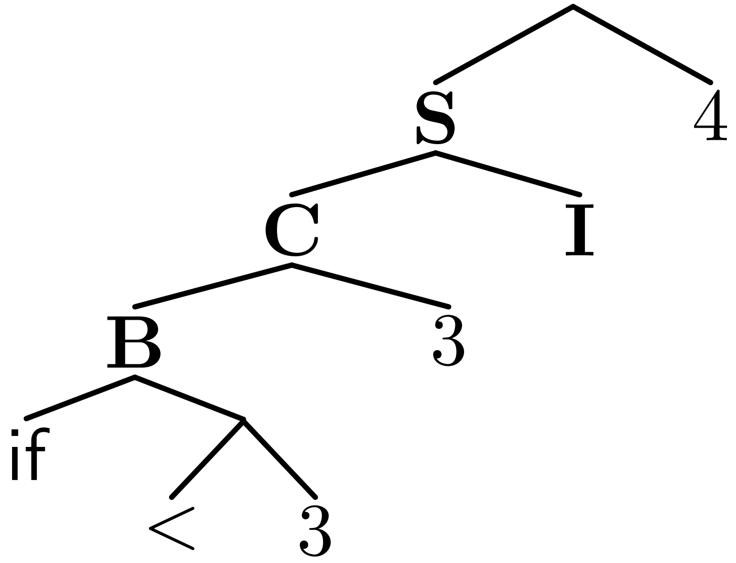

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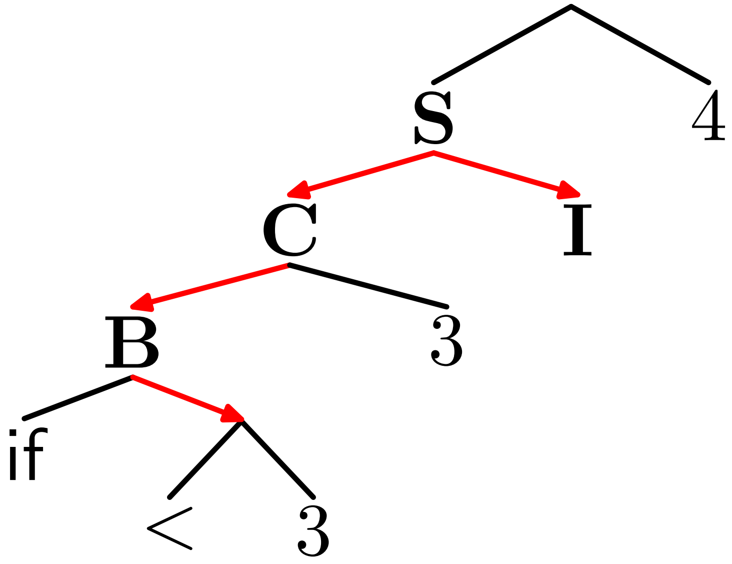

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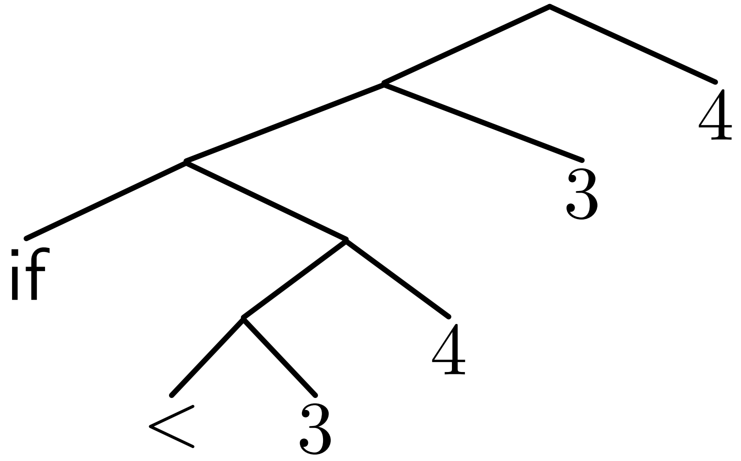

Using Combinators for Refactoring

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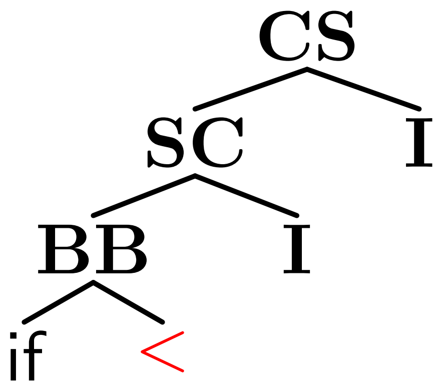
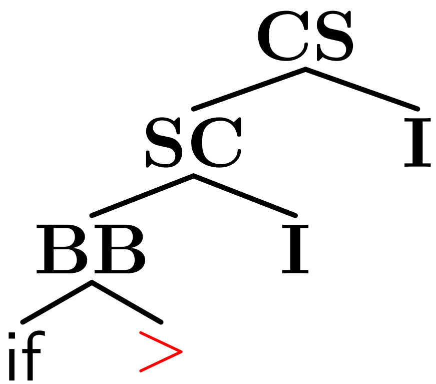


max



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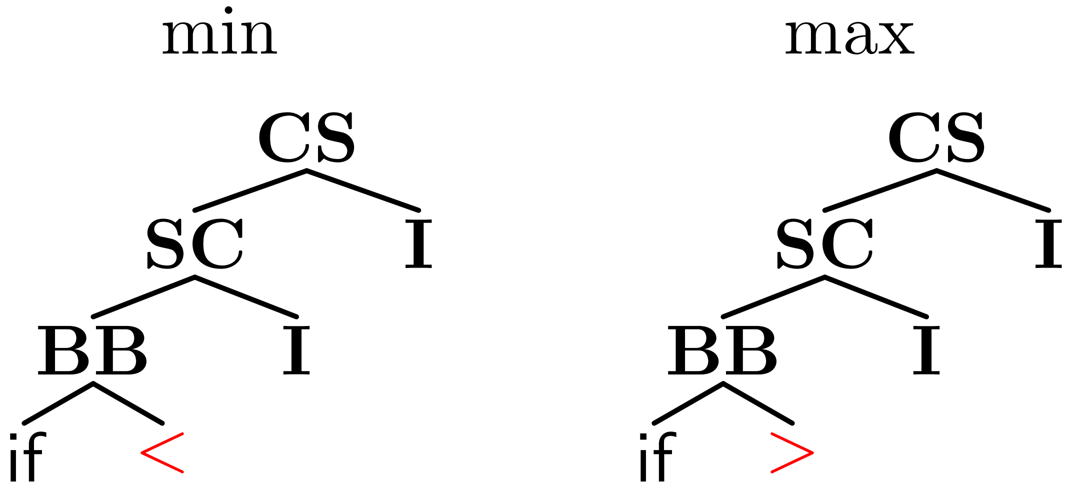

No significant sharing of subtrees (subprograms)

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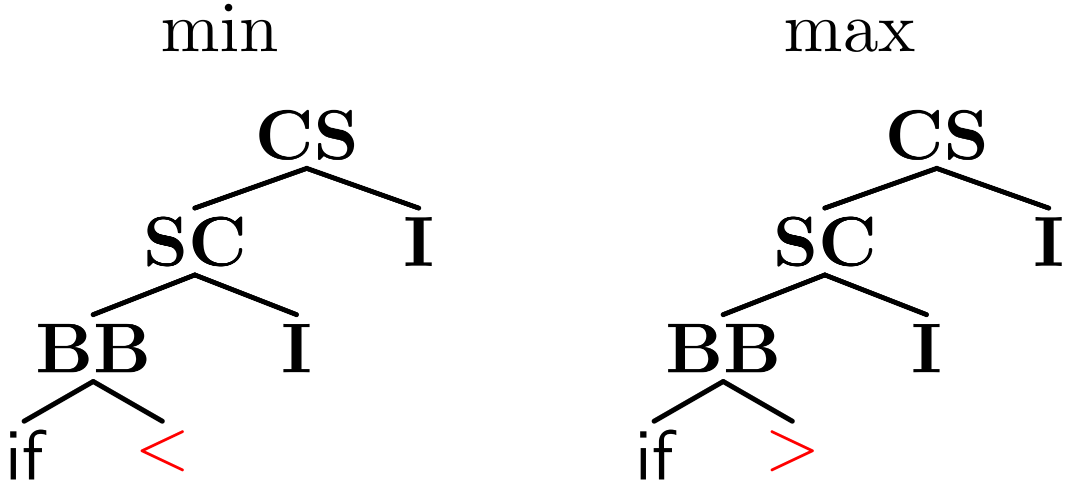

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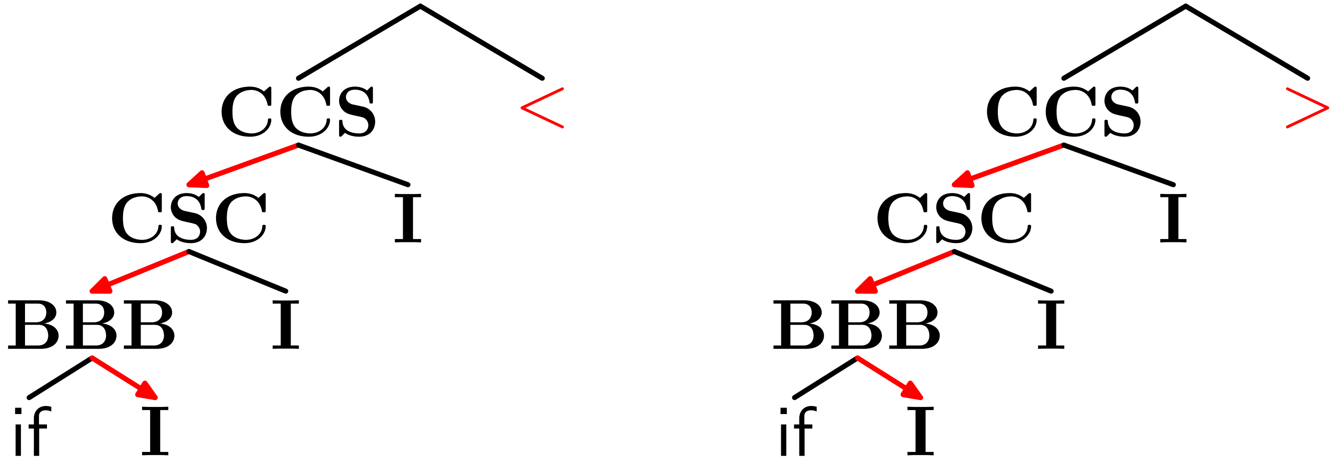
Refactored:



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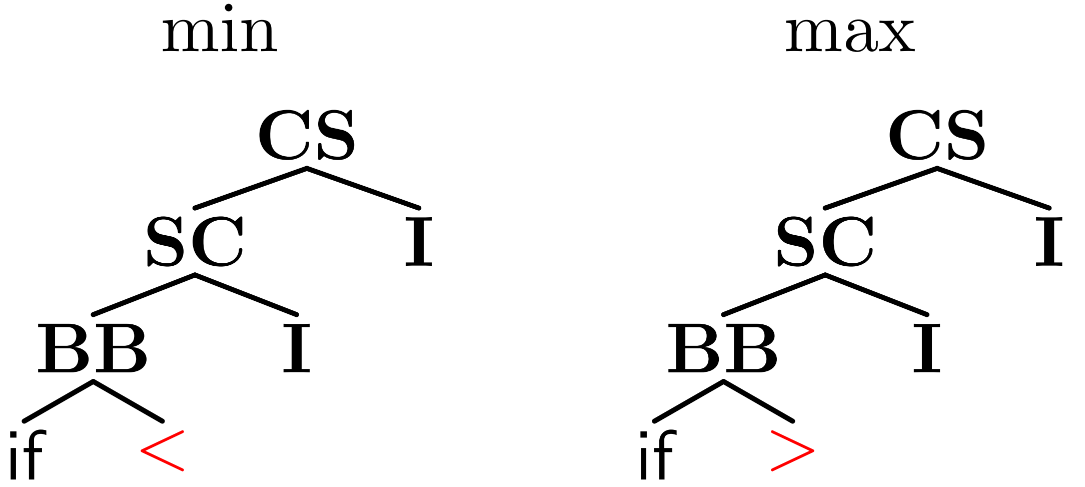

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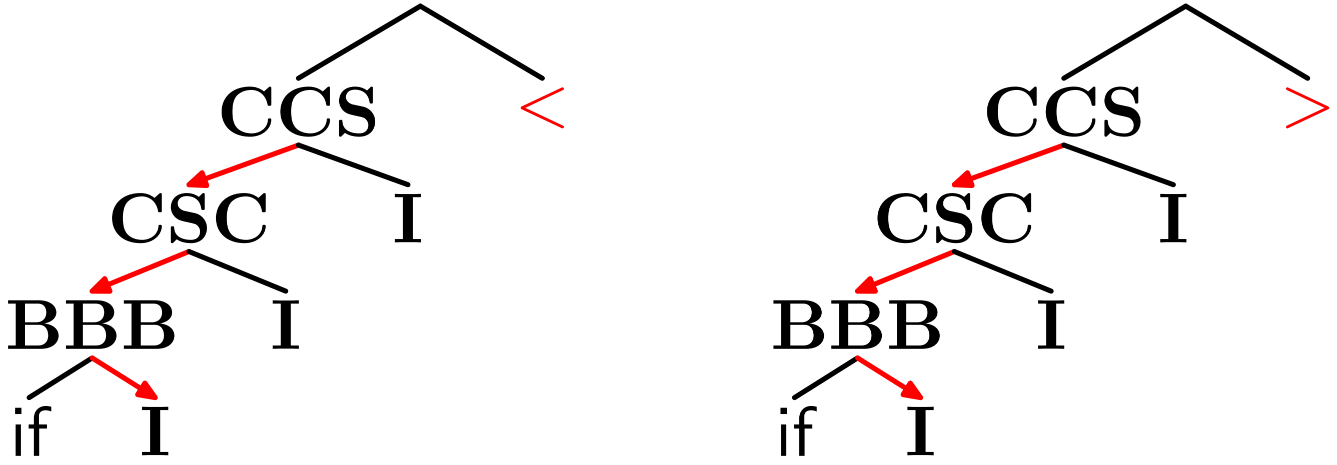

Fruitful sharing of subtrees (subprograms)

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Introduced new combinatory logic basis (intuition: routing)

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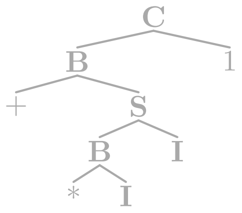
Achieved uniformity: Every subtree is a subprogram

Outline of Proposed Solution

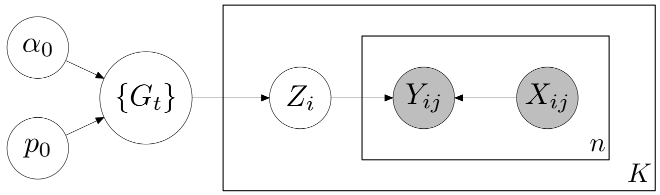
Program representation: What are subprograms?

Combinatory logic



Probabilistic model: Which programs are favorable?

Nonparametric Bayes



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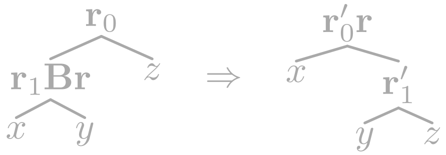
MCMC



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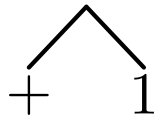
$\textsc{GenIndep}(\text{int} \to \text{int}) \quad \Longrightarrow$



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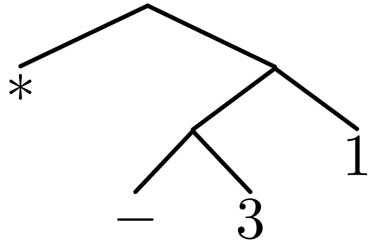

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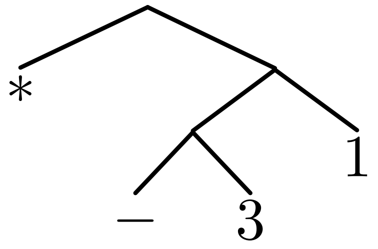

Problem: No encouragement to share subprograms

Adaptor Grammars [Johnson, 2007]

$C_t \leftarrow [\,]$ for each type t [cached list of combinators]

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Interpretation of cache C_t: library of generally useful (unnamed) subroutines which are reused.

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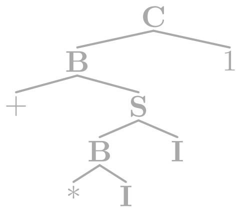

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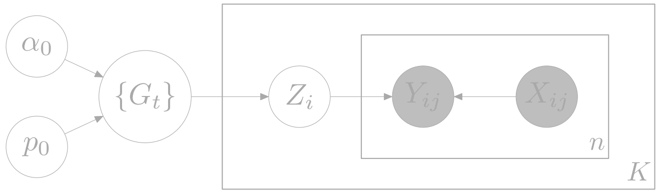

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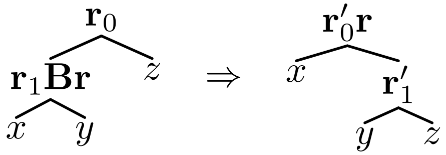

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User provides tree structure that encodes set of programs U

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Two types of transformations:

1. Switching
2. Refactoring

Program transformations (MCMC moves)

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Data: $\{(2, 8)\}$

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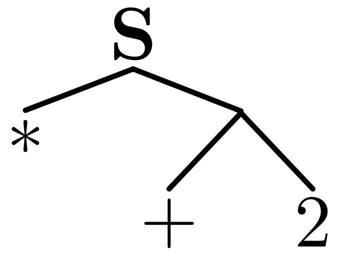
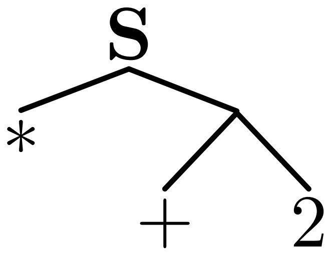


$[x \mapsto x(x+2)]$

Program transformations (MCMC moves)

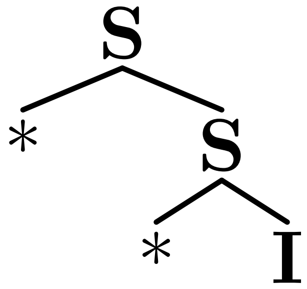
Switching: Change content, preserve empirical semantics

Data: $\{(2, 8)\}$



$[x \mapsto x(x + 2)]$

$\Leftrightarrow$

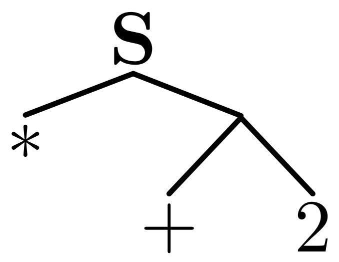


$[x \mapsto x^3]$

Program transformations (MCMC moves)

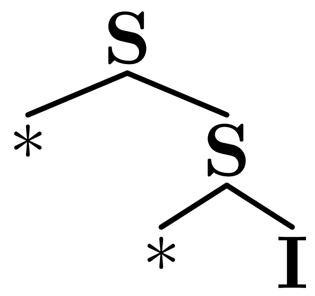
Switching: Change content, preserve empirical semantics

Data: $\{(2,8)\}$



$[x \mapsto x(x+2)]$

$\Leftrightarrow$



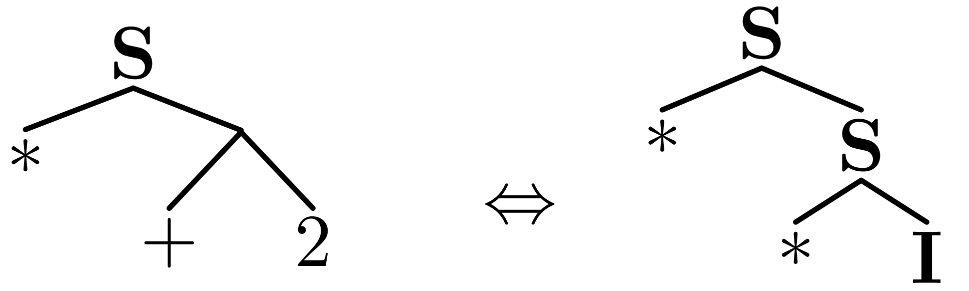
$[x \mapsto x^3]$

Purpose: change generalization

Program transformations (MCMC moves)

Switching: Change content, preserve empirical semantics

Data: $\{(2, 8)\}$



$[x \mapsto x(x+2)]$ $[x \mapsto x^3]$

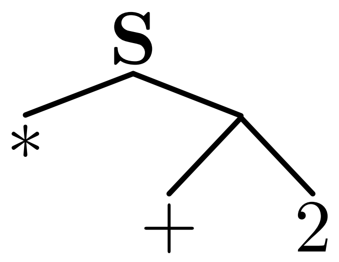
Purpose: change generalization

Refactoring: Change form, preserve total semantics

Program transformations (MCMC moves)

Switching: Change content, preserve empirical semantics

Data: $\{(2, 8)\}$



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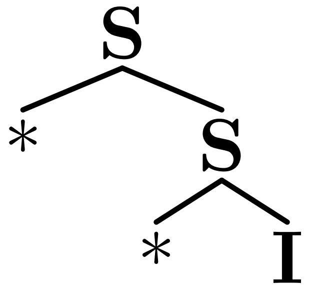

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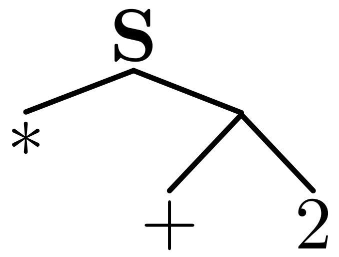

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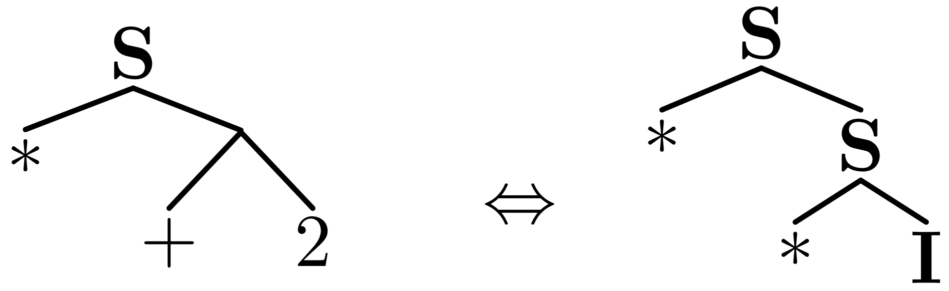

$[x \mapsto x(x+2)]$ $\quad$ $[x \mapsto x^3]$

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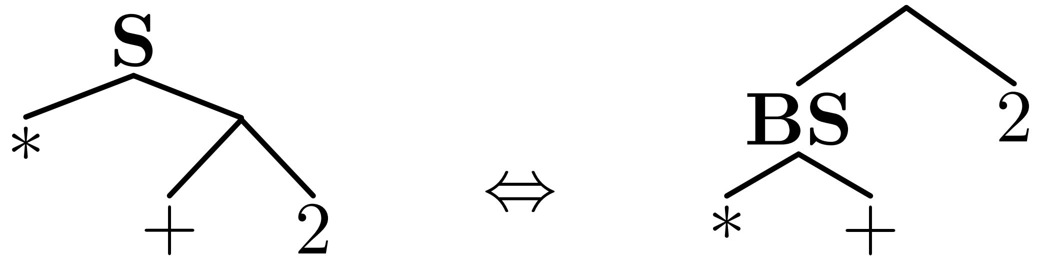

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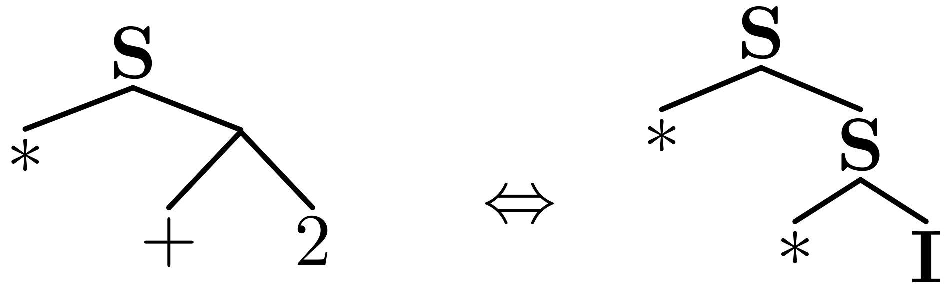

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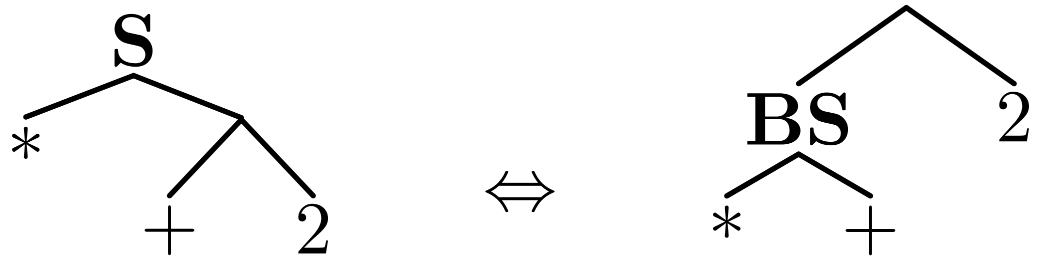

$[x \mapsto x(x+2)]$ $\quad$ $[x \mapsto x(x+2)]$

Purpose: expose different subprograms for sharing

Text Editing Experiments

Setup:

Dataset of [Lau et al., 2003]

$K = 24$ tasks

Each task: train on 2–5 examples, test on $\approxeq 13$ examples

10 random trials

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Dataset of [Lau et al., 2003]
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Example task:

Cardinals 5, Pirates 2.

$$\Downarrow$$

GameScore[winner 'Cardinals'; loser 'Pirates'; scores [5, 2]].

Experimental Results

🟧 Uniform prior

🟥 Independent prior

🟦 Joint prior

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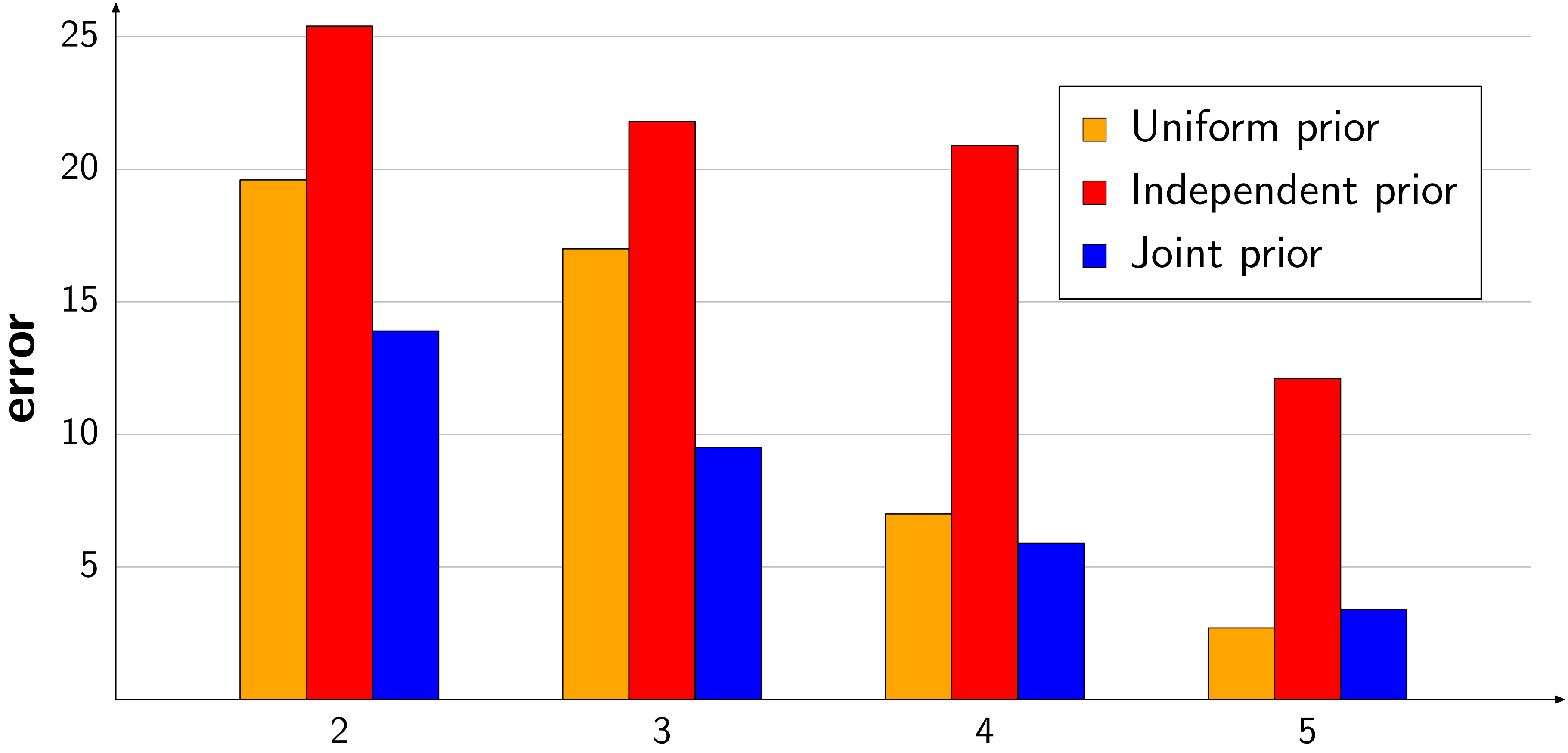

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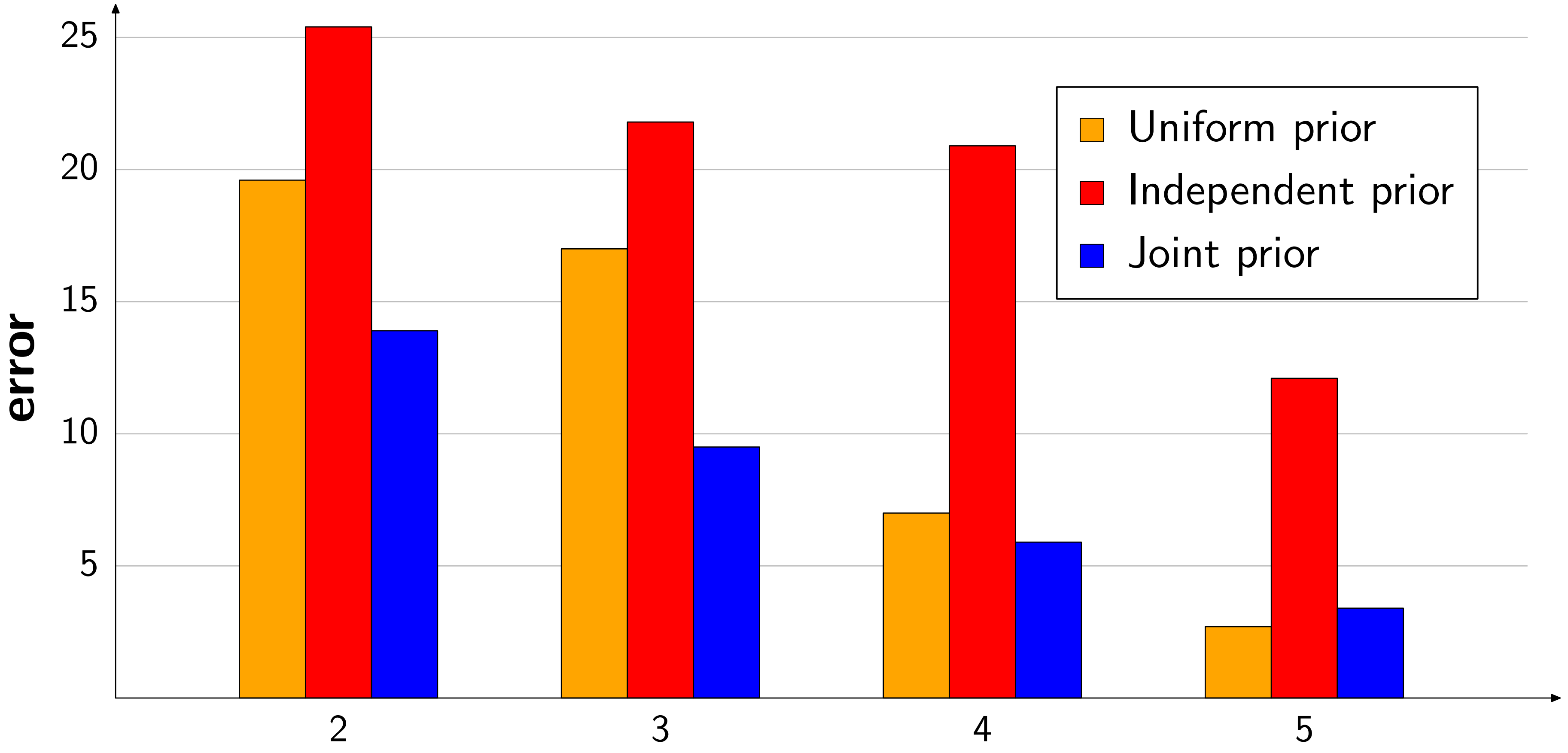

Observations:

- Independent prior is even worse than uniform prior

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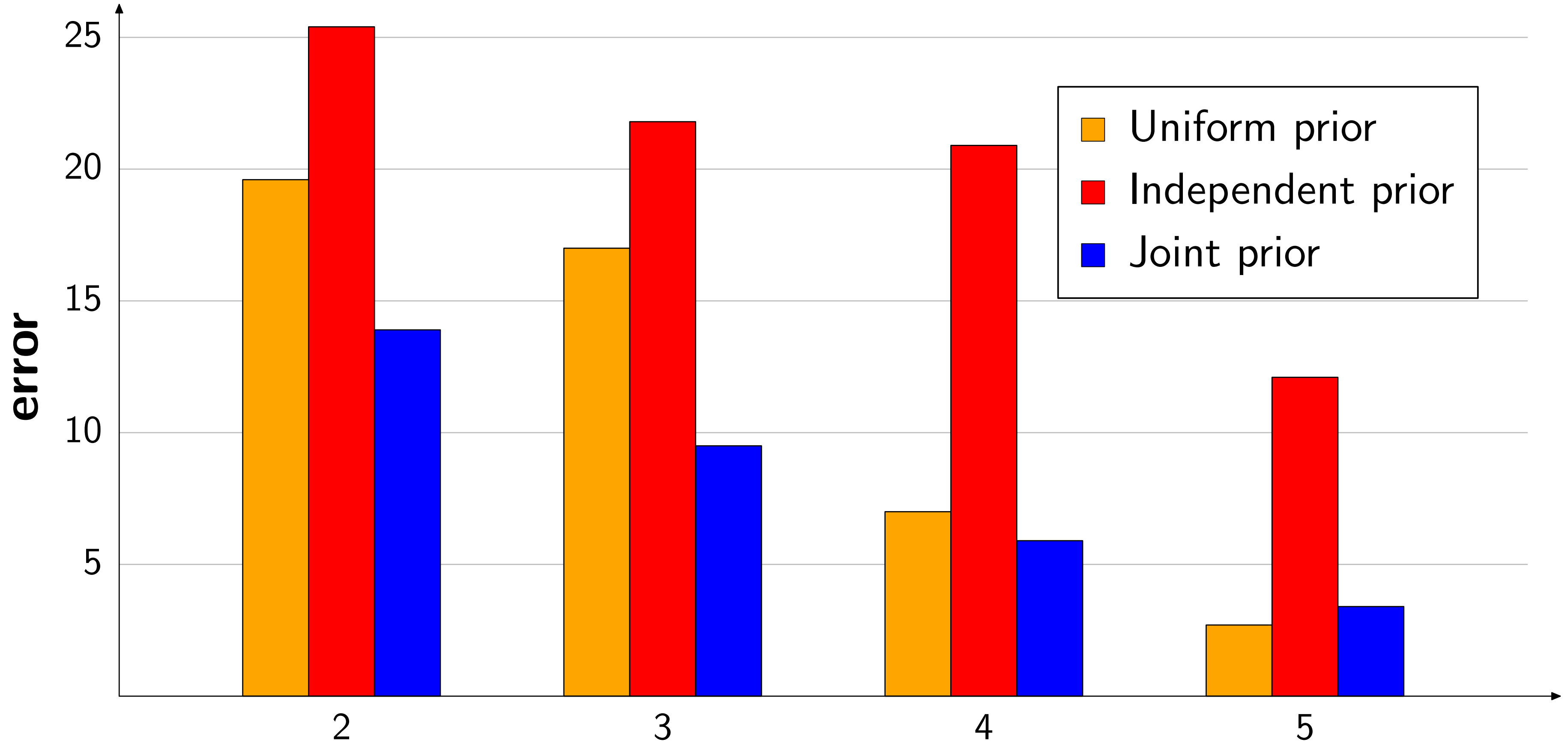

Observations:

- Independent prior is even worse than uniform prior
- Joint prior (multi-task learning) is effective

Summary

$$X \Rightarrow \boxed{} \Rightarrow Y$$

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Key challenge: learn programs from few examples

Main idea: share subprograms across multiple tasks

Tools:

- Combinatory logic: expose subprograms to be shared
- Adaptor grammars: encourage sharing of subprograms
- Metropolis-Hastings: proposals are program transformations