PARSING AND HYPERGRAPHS

Dan Klein and Christopher D. Manning
Computer Science Department
Stanford University
Stanford, CA 94305-9040
{klein, manning}@cs.stanford.edu

Abstract

While symbolic parsers can be viewed as deduction systems, this view is less natural for probabilistic parsers. We present a view of parsing as directed hypergraph analysis which naturally covers both symbolic and probabilistic parsing. We illustrate the approach by showing how a dynamic extension of Dijkstra's algorithm can be used to construct a probabilistic chart parser with an $O(n^3)$ time bound for arbitrary PCFGs, while preserving as much of the flexibility of symbolic chart parsers as allowed by the inherent ordering of probabilistic dependencies.

1 Introduction

An influential view of parsing is as a process of logical deduction, in which a parser is presented as a set of parsing schemata. The grammar rules are the logical axioms, and the question of whether or not a certain category can be constructed over a certain span becomes the question of whether that category can be derived over that span, treating the initial words as starting assumptions (Pereira and Warren 1983, Shieber et al. 1995, Sikkel and Nijholt 1997). But such a viewpoint is less natural when we turn to probabilistic parsers, since probabilities, or, generalizing, scores, are not an organic part of logical systems.[1] There is also a deep connection between logic, in particular propositional satisfiability, and directed hypergraphs (Gallo et al. 1993). In this paper, we develop and exploit the third side of this triangle, directly connecting parsing with directed hypergraph algorithms. The advantage of doing this is that scored arcs *are* a central and well-studied concept of graph theory, and we can exploit existing graph algorithms for probabilistic parsing. We illustrate this by developing a concrete hypergraph-based parsing algorithm, which does probabilistic Viterbi chart parsing over word lattices. Our algorithm offers the same modular flexibility with respect to exploration strategies and grammar encodings as a categorical chart parser, in the same cubic time bound, and in an improved space bound.

2 Hypergraphs and Parsing

First, we introduce directed hypergraphs, and illustrate how general-purpose hypergraph algorithms can be applied to parsing problems.

The basic idea underlying all of this work is rather simple, and is illustrated in figure 1. There is intuitively very little difference between (a) combining subtrees to form a tree, (b) combining hypotheses to form a conclusion, and (c) visiting all tail nodes of a hyperarc before traversing to a head node. We will be building hypergraphs which encode a grammar and an input, and whose paths correspond to parses of that input.

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[1]This is not to say that there is *no* way to incorporate probabilities into a deductive framework, for example by reifying the probabilities.

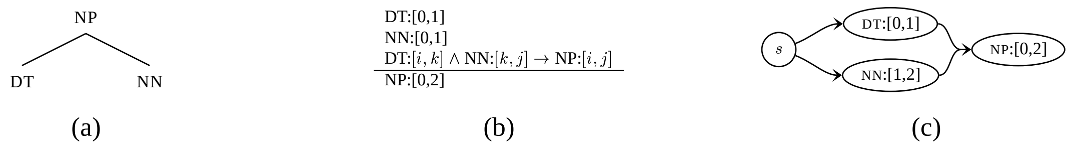


Figure 1: Three views of parsing: (a) tree construction, (b) logical deduction, and (c) hypergraph exploration.

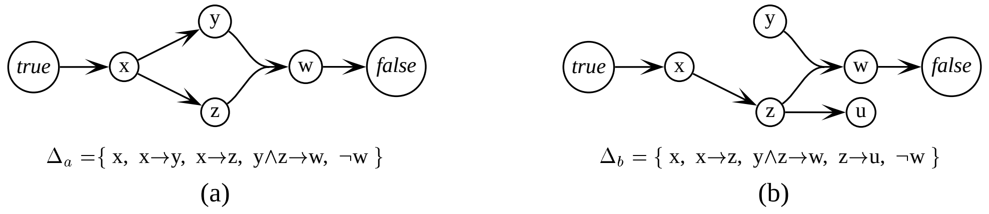

true
x
y
z
w
false
Δ_a ={ x, x→y, x→z, y∧z→w, ¬w }
(a)
true
x
y
z
w
u
false
Δ_b = { x, x→z, y∧z→w, z→u, ¬w }
(b)

Figure 2: Two hypergraphs. Graph (a) is a B-path from *true* to *false*, while (b) is not. Also shown are propositional rule sets to which these graphs correspond. Δ_b is satisfiable, while Δ_a is not.

2.1 Directed Hypergraph Basics

We give some preliminary definitions about directed hypergraphs, objects like in figure 2 , weaving in a correspondence to propositional satisfiability as we go. For a more detailed treatment, see (Gallo et al. 1993).

Directed hypergraphs are much like standard directed graphs. However, while standard arcs connect a single tail node to a single head node, hyperarcs connect a set of tail nodes to a set of head nodes. Often, as in the present work, multiplicity is needed only in the tail. When the head contains exactly one node, we call the hyperarc a *B-arc*.

Definition 1 *A* directed hypergraph $\mathbf{G}$ *is a pair* (N, A) *where* N *is a set of nodes and* A *is a set of directed hyperarcs. A* directed hyperarc *is a pair* (T, H) *where the tail* T *and head* H *are subsets of* N.

Definition 2 *A* B-arc *is a directed hyperarc for which* H *is a singleton set. A* B-graph *is a directed hypergraph where the hyperarcs are B-arcs.*

It is easy to see the construction which provides the link to satisfiability. Nodes correspond to propositions, and directed hyperarcs $\{t_1, \ldots, t_m\} \Rightarrow \{h_1, \ldots, h_n\}$ correspond to rules $t_1 \wedge \cdots \wedge t_m \rightarrow h_1 \vee \cdots \vee h_n$. In the case of B-arcs, the corresponding rules are Horn clauses. The construction also requires two special nodes, *true* and *false*. For the Horn clause case, it turns out that satisfiability is equivalent to the non-existence of a certain kind of path from *true* to *false*.

With the notion of arc generalized, there are multiple kinds of paths. The simplest is the kind inherited from standard graphs, in which a path can enter a hyperarc at any tail node and leave from any head node.

Definition 3 *A* simple path $p = s \rightsquigarrow t$ *is a sequence* $(s = v_0, a_1, v_1, \ldots a_n, v_n = t)$ *of alternating nodes and hyperarcs where: (1) each hyperarc* a_i *is distinct, (2)* $\forall i \in \{0, \ldots, n-1\}, v_i \in tail(a_{i+1})$*, and (3)* $\forall i \in \{1, \ldots, n\}, v_i \in head(a_i)$ *A node* t *is* simply reachable *from a node* s *if a simple path exists from* s *to* t.

In the more important kind of path, each tail node must be reachable before the arc is traversable.

Definition 4 *A* B-path P *in a B-graph* $\mathbf{G}$ *from a node* s *to a node* t *is a minimal subgraph*[2] $(N_P, A_P) < \mathbf{G}$ *in which: (1)* $s, t \in N_P$*, and (2)* $\forall v \in N_P - \{s\}, \exists p = s \rightsquigarrow t$*,* p *a simple path in* P*. A node* t *is* B-reachable *from a node* s *if a B-path exists from* s *to* t.

This fits well with the logical rule interpretation: each hypothesis must be true before a conclusion is implied. It is perhaps not surprising, then, that B-paths from *true* to *false* are what correspond to non-satisfiability.

As an example, the B-graph in figure 2(a) is a valid B-path from *true* to *false*. However, the one in figure 2(b) is not a B-path, for two reasons. First, not all nodes are simply reachable from s (y is not). Second, even

[2]An important point to note is that one cannot choose only part of a hyperarc to include in a subgraph; once the arc is included, all nodes in its head and tail must also be included.

if we added a B-arc $\{x\} \Rightarrow y$, the graph would not be minimal (the B-arc $\{z\} \Rightarrow u$ could be removed). Correspondingly, there is no satisfying assignment to the rule sets in figure 2(a), while $\{x$=*true*, y=*false*, z=*true*, w=*false*, u=*true*$\}$ is a satisfying assignment for figure 2(b).

For the remainder of this paper, we will often drop the "B-" when it is clear from context what kind of graph, arc, path, or reachability is meant.

2.2 Symbolic Parsing and Reachability

We now show how reachability in a certain hypergraph corresponds to parse existence.

In chart parsing terminology, the core declarative object is the *edge*, which is a labeled span. For example, NP:[0,2] represents an NP spanning position 0 to 2. Parsing requires a grammar and an input. Here, we take the input to be a *lattice*, which is a collection of edges stating which words can occur over which spans. The grammar is a set of context-free productions of the form $C \rightarrow X_1 \ldots X_n$. These productions state that edges with labels X_i can be combined to form an edge with label C, subject to adjacency constraints on the edges' spans. When a production is instantiated with specific edges, it is called a *traversal*. Traversals state a particular way an edge can be constructed, for example, that S:[0,8] can be composed of NP:[0,2] and VP:[2,8].

There is an unfortunate clash between chart parsing and hypergraph terminology. A chart (see figure 3c) is typically seen as an undirected graph with numbers as nodes and edges as arcs. Traversals, which record how an edge was constructed, are not part of the graph, but stored in an auxiliary data structure, if at all. However, in the present context, the numbers are not represented graphically (their relative structure is self-evident), edges are nodes in the hypergraph, and traversals are B-arcs in the hypergraph. For this paper, we use "edge" to refer only to labeled spans, and "arc" when we mean a (hyper)graph connection.

Given a grammar G and a lattice L, we wish to construct a hypergraph in which node reachability corresponds to edge parsability. This graph, which we call the *induced B-graph of* G *and* L, is given as follows. For each instantiation of category C in G as an edge C:$[i, j]$, create a node. For each instantiation of a production $C \rightarrow X_1 \ldots X_n$ in G as a traversal C:$[i, j] \rightarrow X_1$:$[i, k_1]$ X_2:$[k_1, k_2]$ $\ldots$ X_n:$[k_{n-1}, j]$, create a B-arc $\{X_1$:$[i, k_1],\ X_2$:$[k_1, k_2],\ \ldots X_n$:$[k_{n-1}, j]\} \Rightarrow C$:$[i, j]$. This much of the construction represents the connectivity of the grammar. To represent the data, create a special source edge s, and add arcs of the form $\{s\} \Rightarrow C$:$[i, j]$ for each word edge C:$[i, j]$ and $\{s\} \Rightarrow C$:$[i, i]$ for each category C with an empty production.

Similarly, we define a mapping π which takes parse trees T to B-graphs $\pi(T)$, where the nodes of $\pi(T)$ are the edges of T (along with s), and the B-arcs in $\pi(T)$ are the traversals of T (along with s arcs to terminals in T). For example, figure 1(c) is the image of figure 1(a). For any tree T, $\pi(T)$ is not only a B-graph, but a B-path from s to the root edge of T.

For a given G and L, this mapping is onto the set of B-paths in the induced graph with source s: any B-path from s is $\pi(T)$ for some tree T which can be constructed over L using G. It is not necessarily one-to-one, because of cyclic same-span constructions.[3] However, this does not matter for determining parse *existence*; it is enough that the inverse image of a B-path be non-empty.

The reduction between parse existence and hypergraph reachability is expressed by the following theorem.

Theorem 1 *For a grammar G and a lattice L, a node e in the induced B-graph is B-reachable from s iff a parse of the edge e exists. Each parse T of e corresponds to a particular B-path $\pi(T)$ from s to e, and for each B-path P, there is a unique canonical tree $\pi^{-1}(P)$ in which no edge dominates itself.*

For instance, in figure 3(b), the NP node is B-reachable, but the PP node is not (because IN:[1,2] is not). Thus, over the span [0,2], an NP can be parsed, while a PP cannot.

Therefore, if we wish to know if some edge e can be parsed over L, we can construct the induced graph and use any B-reachability algorithm to ask whether e is reachable from s. For example, Gallo et al. (1993) describe an algorithm which generalizes depth-first search, and which runs in time linear in the size of the graph. Moreover, since π is easy to invert, any B-path produced can be turned into a concrete parse of e.

[3] It will be one-to-one if the grammar contains neither empty nor unary productions; not much is lost if this case is used for intuition.

2.3 Viterbi Parsing and Shortest Paths

A more complex problem than parse existence is the problem of discovering a best, or Viterbi, parse for an edge e, where "best" is given by some scoring function over trees. For the present work, we assume that the scoring function takes a particular form. Namely, for the proofs to go through, it must be that combining trees (1) cannot yield a score better than the best scored component, and (2) replacing a subtree with a lower scoring subtree will worsen the score of any containing tree. These assumptions are acheived if each production (and category) is associated with an element of a $\preceq_S$-ordered c-semiring[4] (Bistarelli et al. 1997), and a tree's score is the semiring product of its production (and node) scores. In particular, maximizing multiplied production probabilities, minimizing the number of tree nodes, and many other scoring functions are of this form.

In the case of such scoring functions, the same induced graph used for parse existence can be used for finding a best parse. We simply score each arc with the score of the local tree to which it corresponds. The score of a B-path[5] is then the score of at least one parse which maps to that B-path. Furthermore, the canonical parse mentioned in theorem 1 for that B-path is a best parse and has the same score as its B-graph.

Thus, any algorithm for finding shortest (or, more generally, best) paths in B-graphs can be used to find best parses, using the construction and mapping above. For example, Gallo et al. (1993) describe an extension of Dijkstra's algorithm to B-graphs, which runs in time linear in the size of the graph.[6]

2.4 Practical Issues

At this point, one might wonder what is left to be done. We have a reduction which, given a grammar G and a lattice L, allows us to build and score the induced graph. From this graph, we can use reachability algorithms to decide parse existence, and we can use shortest-path algorithms to find best parses. Furthermore, this view can be extended to other problems of parsing. For example, algorithms for summing paths can be adapted to calculate inside probabilities (see Klein and Manning (2001a)).

However, there are two primary issues which remain. First, there is the issue of efficiency. Reachability and shortest-path algorithms, such as those cited above, generally run in time linear in the size of the induced graph. However, the size of the induced graph, while polynomial in the size of the lattice L, is exponential in the arity of the grammar G, having a term of $|L|^{arity(G)+1}$ in its size. The implicit binarization of the grammar done by chart parsers is responsible for their cubic bounds, and we wish to preserve this bound for our Viterbi parsing.

Second, one does not, in general, wish to construct the entire induced graph in advance, or even at all. Rather, one would like to dynamically create only the portions which are needed, as they are needed. Various factors which can affect what regions of the graph are built at what times include:

- Structural search strategies, such as bottom-up, top-down, left-corner, and so on.
- Lattice scanning strategies, such as scanning the lattice from left to right, or in whatever order it becomes available from previous processing.
- Rule encodings. Practical grammars are often encoded in a variety of ways, such as tries or fully minimized DFSAs (Klein and Manning 2001b), rather than simply as linear rewrite rules as in theoretical presentations.

Therefore, in section 3, we present a chart parser for arbitrary PCFGs which can be seen as dynamically constructing the reachable regions of an induced graph and doing a Dijkstra's algorithm style shortest-path computation over it. This parser preserves the time bounds of categorical chart parsers and allows a variety of introduction strategies and rule encodings. We discuss the kinds of subtle errors that can arise in a naive implementation and present simple conditions that ensure the correctness of various parsing strategies.

[4]These semirings have been used in work on soft constraint satisfaction, hence the "c-" prefix. They are semirings $\langle A, \oplus, \otimes, 0, 1\rangle$ where $\oplus$ is idempotent and $1 \oplus x = 1$ for all x. The order $\preceq_S$ required is that $a \preceq_S b$ iff $a \oplus b = b$. $\langle [0, 1], \max, \times, 0, 1\rangle$ is an example.

[5]The score of a node in a B-path is defined recursively as the semiring product of the score of the arc entering that node with the scores of the nodes in that arc's tail. The source has score 1 (the maximum score in the semiring).

[6]The choice of algorithm may impact the permitted generality of the scoring function. The algorithm in (Gallo et al. 1993) actually does work for all $\preceq_S$-ordered c-semiring scoring functions, though their presentation does not state this; one must also note that their score addition corresponds to the semiring multiplication. In any case, the algorithm in section 3 is entirely self-contained.

2.5 Relation to Superior Grammars

Knuth (1977) introduces a formalism of *superior grammars*, where the terminals are superior functions, which calculate a score for a rule as a function of the scores of nonterminals on the righthand side. The formalism is closely related to the above hypergraph formalism, and can also be seen as a generalization of PCFGs. He also presents a generalization of Dijsktra's algorithm for the problem of finding the best cost string in the language defined by such a grammar (there is no concept of comparing parses for a single string). Rather than constructing B-graphs, we could cast the present work in terms of superior grammars, building a large grammar isomorphic to our B-graph, with a non-terminal for each edge and a terminal for each lattice element. Knuth's superiority criterion would then be used in place of the $\preceq_S$-ordered c-semiring property.[7] When the superior functions are uniform across productions, superior grammars reduce to c-semiring scored B-graphs. The choice of which formalism to base our work on is thus more aesthetic than substantive, but we believe that the hypergraph presentation allows easier access to a greater variety of algorithmic tools, and presents a clearer, more visually appealing intuition. At any rate, the practical issues described above, and their solutions, which form the bulk of this paper, would be unchanged under either framework.

3 Viterbi Parsing Algorithm

Agenda-based active chart parsing (Kay 1980, Pereira and Shieber 1987) is an attractive presentation of the central ideas of tabular methods for CFG parsing. Earley (1970)-style dotted items combine via deduction steps ("the fundamental rule") in an order-independent manner, such that the same basic algorithm supports top-down, bottom-up, and left-corner parsing, and the parser deals naturally and correctly with the difficult cases of left-recursive rules, empty elements, and unary rules.

However, while $O(n^3)$ methods for parsing PCFGs are well known (Baker 1979, Jelinek et al. 1992, Stolcke 1995), a $O(n^3)$ probabilistic parser corresponding to active chart parsing for categorical CFGs, has not yet been provided. Producing a probabilistic version of an agenda-driven chart parser is not trivial. A central idea of such parsers is that the algorithm is correct and complete regardless of the order in which items on the agenda are processed. Achieving this is straightforward for categorical parsers, but problematic for probabilistic parsers. For example, consider extending an active edge VP$\rightarrow$V.NP PP:[1,2] with an NP:[2,5] to form an edge VP$\rightarrow$V NP.PP over [1,5]. In a categorical chart parser (CP), we can assert the *parsability* of this edge as soon as both component edges are built. Any NP edge will do; it need not be a best NP over that span. However, if we wish to score edges as we go along, there is a problem. In a Viterbi chart parser, if we later find a better way to form the NP, we will have to update not only the score of that NP, but also the score of any edge whose current score depends on that NP's score. This can potentially lead to an extremely inefficient upward propagation of scores every time a new traversal is explored.[8]

Most exhaustive PCFG parsing work has used the bottom-up CKY algorithm (Kasami 1965, Younger 1967) with Chomsky Normal Form (CNF) Grammars (Baker 1979, Jelinek et al. 1992) or extended CKY parsers that work with n-ary branching grammars, but still not with empty constituents (Kupiec 1991, Chappelier and Rajman 1998). Such bottom-up parsers straightforwardly avoid the above problem, by always building all edges over shorter spans before building edges over longer spans which make use of them. However, such methods do not allow top-down grammar filtering, and often do not handle empty elements, cyclic unary productions, or n-ary rules. Stolcke (1995) presents a top-down parser for arbitrary PCFGs, which incorporates elements of

[7]This has the theoretical – but not clearly useful – advantage of allowing the score combination function to vary per production.

[8]Goodman (1998) provides an insightful presentation unifying many categorical and probabilistic parsing algorithms in terms of the problem's semiring structure, but he merely notes this problem (p. 172), and on this basis puts probabilistic agenda-based chart parsers aside. The agenda-based chart parser of Caraballo and Charniak (1998) (used for determining inside probabilities) suffers from exactly this problem: In Appendix A (p. 293), they note that such updates "can be quite expensive in terms of CPU time", but merely suggest a method of thresholding which delays probability propagation until the amount of unpropagated probability mass has become significant, and suggest that this thresholding allows them to keep the performance of the parser "as $O(n^3)$ empirically." We do not present an inside probability algorithm here, but the hypergraphical view of parsing can be developed to give an inside parsing algorithm, as discussed in (Klein and Manning 2001a).

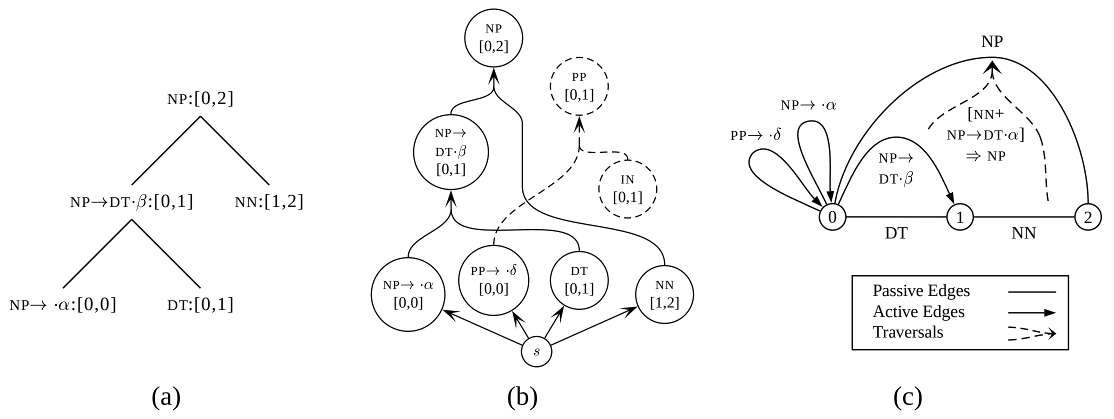


Figure 3: Various representations of a parse: (a) a binary tree of chart edges, (b) a path in an induced hypergraph, and (c) a collection of edges and traversals entered into a standard chart.

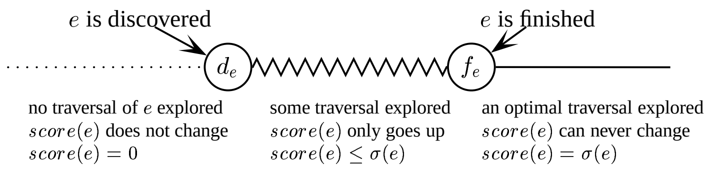
the control strategies of Earley's (1970) parser and Graham et al.'s (1980) parser. Stolcke provides a correct and efficient solution for parsing arbitrary PCFGs, avoiding the problem of left-recursive predictions and unary rule completions through the use of precomputed matrices giving values for the closure of these operations. However, the add-ons for grammars with such rules make the resulting parser rather complex, and again we have a method only for a single parsing regimen, rather than a general tabular parsing framework.

In the current hypergraphical context, we can interpret these effects as follows. A CP is performing a single-source reachability search. Any path from the source is as good as any other for reachability, and the various processing orders all eventually explore the entire region of the graph which is accessible given the rule introduction strategy (and goal). However, once scores are introduced, one cannot simply explore traversals in an arbitrary order, just as how, in relaxation-based shortest path algorithms, one cannot relax arcs in an arbitrary order. CKY parsers ensure a correct exploration order by exploring an entire tier of the graph before moving on to the next. For CNF grammars, all parses of the same string have the same number of productions in them, and so this tiering strategy works. However, in general, we will have to follow the insight behind Dijkstra's algorithm: always explore the current best candidate, leaving the others on a queue until later.

3.1 The Algorithm

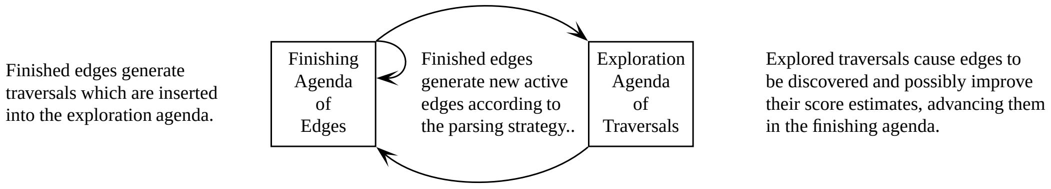
Our algorithm has many of the same data structures of a standard CP. The fundamental data structure is the *chart*, which is composed of numbered *vertices* placed between words, edges, and traversals (see figure 3(c)). Unlike in the general presentation above, there are two kinds of edges, active and passive. *Passive edges* are identified by a span and a category, such as NP:[2,5], and represent that there is some parse of that category over the span. *Active edges* are identified by a span and a grammar state, such as VP→V.NP PP:[1,2], and indicate that that grammar state is reachable over that span. In the case where grammar rules are encoded as lists, this state is simply an Earley-style dotted rule, and to reach it one must have been able to parse the sequence of categories to the left of the dot. However, grammar rules can be compacted in various ways, and so the label of the active edges for this parser is in general a deterministic finite state automaton (DFSA) state. List rules denote particularly simple, linear DFSAs, whereas trie DFSAs are equivalent to left-factoring the grammar. The "fundamental rule" states that new edges are produced by combining active edges with compatible passive edges, advancing the active edge. For example, the two edges described above can combine to form the active edge VP→V NP.PP:[1,5]. This information is recorded in a traversal, which, due to the active/passive binarization, is simply an (active edge, passive edge, result edge) triple.[9] As each edge can potentially be formed by many different traversals, this distinction between an edge and a traversal of an edge is crucial to parsing efficiently (but often lost in pedagogical presentations: e.g., Gazdar and Mellish (1989)).

[9]The result edge is primarily to simplify proofs and pseudocode; it need not ever be stored in a traversal's coded representation.



e is discovered
e is finished
d_e
f_e
no traversal of e explored
$score(e)$ does not change
$score(e) = 0$
some traversal explored
$score(e)$ only goes up
$score(e) \leq \sigma(e)$
an optimal traversal explored
$score(e)$ can never change
$score(e) = \sigma(e)$

Figure 4: The life cycle of an edge e



Finished edges generate traversals which are inserted into the exploration agenda.
Finishing Agenda of Edges
Finished edges generate new active edges according to the parsing strategy..
Exploration Agenda of Traversals
Explored traversals cause edges to be discovered and possibly improve their score estimates, advancing them in the finishing agenda.

Figure 5: The core loop of the parser

The core cycle of a CP is to process traversals into edges and to combine new edges with existing edges to create new traversals. Edges which are not formed from other edges via traversals (for example, the terminal edges in figure 3(a)) are *introduction edges*. *Passive introduction edges* are words from the lattice and are often all introduced during initialization. *Active introduction edges* are the initial states of rules and are introduced in accordance with the grammar strategy (top-down, bottom-up, etc.). To hold the traversals or edges which have not yet been processed, a CP has a data structure called an *agenda*, which holds both traversals and introduction edges. Items from this agenda can be processed in any order whatsoever, even arbitrarily or randomly, without affecting the final chart contents.

In our probabilistic chart parser (PCP), the central data structures are augmented with scores. Grammar rules, which were previously encoded as symbolic DFSAs are scored DFSAs, as in Mohri (1997), with a score for entering the initial state, a score on each transition, and, for each accepting state, a score for accepting in that state. Each edge e is also scored at all times. This value, $score(e)$ (or $score(e, t)$ at a time t), is the best estimate to date of that edge's true best score, $\sigma(e)$. In our algorithm, the estimate will always be conservative: $score(e)$ will always be worse than or equal to $\sigma(e)$.

The full algorithm is shown in pseudocode in figure 6. It is broadly similar to a standard categorical chart parsing algorithm. However, in order to solve the problem of entering edges into the chart before their correct score is known, we have a more articulated edge life cycle (shown in figure 4).[10] We crucially distinguish edge *discovery* from edge *finishing*. A non-introduction edge is discovered the first time we explore a traversal which forms that edge (in exploreTraversal). An introduction edge is discovered at a time which depends on our parsing strategy (during initialize or another edge's finishEdge). Discovery is the point when we know that the edge *can* be parsed. An edge is finished when it is inserted into the chart and acted upon (in finishEdge). The primary significance of an edge's finishing time is that, as we will show, our algorithm maintains the Dijkstra's algorithm property that when an edge is finished, it is correctly scored, i.e., $score(e) = \sigma(e)$.

A CP stores all outstanding computation tasks in a single agenda, whether the tasks are unexplored traversals or uninserted introduction edges. We have two agendas and stronger typing. To store edges which have been discovered but not yet finished, we have a *finishing agenda*. To store traversals which have been generated but not explored, we have an *exploration agenda*.

The algorithm works as follows. During initialization, all passive introduction edges (one per word in the lattice) are discovered, along with any initial active edges (for example, all S$\rightarrow.\alpha$:[0,0] edges if we are using a top-down strategy and S:[0,n] is the goal edge).[11] Passive introduction edges get their initial scores from the lattice, while active introduction edges get their initial scores from the grammar (often all are simply given the

[10]Note that the comments in the figure apply only to non-introduction edges, but the timeline applies to all edges.
[11]Other word introduction strategies are possible, such as scanning the words incrementally in an outer loop from left-to-right whenever the finishing agenda is empty. A sufficient constraint on scanning strategies is presented in section 4.

```
parse(Lattice sentence, Edge goal)
  initialize(sentence, goal)
  while finishingAgenda is non-empty
    while explorationAgenda is non-empty
      get a traversal $t$ from the explorationAgenda
      exploreTraversal($t$)
    get a best edge $e$ from the finishingAgenda
    finishEdge($e$)

initialize(Lattice sentence, Edge goal)
  create a new chart and new agendas
  for each word w:[start,end] in the sentence
    discoverEdge(w:[start,end])
  for each vertex $x$ in the sentence
    if allow-empties
      discoverEdge(empty:[$x$,$x$])
  doRuleInitialization(goal)

exploreTraversal(Traversal $t$)
  $e$ = $t$.result
  if notYetDiscovered($e$)
    discoverEdge($e$)
  relaxEdge($e$, $t$)

relaxEdge(Edge $e$, Traversal $t$)
  newScore = combineScores($t$)
  if (newScore is better than $e$.score)
    $e$.score = $t$.score
    $e$.bestTraversal = $t$

discoverEdge(Edge $e$)
  add $e$ to the finishingAgenda

finishEdge(Edge $e$)
  add $e$ to the chart
  doFundamentalRule($e$)
  doRuleIntroduction($e$)

doFundamentalRule(Edge $e$)
  if $e$ is passive
    for all active edges $a$ which end at $e$.start
      for active and/or passive result edges $r$
        create the traversal $t$ = ($a$, $e$, $r$)
        add $t$ to the explorationAgenda
  if $e$ is active
    for all passive edges $p$ which start at $e$.end
      for active and/or passive result edges $r$
        create the traversal $t$ = ($e$, $p$, $r$)
        add $t$ to the explorationAgenda

doRuleIntroduction(Edge $e$)
  if top-down and $e$ is active
      for all categories $c$ that can follow $e$.label
        for all intro active edges $a$ at $e$.end with LHS $c$
          if notDiscovered($a$) then discoverEdge($a$)
  if bottom-up and $e$ is passive
      for all categories $c$ with a RHS beginning with $e$.label
        $a$ = $c$:[$e$.start, $e$.start]
        if notDiscovered($a$) then discoverEdge($a$)

doRuleInitialization(Edge goal)
  if top-down
    for all intro active edges $a$ at goal.start with LHS goal.label
      discoverEdge($a$)
```

Figure 6: Pseudocode for our probabilistic chart parser

maximum score). Introduction edges are correctly scored at discovery and their scores never change afterwards.

The core loop of the algorithm is shown in figure 5. If there are any traversals to explore, a traversal t is removed from the exploration agenda and processed with exploreTraversal. Any removal order is allowed. In exploreTraversal, t's result edge e is calculated. If e is an undiscovered edge, then it becomes discovered (and given the minimum score). In any case, e's score is checked against t (relaxEdge). If t forms e with a better score than previously known for e, e's score (and best traversal) is updated.

If the exploration agenda is empty, the finishing agenda is checked. If it is non-empty, the edge with the best current score estimate is finished – removed and processed with finishEdge. This is the point at which the fundamental rule is applied (doFundamentalRule) and new active edges are introduced (in accordance with the active edge introduction strategy).[12]

4 Analysis

We outline the completeness of the algorithm: that it will discover and finish all edges and traversals which the grammar, goal, and words present allow. Then we argue correctness: that every edge e which is finished is, at its finishing time, assigned the correct score. Finally, we give tight worst-case bounds on the time and memory usage of the algorithm.

[12]In the application of the fundamental rule, an (active, passive) pair can potentially create two traversals. In categorical DFSA chart parsing, edges may be active, passive, or both. However, the passive and active versions of what would have been a single active/passive edge in a categorical parser will not in general have the same score, because the passive one is assessed an acceptance cost, and so the algorithm introduces separate edges.

4.1 Completeness

For space reasons, we simply sketch a proof of the reduction of the completeness of our PCP to the known completeness of a CP. We state this reduction rather than prove completeness directly in order to stress the parallelism between the two parser types. To argue completeness for a variety of word and rule introduction strategies, it is important to have a concrete notion of what such strategies are. Constraints on the word introduction strategy are only needed for correctness, and so we defer discussion until then. Let E be the set of edges, P the set of passive introduction edges (i.e., word edges), and A the set of active introduction edges (i.e., rule introductions).

Definition 5 *An* edge-driven *rule introduction strategy is a mapping $R\colon E \rightarrow 2^A$ which takes an edge e to a set A_e of active introduction edges which are to be immediately discovered when e is finished.*

The standard top-down and bottom-up strategies are both edge-driven.[13]

Theorem 2 *For any edge-driven active edge introduction strategy R, any DFSA grammar G, and any input lattice L, and goal edge g, there exists some agenda selection function S for which the sequence of edge insertions I made by a categorical chart parser and the sequence of edge finishings F made by our probabilistic chart parser are the same.*

The proof is by simulation. We run the two parsers in parallel, showing by induction over corresponding points in their execution that $I = F$ and that every edge in the PCP's finishing agenda is backed by some edge or traversal from the CP's agenda. The selection function is chosen to make the CP process agenda items which will cause the insertion of whichever edge the PCP will next select from its finishing agenda.

The completeness reduction means that the edges found (i.e., *finished*) by both parsers will be the same. From the known completeness of a CP under weaker conditions (Kay 1980), this means that the PCP will find every edge which has a parse allowed by the grammar, words, and goal, *not* that it will score them correctly.

4.2 Correctness

We now show that any edge which is finished is correctly scored when it is finished.

First, we need some terminology about traversal trees. A traversal tree T is a binary tree of edge tokens, as in figure 3(a). A leaf in this tree is a token of an introduction edge, either a word (if passive) or a rule introduction (if active). A non-leaf x is a token of a non-introduction edge and has two children, a and p, which are tokens of an active edge and a passive edge, respectively, forming a traversal token (a, p, x). The reason we must make a type/token distinction is that a given edge or traversal may appear more than once in a traversal tree. For example, consider empty words which may be used several times over the same zero-span, or an introduction active edge for a left-recursive rule. We use $type(x)$ to denote the type of an edge token x.

The basic idea is to avoid finishing incorrectly scored edges by always finishing the highest-scored edge available. This will cause us to work in an inside-outwards fashion when necessary to ensure that score propagation is never needed. The chief difficulties therefore occur when what should have been a high-scoring edge is unavailable for some reason. A subtle way this can occur is if an introduction edge is discovered too late. If this happens, we may have already mistakenly finished some other edge, assigning it the best score that it could have had *without* that introduction edge's presence in the grammar or input. Therefore, we need tighter constraints on word and rule introduction strategies to prove correctness than those needed for completeness.

The condition on word introduction is simple.

Definition 6 *The* Word Introduction Condition *(or "no internal insertion"): Whenever an edge e with span S is finished at time f_e, all words (passive introduction edges) contained in S must have been discovered at f_e.*

[13] An example of a non-edge-driven strategy would be if we introduced an arbitrary undiscovered edge from A into an arbitrary zero-span whenever the finishing agenda was empty. It appears very difficult to state a criterion for non-edge-driven strategies which guarantees both their completeness and correctness.

This is satisfied by any reasonable lattice scanning algorithm and any sentence scanning algorithm whatsoever. The only disallowed strategy is to insert words from a lattice into a span which has already had some covering set of words discovered and parsed. It should be fairly clear that this kind of internal-insertion strategy will lead to problems.

Now we supply some theoretical machinery for a condition on rule introductions.

Definition 7 *A (possibly partial) ordering $\prec_T$ of nodes (edge tokens) in a traversal tree T* allows descent *if whenever a node x dominates a set of children C, for any $c \in C$, $c \preceq_T x$.*

Definition 8 *The* Rule Introduction Condition *(or "no rule blocking"): In any parse T of an edge e, there is some ordering $\prec_T$ of nodes which allows descent and such that for any active introduction edge $a \in T$,* EITHER

(1) there is some edge x with a token x' in T which does not dominate any token a' of a and whose finishing will cause the introduction of a (i.e., $a \in R(x)$) and $x' \prec_T a'$, OR

(2) any parse of e will contain either a or an edge b whose discovery would be simultanous with that of a.

This is wordy, but the key idea is that active introduction edges must "depend" on some other edge in the parse in such a way that if an active introduction edge is undiscovered, we can track back to find another edge earlier in the parse which must also be undiscovered.

The last constraint we need is one on the scores of the DFSA rules. If a prefix of a rule is bad, its continuations must be as bad or worse. Otherwise, we may incorrectly delay extending a low scoring prefix.

Definition 9 *The* Grammar Scoring Condition *(or "no score gain"): The grammar DFSAs are scored by assigning an element of a $\preceq_S$-ordered c-semiring to each initial state, transition, and accepting state. The score of a trajectory (sequence of DFSA states) is the semiring product of the scores of the initial state and the transitions, along with the accepting cost (for complete trajectories).*

If this is met, then the score of a traversal tree is then simply a product of scores for each introduction edge token and traversal token. Therefore, the score of an entire traversal tree is no better than the score of any subtree, and monotonically increasing under substitution of a better scored subtree.

A subtle concrete implication of this condition is that, for example, if grammar productions are going to be compressed into a DFSA which transduces rule RHSs to sums of log-probabilities, not only must the log-probabilities of the full productions all be non-positive, but so must each starting cost, transition, and accepting cost.[14] Otherwise, the scoring of the underlying n-ary grammar trees might have the property that adding structure reduces the score, but the scoring of the traversal trees will not.

Now we are ready to state the completeness theorem.

Theorem 3 *Given any DFSA grammar G, lattice L, and introduction strategies obeying the conditions above, any edge e which is finished by the algorithm at some time f_e has the property that $score(e, f_e) = \sigma(e)$.*

The proof is by contradiction. Take the first edge e which is selected from the finishing agenda and finished with an incorrect score estimate, so at e's finishing time f_e, $score(e, f_e) \neq \sigma(e)$.

Perhaps $score(e, f_e) > \sigma(e)$, contrary to our earlier claim that scoring was always conservative (see figure 4). For e to ever be scored incorrectly, e must be a non-introduction edge. Its current incorrect score was then either set at discovery (when it was initialized to the minimum score, which is not greater than $\sigma(e)$ or anything else, a contradiction) or by relaxing a traversal (a, p, e). But such a traversal cannot be created until both a and p are already finished. By choice of e, a and p were correctly scored at their finishing. Consider the traversal tree T formed by taking best parses of a and p, and joining them under a root token of e. After relaxation, we had $score(e) = \sigma(T)$.[15] But, since T is a parse of e, $\sigma(T) \leq \sigma(e)$. Since that relaxation gave e the score it still has at f_e, we have $score(e, f_e) \leq \sigma(e)$, again a contradiction.

Therefore, $score(e, f_e) < \sigma(e)$. Since e has a parse (by completeness), it has at least one best parse. Choose one and call it B. By virtue of being a best parse, $\sigma(B) = \sigma(e)$. We claim that there is some edge x in B which, at f_e, has been discovered, is correctly scored, yet has not been finished. Assume such an x exists. Since x is

[14]The condition implies this because positive elements are not in the relevant semiring: $\langle[-\infty, 0], \max, +, -\infty, 0\rangle$.

[15]We define the score function σ for a traversal tree T to be the score of that specific parse.

discovered but not finished at f_e, it was in the finishing agenda with its current score just before e was chosen to be finished. But e was chosen from the finishing agenda, not x, so it must be that $score(x, f_e) \leq score(e, f_e)$.

On the other hand, since x is contained in B, some best parse of P of x is a subtree of B. But then by "no score gain" it must be that $\sigma(e) = \sigma(B) \leq \sigma(P) = \sigma(x)$. Thus, if we find such an edge x, then $\sigma(e) \leq \sigma(x) = score(x, f_e) \leq score(e, f_e) < \sigma(e)$, a contradiction.

The rest of the proof involves showing the existence of such an x. Consider the nodes in B. Since e is unfinished, there is a non-empty set of unfinished nodes, call it U. We want some $u \in U$ which both has no unfinished children and which is minimal by $\prec_T$. Clearly some elements are minimal since U is non-empty and finite. Call the set of minimal elements M. For any $u \in M$ which has an unfinished child, that child must also be minimal since $\prec_T$ allows descent. Therefore, removing all elements from M which have an unfinished child leaves a non-empty set. Choose any u from this set.

If u dominates two finished children (call them a and p), then since e is the first incorrectly finished edge, $type(a)$ and $type(p)$ had their correct scores at their finishing times. Whenever the later of $type(a)$ and $type(p)$ was finished, the traversal $t = (type(a), type(p), type(u))$ was generated. And before anything else could have been finished, t was explored. Thus, $type(u)$ has been discovered and has been relaxed by t, say at time r_t. Therefore, at r_t, and therefore still at f_e, $score(type(u))$ can be no worse than its score in B, which of course means its score has been correct since r_t, so $type(u)$ is correctly scored at f_e. But recall that $type(u)$ is unfinished, so we are done.

If u dominates no finished nodes, then it is a leaf. If $type(u)$ is a passive introduction edge, then by "no internal insertion" $type(u)$ has been discovered. Since passive introduction edges are correctly scored at discovery, we are done. If $type(u)$ is an active introduction edge, then we need only show that it has been discovered, since these are also correctly scored on discovery. To be sure it has been discovered, we must appeal to "no rule blocking." It is possible that any parse of e contains an edge whose discovery would be simultaneous with that of $type(u)$. If so, since there is some finished parse of e, $type(u)$ must be discovered, and we are done. If not, then let x be an edge from (2) whose finishing would guarantee $type(u)$'s discovery. If x is unfinished, then some instance of x is $\prec_T u$ and unfinished, contradicting u's minimality. Thus we are done.

We have now proven the correctness of the algorithm for strategies meeting the given criteria. The traditional bottom-up, top-down, and left-corner strategies satisfy the Rule Introduction Condition. We prove this for only the top-down strategy here; the other proofs are similar.

Theorem 4 *The top-down rule introduction strategy satisfies the Rule Introduction Condition.*

Order the nodes in T by the order in which they would be built in a top-down stack parse of T. Since no node is completed before its children in such a parse, this allows descent. In a top-down parse, for every active introduction token a' except for leftmost node in the tree, there is another (active) node x' which is a left sibling of a node dominating a' and for which $type(x')$'s finishing will introduce $type(a')$. For any active introduction edge a, let a' be its leftmost token in T. Because it is leftmost, its x' will not dominate any token of a. Therefore, (1) holds unless a' is the leftmost node. Assume it is leftmost. Since T is a parse of some edge e either of category C or with a label with LHS C, a has a label with LHS C. Thus, if any parse S of e whatsoever is found, its leftmost leaf is a token of some active introduction edge b with a label with LHS C. But then, whenever b was discovered, so was a, since the top-down introduction strategy always simultaneously introduces the initial states of all rules with the same LHS.

4.3 Asymptotic Bounds and Performance

We briefly motivate and state the complexity bounds. Let n be the number of nodes in the input lattice, C the number of categories in the grammar, and S the number of states in the grammar. $C \leq S$ since each category's encoding contains at least one state. The maximum number of edges E is $O((C+S)n^2) = O(Sn^2)$, and the maximum number of traversals T is $O(SCn^3)$. Time is dominated by the work per traversal, which can be made amortized $O(1)$ (with a Fibonacci heap-backed priority queue), so the total time is $O(T) =$

$O(SCn^3)$. For memory, there are several $O(E)$ data structures holding edges. The concern is the exploration agenda which holds traversals. But everything on this agenda at any one time resulted from a single call to doFundamentalRule, and so its size is also $O(E)$. Therefore, the total memory is $O(E) = O(Sn^2)$. This is not necessarily true for a standard CP, which can require $O(T)$ space for its agenda.

We have implemented the parser in Java, and tested it with various rule encodings on parsing of Penn Treebank Wall Street Journal sentences. With efficient rule encodings, sentences of up to 100 words can be parsed in 1 Gb of memory. Graphs of runtime performance can be found in (Klein and Manning 2001b).

5 Conclusion

We have pointed out a deep connection between parsing and hypergraphs. Using that connection, we presented an agenda-based probabilistic chart parser which naturally handles arbitrary PCFG grammars and works with a variety of word and rule introduction strategies, while maintaining the same cubic time bounds as a categorical chart parser.

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