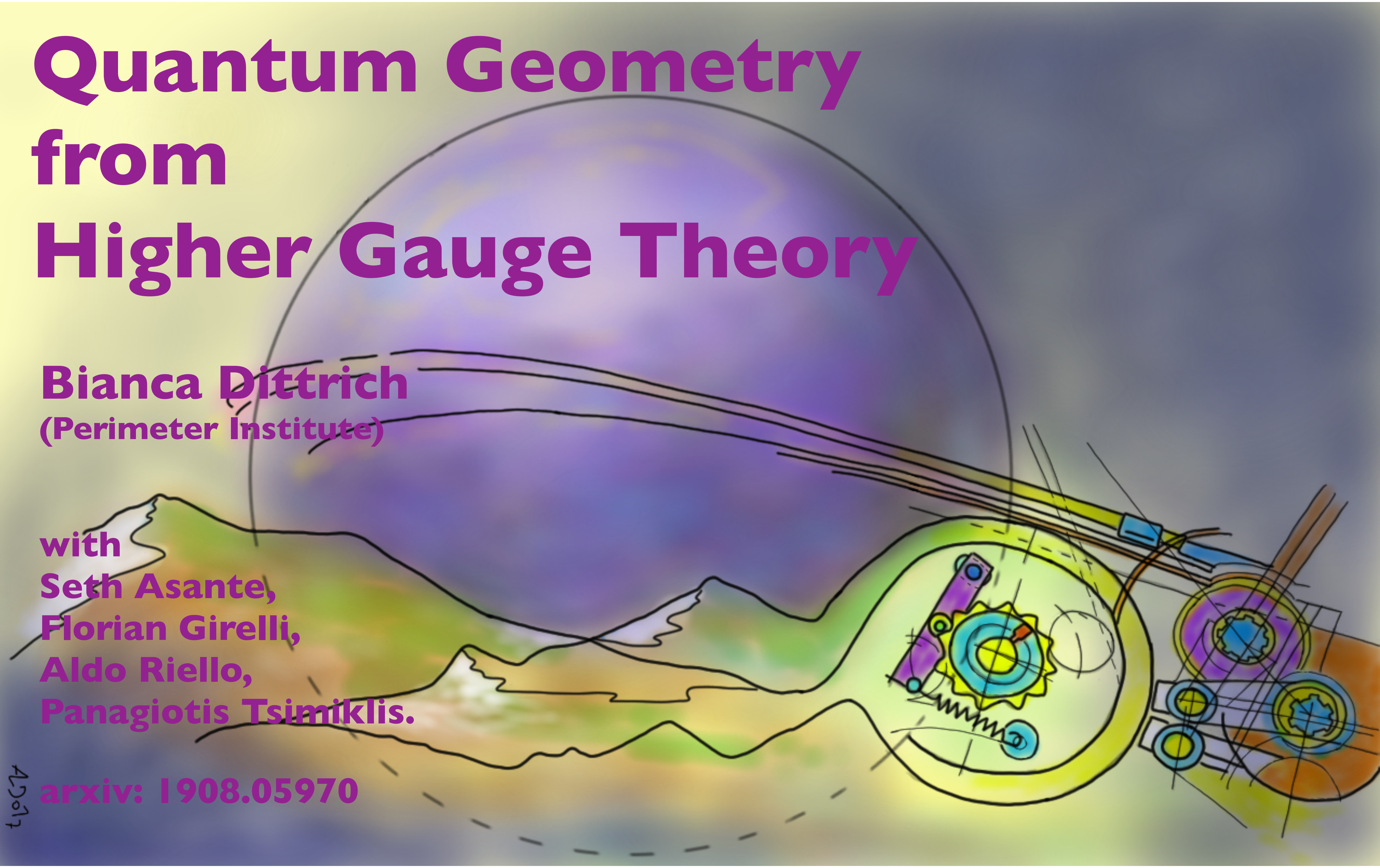
Quantum Geometry from Higher Gauge Theory



Bianca Dittrich
(Perimeter Institute)

with
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Florian Girelli,
Aldo Riello,
Panagiotis Tsimiklis.

arxiv: 1908.05970

ILQGS *Sep 10, 2019*

Main Message:

Prospect for a new 4d quantum geometry

- Includes tetrads as basic variables (as opposed to just bi-vectors)
- Conjugated momenta: 2-connection, which is a two-form
- 2-curvature is a three-form: (space-time) diffeomorphism generator
- 2-curvature generates vertex translations (as opposed to edge translations)

→ Offers many advantages on the kinematic and dynamic level.

Overview

1. Motivation: 3D path integral for quantum gravity:

   Trick: Enlarge configuration space

2. 4D quantum gravity - does it work there?

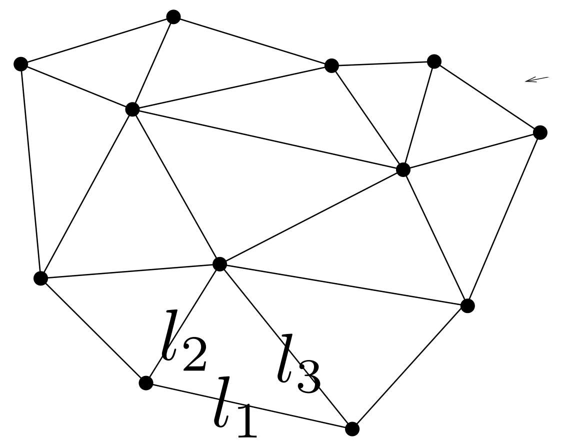
3. We should enlarge even more: BFCG action

4. Quantization, spin bubble model and quantum geometry

5. Outlook

Almost no (2-category) abstract non-sense

Motivation: 3D gravity - path integral



$$Z = \int \mathcal{D}l \, e^{\frac{i}{\hbar} S_{\mathrm{grav}}(l)}$$

Domain of integration very complicated to impose:

- lengths positive
- triangle inequalities
- tetrahedral inequalities

} non-local constraints + complicated action

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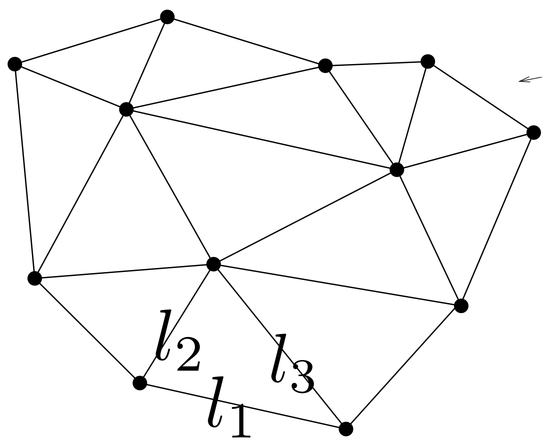

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Trick: enlarge configuration space … (and constrain again)

SU(2) gauge freedom

Triads and spin connection: $g = e \cdot e \geq 0 \;, \quad \omega$ (su(2)-valued one-forms)

Palatini action: $$S = \int \mathrm{Tr}(e \wedge F(\omega))$$

example for a BF action: $$S = \int \mathrm{Tr}(B \wedge F(A))$$

$(d-2)$-form

defined in dimensions >1

[Horowitz 89]

Path integral for first order 3D gravity

path integral for Palatini action $\rightsquigarrow$ path integral over flat connections

$$Z = \int \mathcal{D}[e]\mathcal{D}[\omega]\ e^{\frac{i}{2\ell_{\rm Pl}} \int \mathrm{Tr}(e \wedge F(\omega))} \quad \rightsquigarrow \quad Z = \int \mathcal{D}[\omega] \prod_x \delta(F_x[\omega])$$

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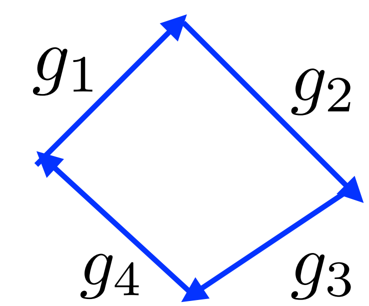
Regularize on a lattice (dual to a triangulation):

$$Z_{\text{PR-group}} = \left[\prod_l \int_{\mathrm{SU}(2)} \mathrm{d}g_l\right] \prod_f \delta\left(\overleftarrow{\prod}_{l\ni f} g_l^{\epsilon(l,f)}\right),$$



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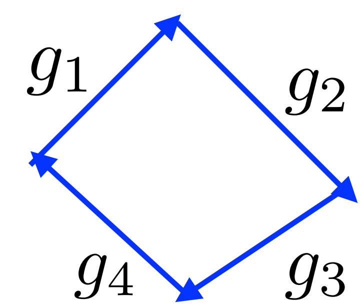


Variable transformation via group Fourier transform

$$\psi(g) = \sum_{j,m,n} \tilde{\psi}(j,m,n) D^j_{mn}(g)$$

Wigner (representation) matrices

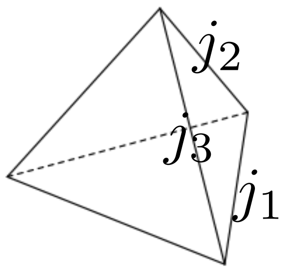
into path integral

Path integral for first order 3D gravity

"State sum" over spin configurations (representation labels):

$$Z_{\mathrm{PR-spin}} = \sum_{\{j_e\}} \prod_{\mathrm{tetra}} \{6j_{e \in \mathrm{tetra}}\}$$

6j symbols (spin network evaluation) associated to tetrahedra



[Ponzano-Regge 1968;
with positive CC: Turaev-Viro 90's]

Spins give length of edges.
All inequalities implemented.

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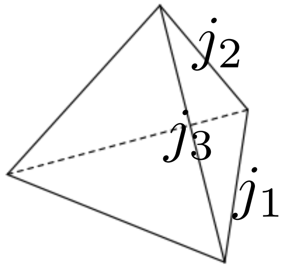

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A very reasonable 3D quantum gravity theory:

- Well defined (after gauge fixing) path integral
- Diffeo-symmetry and consistent Hamiltonians / constraint algebra (translations of vertices)
- canonical quantization = path integral quantization
- equivalence to Chern-Simons quantization
- …

[Barrett, Bonzom, Crane, BD, Freidel, Livine, Louapre, Meusburger, Noui, Perez, Rovelli, …]

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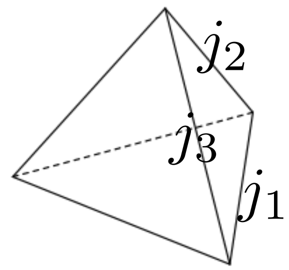

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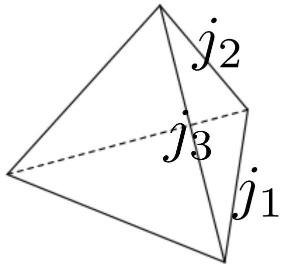

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Open issues: sum over orientations, winding numbers (from holonomies), …

(2+1)D quantum gravity

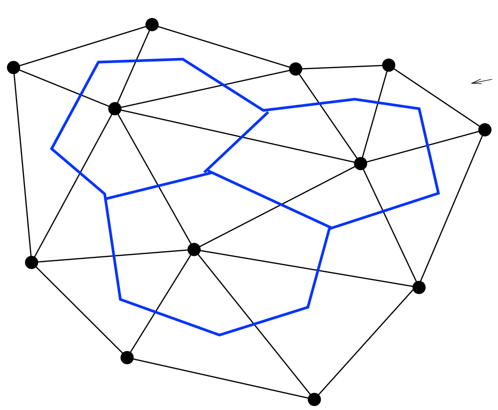
- Hilbert space spanned by spin network functions:

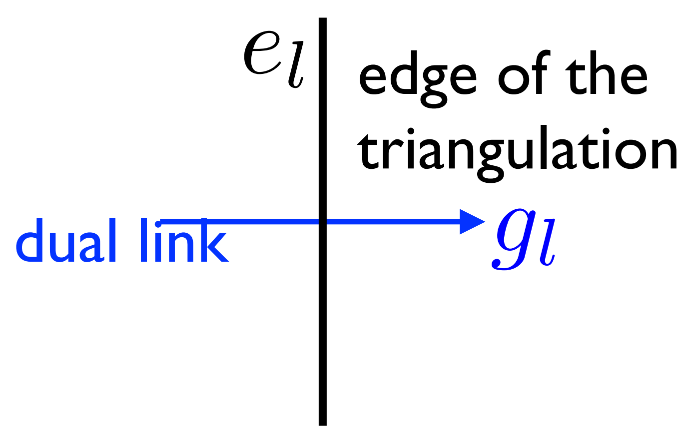
$$\psi(\{g_l\}) = \sum_{\{j_l\},\{\iota_\nu\}} \tilde{\psi}(\{j_l\},\{\iota_\nu\}) \prod_l D^{j_l}_{m_l n_l}(g_l) \prod_\nu \iota_\nu^{mm'n\ldots}$$

- Quantum geometry operators, that measure lengths and (extrinsic curvature) angles.

- Arises also from quantization of (lattice) phase space:

$$\otimes_l T^* SU(2)$$





$$\{e_l^a, g_l\} = g_l \tau^a$$

Dimensions:

$$1 + 1 = 2$$

4D gravity?

Can we do a similar trick?

Palatini action

non-linear in tetrads

$$S = \int \mathrm{Tr} * (e \wedge e) \wedge F(\omega)$$

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BF action

(topological)

$$S = \int \mathrm{Tr}\, B \wedge F(A)$$

two-form

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two-form

Plebanski action

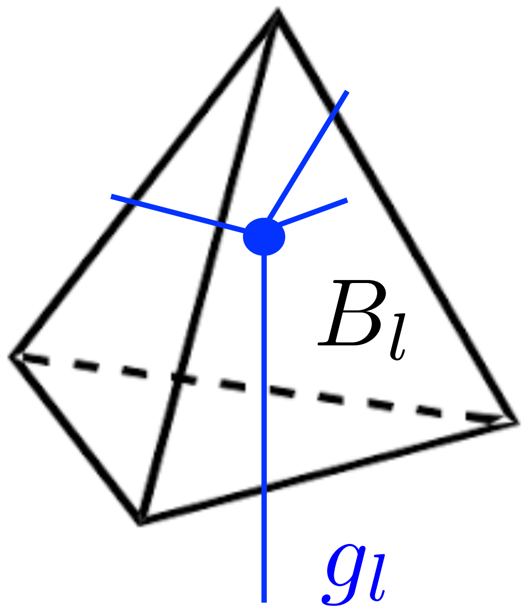
$$S = \int \mathrm{Tr} \, B \wedge F(A) + \mathrm{Tr} \, \phi B \wedge B$$

Simplicity constraints impose*:

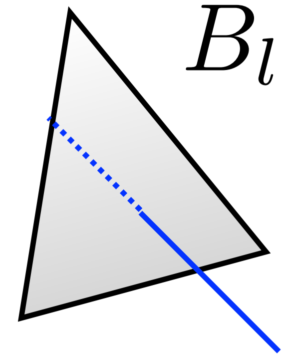
$$B = \star e \wedge e$$

BF action, Plebanski action, Palatini (-Holst) action and Ashtekar (-Barbero) variables lead to similar phase space and quantum geometry.

(3+1)D Phase space and quantum geometry



$$\otimes_l T^* SO(4) \qquad \text{or} \qquad \otimes_l T^* SU(2)$$

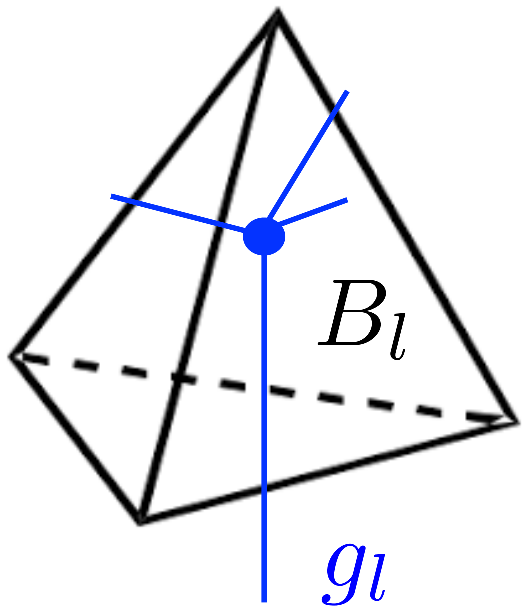


B_l encodes normal to triangle

g_l encodes extrinsic curvature angle (with caveat*)

Dimensions:
$2 + 1 = 3$

(3+1)D Phase space and quantum geometry



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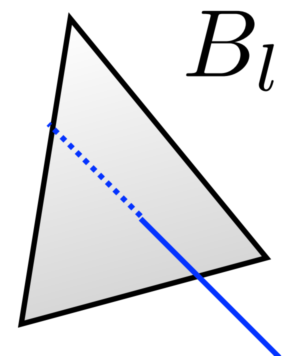

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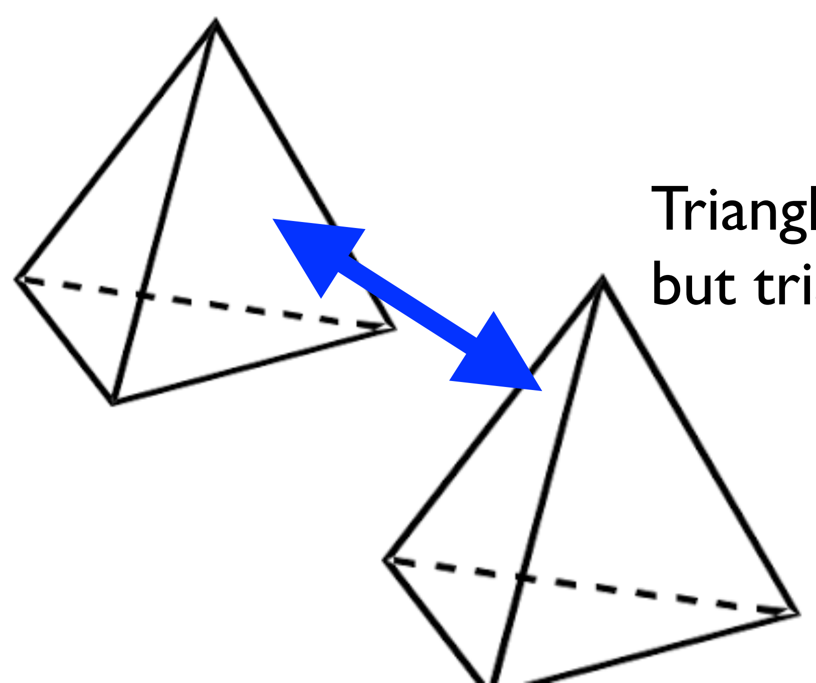
Tetrads need to be reconstructed from normals.
(Issue for matter couplings.)

Problem (leading to caveat*):

Reconstructions from different tetrahedra do not fit (in shape).

[BD, Speziale '07; BD, Ryan 08,]



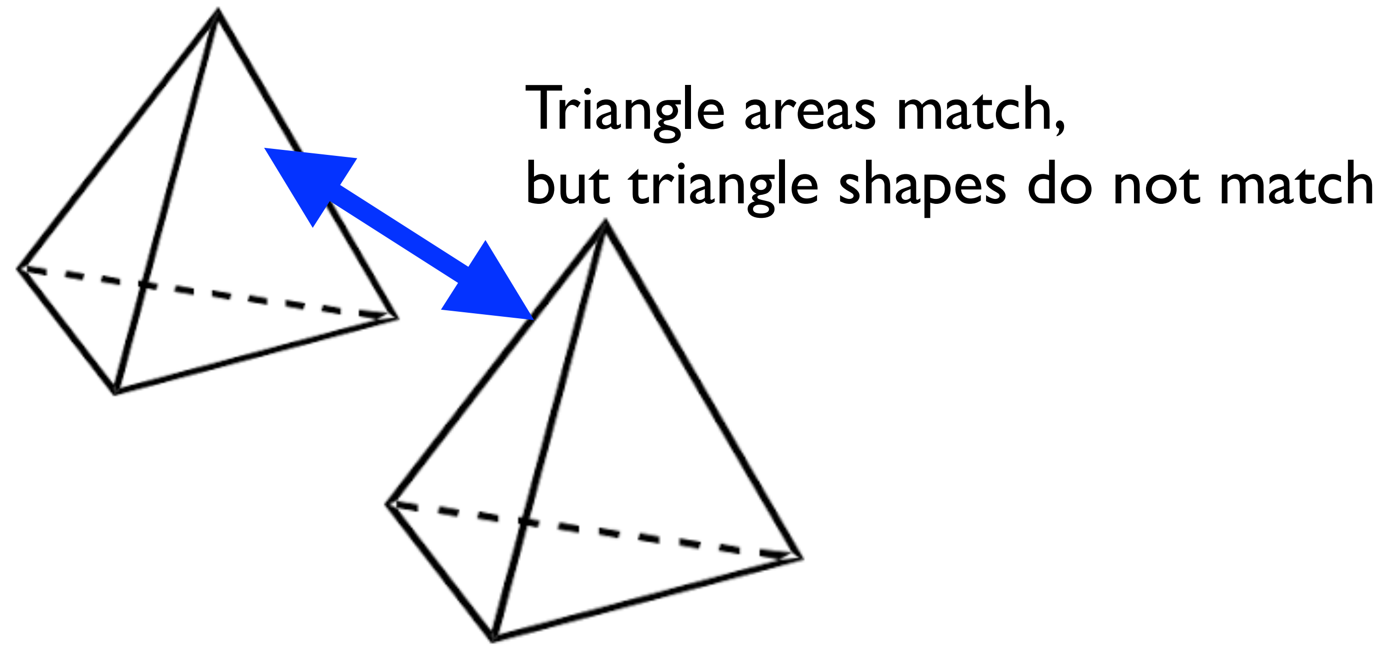
Triangle areas match,
but triangle shapes do not match.

Simplicity constraints

Reconstructions from different tetrahedra do not fit (in shape).

[BD, Speziale '07; BD, Ryan '08,]

[Freidel, Speziale '10] Twisted Geometries



[BD, Ryan '07] Due to: Edge simplicity constraints are not imposed.

$$\mathrm{hol}_{\mathrm{edge}} \triangleright e(B) = e(B)$$

[BD, Ryan '10] Are equivalent to secondary simplicity constraints (and to reality conditions on Ashtekar connection). Needed to reconstruct Levi-Civita connection.

Issue for spin foam/LQG- dynamics?

[Alexandrov, Anza, Bonzom, Han, Hellmann, Kaminski, Oliveira, Speziale, …]

Summary: Phase space from BF theory

Basic variables:

	A	B
	so(4) valued	so(4) valued
	connection one-form	two-form

$$1+2=3$$

conjugated to each other

Geometry:	(extrinsic) curvature*	Areas (normals to triangles)

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Geometry:

	(extrinsic) curvature*	Areas (normals to triangles)

Hamiltonian:

$$F(A)=0$$

Problem: Generates edge translations (instead of vertex translations).
Reason: is a two-form.

[Waelbroeck, Zapata 94]: Propose to break symmetry down to vertex translations. Would result in non-local expressions.

[Thiemann 96]: Action based on tetrahedra. Geometric interpretation lost.

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Would like

a (tetrad) one-form.

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Trick: enlarge phase space … (and constrain again)

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Σ
$\mathbb{R}^4$ valued two-form
(2-connection)

E
$\mathbb{R}^4$ valued one-form
(tetrads)

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Basic geometric variables.

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2-curvature: $G = d_A \Sigma$ is a three-form. Will be important.

The BFCG action

[Girelli, Pfeiffer, Popescu '07]

$$S = \int \mathrm{Tr}_{so(4)} B \wedge F(A) \; + \; \mathrm{Tr}_{\mathbb{R}^4} C \wedge G(A, \Sigma)$$

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[Mikovic, Vojinovic '11]

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$$= \int \mathrm{Tr}_{so(4)} B \wedge F(A) \; - \; \mathrm{Tr}_{\mathbb{R}^4} T(A, E) \wedge \Sigma$$

$$T(A, E) = d_A E$$ Torsion

Torsion freeness and vanishing curvature imposed by Lagrange multiplyers.

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EOM:

$$F(A) = 0 \qquad d_A B - 2E \wedge \Sigma = 0$$

$$G(A, \Sigma) = 0 \qquad T(A, E) = 0$$

Topological theory.

The BFCG action

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Constrain:

$$S = \int \mathrm{Tr}_{so(4)} B \wedge F(A) \; + \; \mathrm{Tr}_{\mathbb{R}^4} E \wedge G(A, \Sigma) + \mathrm{Tr}_{so(4)} \, \phi \, (B - *(E \wedge E))$$

[Mikovic, Vojinovic ’11; Mikovic, Oliveira, Vojinovic ’15-'18]
constraint analysis of (continuum) actions

Some EOM:

$$\phi = F$$

$$G = 2 * (F \wedge E)$$

Key problem

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Plan: [Asante, BD, Girelli, Riello, Tsimiklis '19]

- Quantization of the BFCG action and correspondence with KBF model
- Key: solving part of the equations of motions (constraints), including edge simplicity constraints
- Construction of boundary Hilbert space:
  - with consistent constraints
  - G-network functions

Path integral

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Regularize on a lattice: holonomies associated to dual links, four-vectors to edges:

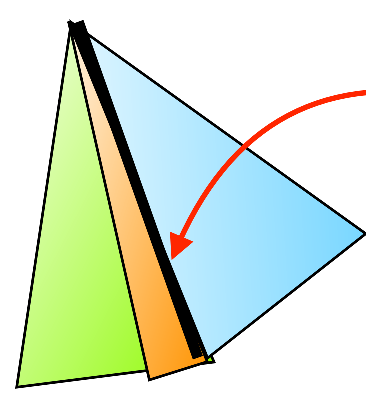
Parallel transport

$$Z_{\mathrm{discr}} = \int \prod_l dg_l \prod_e dE_e \prod_f \delta_{SO(4)} \left(\mathrm{hol}_f(g)\right) \prod_t \delta_{\mathbb{R}^4} \Big(\sum_{e \in t} \pm E'_e \Big)$$

1-Flatness constraint

Triangle closure constraint

Hypersurface triangulation: three tetrahedra meeting at an edge



Edge vector needs to be defined in one of the three adjacent tetrahedral reference systems; parallel transported to the others.

Towards a spin bubble model

One would now (2-)group Fourier-transform:

- SO(4)-group Fourier transform: not useful, as there are parallel transport matrices appearing in triangle closure constraints
- 2-group Fourier-transform: not known (no Peter-Weyl theorem)

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Way forward: [Asante, BD, Girelli, Riello, Tsimiklis '19]

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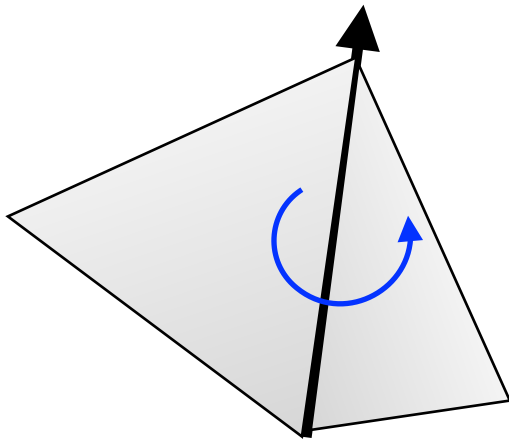
- Key insight: includes edge simplicity constraints (half of 1-flatness constraints)

$$\mathrm{hol}_\mathrm{e}(\mathrm{g}) \triangleright E_e = E_e$$ (on hypersurface)



Continuum: $d_A E = 0 \quad \Rightarrow \quad d_A d_A E = 0 \quad \Rightarrow \quad F \wedge E = 0$

- make use of 1-gauge invariance of path integral

Implementing torsion-freeness

(equivalent to primary and secondary simplicity constraints)

Step 1:

- triangle closure constraint
- edge simplicity constraint

proof ⟹

Holonomies are Levi-Civita:

$$g = \mathrm{Boost}(\theta)\mathrm{Rot}(E)$$

↑ Only free parameter: extrinsic curvature

[BD, Ryan '10;
Asante, BD, Girelli, Riello, Tsimiklis '19]

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Holonomies are Levi-Civita:

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↑ Only free parameter: extrinsic curvature (θ)

Step 2: Use this in the 1-flatness constraints. This is an algebraic calculation and fixes:

$$\theta = \pm\Theta(E) = \pm\Theta(\ell)$$

← Length=norm of E (ℓ)

Just left with (ℓ, θ) variables.

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Just left with (ℓ, θ) variables.

Step 3: U(1) Fourier transform

$$\delta(\theta \pm \Theta(l)) = \sum_{s\in\mathbb{Z}} e^{is(\theta\pm\Theta(l))}$$

Spin bubble model for quantum flat space-time

Obtain amplitudes of the KBF-model:

[Korepanov `02;
Baratin, Freidel `07]
turn it into well-defined model via gauge fixing

$$Z = \int \mathcal{D}[\ell_e] \sum_{\{s_t \in \mathbb{Z}\},\{\epsilon_\sigma = \pm 1\}} \mu(\ell_e, s_t) \prod_\sigma \exp\left(i\epsilon_\sigma \sum_{t\in\sigma} s_t \Theta_t(\ell)\right)$$

Sum over s_t enforces flatness.

Spin bubble model for quantum flat space-time

Obtain amplitudes of the KBF-model:

[Korepanov `02;
Baratin, Freidel `07]
turn it into well-defined model via gauge fixing

$$Z = \int \mathcal{D}[\ell_e] \sum_{\{s_t \in \mathbb{Z}\}, \{\epsilon_\sigma = \pm 1\}} \mu(\ell_e, s_t) \prod_\sigma \exp\left(i\epsilon_\sigma \sum_{t\in\sigma} s_t \Theta_t(\ell)\right)$$

Sum over s_t enforces flatness.

[Baratin, Freidel `14]

This state sum model can be reconstructed from 2-group representation theory.

(Simplex amplitude = Contraction of 2-intertwiners)

l_e and s_t are labels of representations and intertwiners of the Euclidean 2-group.

Obtain (Regge) gravity by constraining $s_t = a_t(l_e)$.

Quantum geometry

[Asante, BD, Girelli, Riello, Tsimiklis '19]

Boundary Hilbert space:

- quantization of all variables: $(B_t, g_l); (E_e, \Sigma_f)$
- consistent quantization of all constraints: 1-Flatness, 2-Flatness, 1-Gauss, 2-Gauss

- (first class) subalgebra: 1-Gauss, 2-Gauss and Edge Simplicity
- reduced wave functions depend only on (ℓ, θ)

G-network functions

Future work: relate explicitly to 2-group representation theory.

- Hamiltonian: 2-Flatness constraint $G_c = \sum_{f \in c} \pm \Sigma'_f$

Generates vertex translations (diffeos) for the tetrad sector.

[Relation to Freidel, Livine, Pranzetti '19]

Summary and Outlook

Higher gauge theory:

An alternative approach to quantum geometry:

- Includes tetrads.
- Geometric transparent Hamiltonian and Diffeomorphism constraints.

Constructed relation between 2-group and 2-representation picture of quantum flat space model.

- Boundary Hilbert space with all constraints.
- G-network functions.

[Asante, BD, Girelli, Riello, Tsimiklis '19]

Summary and Outlook

Higher gauge theory:

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- G-network functions. [Asante, BD, Girelli, Riello, Tsimiklis '19]

Infinite many things to do, both on the mathematical (TQFT) and physical (QG) side:

- (3+1)D flat space holography [Asante, BD, Haggard 18]
- construct explicit 2-group Fourier transform (for more general 2-groups)
- study defects: coupling to strings and particles
- connection to tele-parallel gravity
- generalize to homogeneously curved space-times (McDowell-Mansouri action)
- impose simplicity constraints: in phase space
- quantization: which spectra do survive?
- improve Hamiltonian constraints
- …