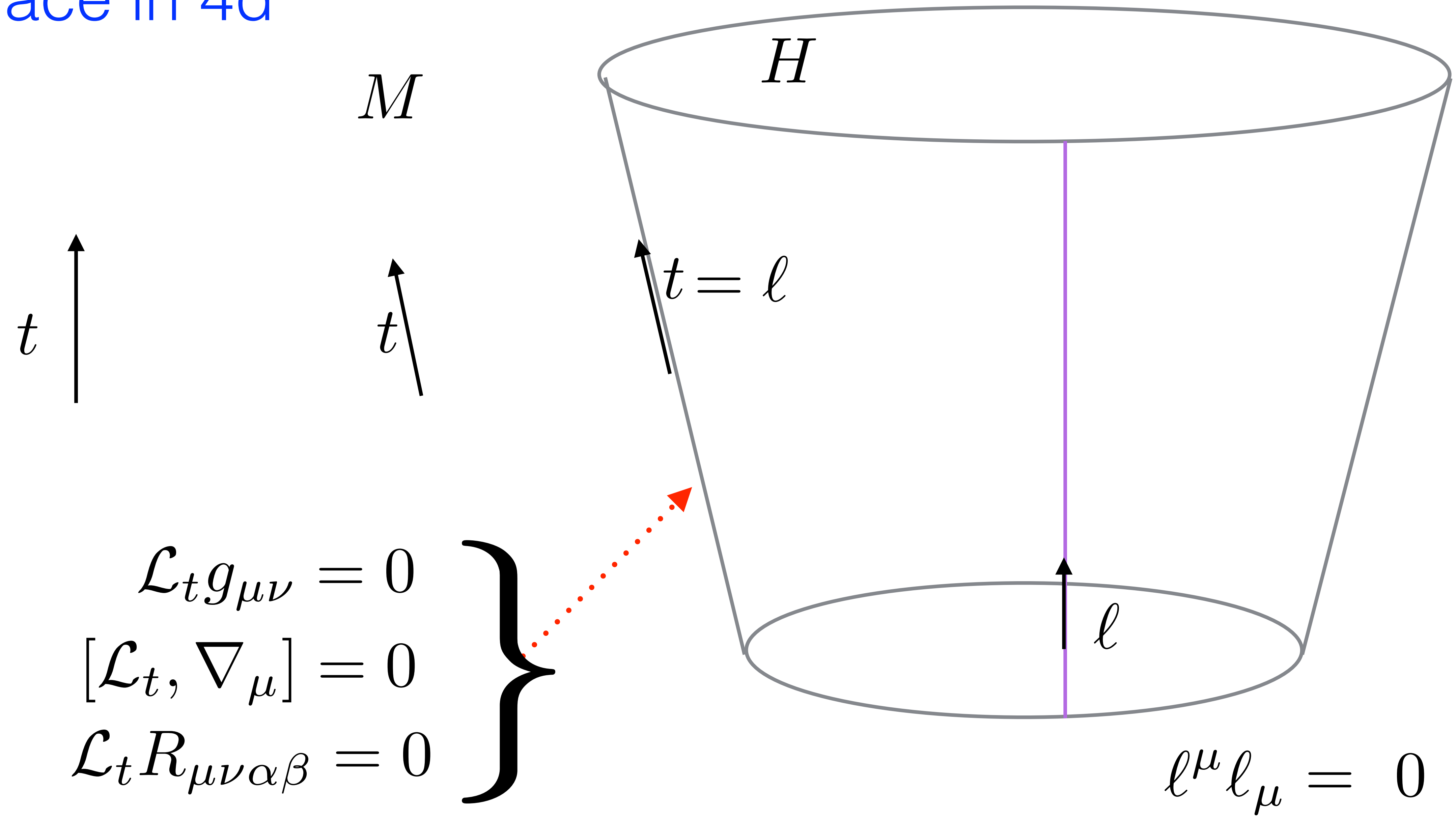
Isolated horizons of the Petrov type D

1, 2). Denis Dobkowski-Ryłko, Jerzy Lewandowski, Tomasz Pawłowski (2018);
3). JL, Adam Szereszewski (2018);
4). DDR, Wojtek Kamiński, JL, AS (2018);
Uniwersytet Warszawski

Null surface stationary to the 2nd order

3d null surface in 4d spacetime

Exists:



Such that

$$\left.\begin{aligned} \mathcal{L}_t g_{\mu\nu} &= 0 \\ [\mathcal{L}_t, \nabla_\mu] &= 0 \\ \mathcal{L}_t R_{\mu\nu\alpha\beta} &= 0 \end{aligned}\right\}$$

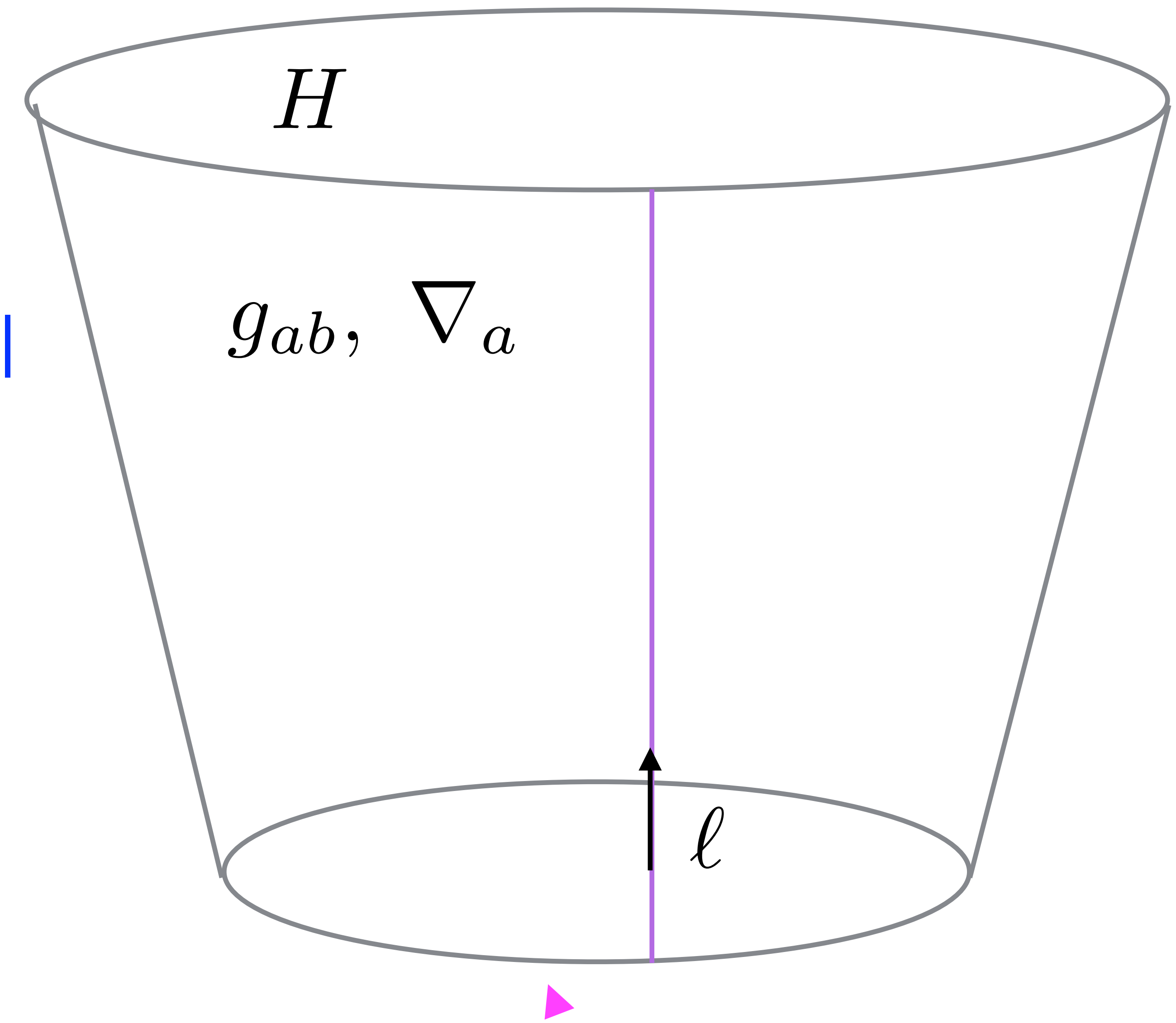
Assumption about M **:** $G_{\mu\nu} + \Lambda g_{\mu\nu} = 0$

Intrinsic geometry

$g_{\mu\nu},\ \nabla_\mu$ - spacetime geometry and derivative

$\mu, \nu - M$

$a, b - H$



$\nabla_a \ell^b = \omega_a \ell^b$ - rotation potential

$\kappa^\ell = \omega_a \ell^a$ - surface gravity

Assumption: $\kappa^\ell \neq 0$

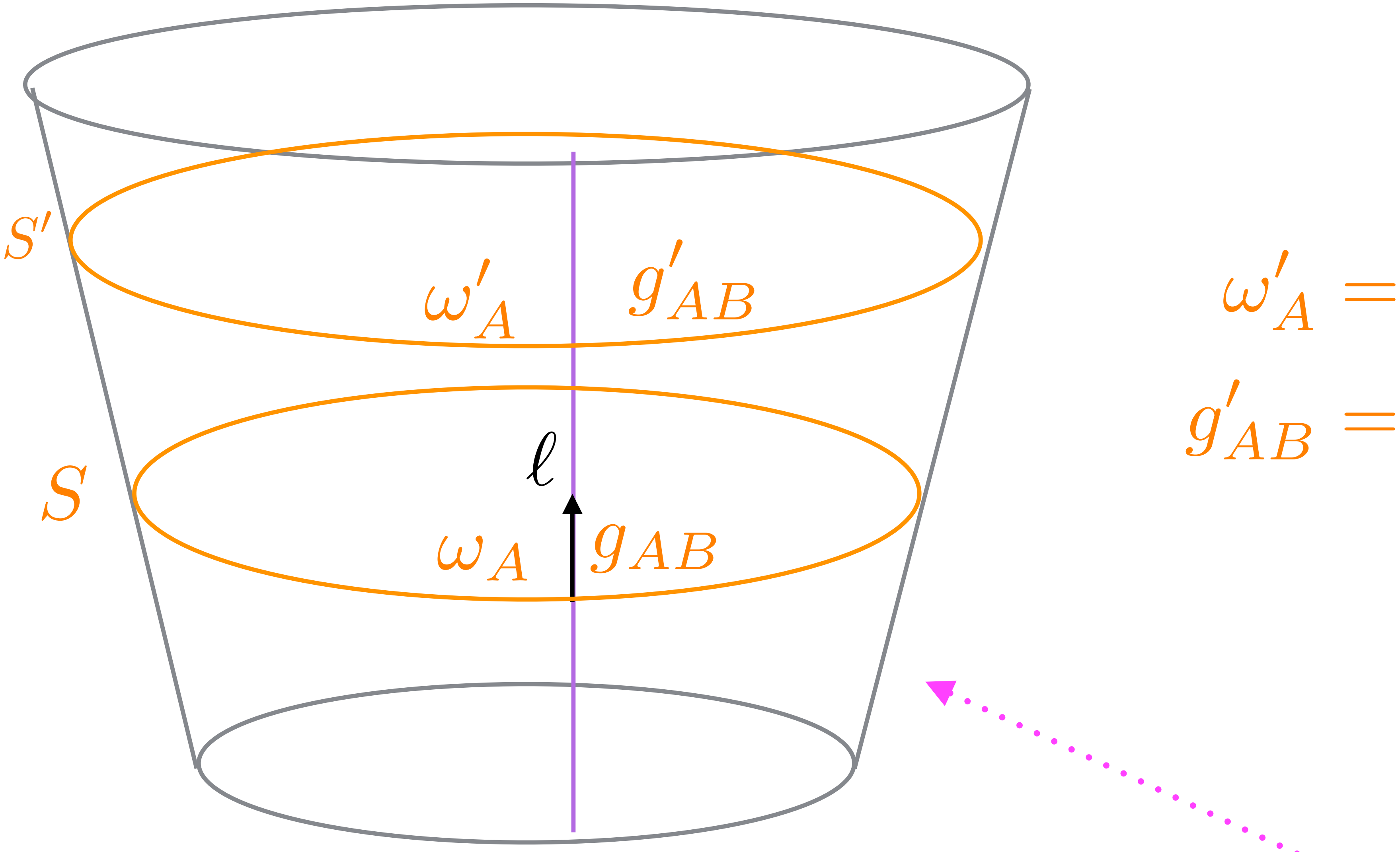
$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 0$$

$$\Downarrow$$

$\nabla_a \kappa^\ell = \mathcal{L}_\ell \omega_a = 0$ - **zeroth law of thermodynamics**

(g_{ab}, ω_a) determine ∇_a determine $g_{\mu\nu}$, ∇_μ, $R_{\mu\nu\alpha\beta}$

Data on a 2d slice



$$\omega'_A = \omega_A + f_{,A}$$

$$g'_{AB} = g_{AB}$$

$$(\omega_A, g_{AB}) \quad \rightarrow \quad (\omega_a, g_{ab}) \quad \rightarrow \quad g_{\mu\nu} \;,\; \nabla_\mu \;,\; R_{\mu\nu\alpha\beta}$$

Data on a 2d slice: summary

2d-surface ———— S

Endowed with (g_{AB}, ω_A) modulo $\omega'_A = \omega_A + f_{,A}$

∇_A - the corresponding derivative $\nabla_A g_{BC} = 0$ $\nabla_{[A}\nabla_{B]} f = 0$

Scalar invariants:

Gaussian curvature: K

$d\omega =: \mathcal{O}\ d\mathrm{Area}$

combined:

$$\Psi := -\frac{1}{2}(K + i\mathcal{O})$$

The Weyl tensor in Newman-Penrose formalism

- Spacetime Weyl tensor in the null frame formalism may be expressed by the following complex valued N-P components:

$$\Psi_0 = C_{4141}, \qquad \Psi_1 = C_{4341} \qquad \Psi_2 = C_{4123},$$
$$\Psi_3 = C_{3432}, \qquad \Psi_4 = C_{3232}$$

- Four components are constant along the null generators of H:

$$D\Psi_I = 0, \qquad I = 0, 1, 2, 3$$

- Additionally we assume: $D\Psi_4 = 0$

- The components Ψ_0 and Ψ_1 vanish due to vanishing of the expansion and shear of ℓ :

$$\Psi_0 = \Psi_1 = 0$$

- Ψ_2 is related to the complex invariant:

$$\Psi_2 = \Psi + \frac{\Lambda}{6}$$

Possible Petrov types

The spacetime Weyl tensor at H is determined by the data

$$(S, g_{AB}, \omega_A)$$

Theorem:

The possible Petrov types of *H* are:

~~I~~, II, D, ~~III~~, ~~N~~, O

wherein:

$$\Psi + \frac{\Lambda}{6} = 0 \quad \Leftrightarrow \quad \textbf{O} \quad \Leftrightarrow \quad K = \frac{\Lambda}{3} \qquad d\omega = 0$$

$$\Psi + \frac{\Lambda}{6} \neq 0 \quad \Rightarrow$$ **generically II, unless…**

The Petrov type D equation

We use a null 2-frame

$$g_{AB} = m_A \bar{m}_B + \bar{m}_A m_B \qquad d\mathrm{Area}_{BC} = i(\bar{m}_B m_C - \bar{m}_C m_B)$$

Theorem:

At *H* the spacetime Weyl tensor is of the Petrov type D iff the following two conditions are satisfied:

$$\Psi + \frac{\Lambda}{6} \neq 0$$

$$\bar{m}^A \bar{m}^B \nabla_A \nabla_B \left(\Psi + \frac{\Lambda}{6}\right)^{-\frac{1}{3}} = 0$$

In local conformally flat coordinates

$$g_{AB}dx^A dx^B = \frac{2}{P^2}dz d\bar{z} \qquad\qquad m^A\partial_A = P\partial_z$$

The Petrov type D equation:

$$\partial_{\bar{z}}(P^2\partial_{\bar{z}}(\Psi + \frac{\Lambda}{6})^{-\frac{1}{3}}) = 0$$

The Petrov type D equation as integrability condition for the near horizon geometry equation

Theorem:

Suppose (g_{AB}, ω_A) satisfy the NHG equation, namely

$$\nabla_{(A}\omega_{B)} + \omega_A\omega_B + \frac{1}{2}(\Lambda - K)g_{AB} = 0$$

Then they also satisfy the Petrov type D equation:

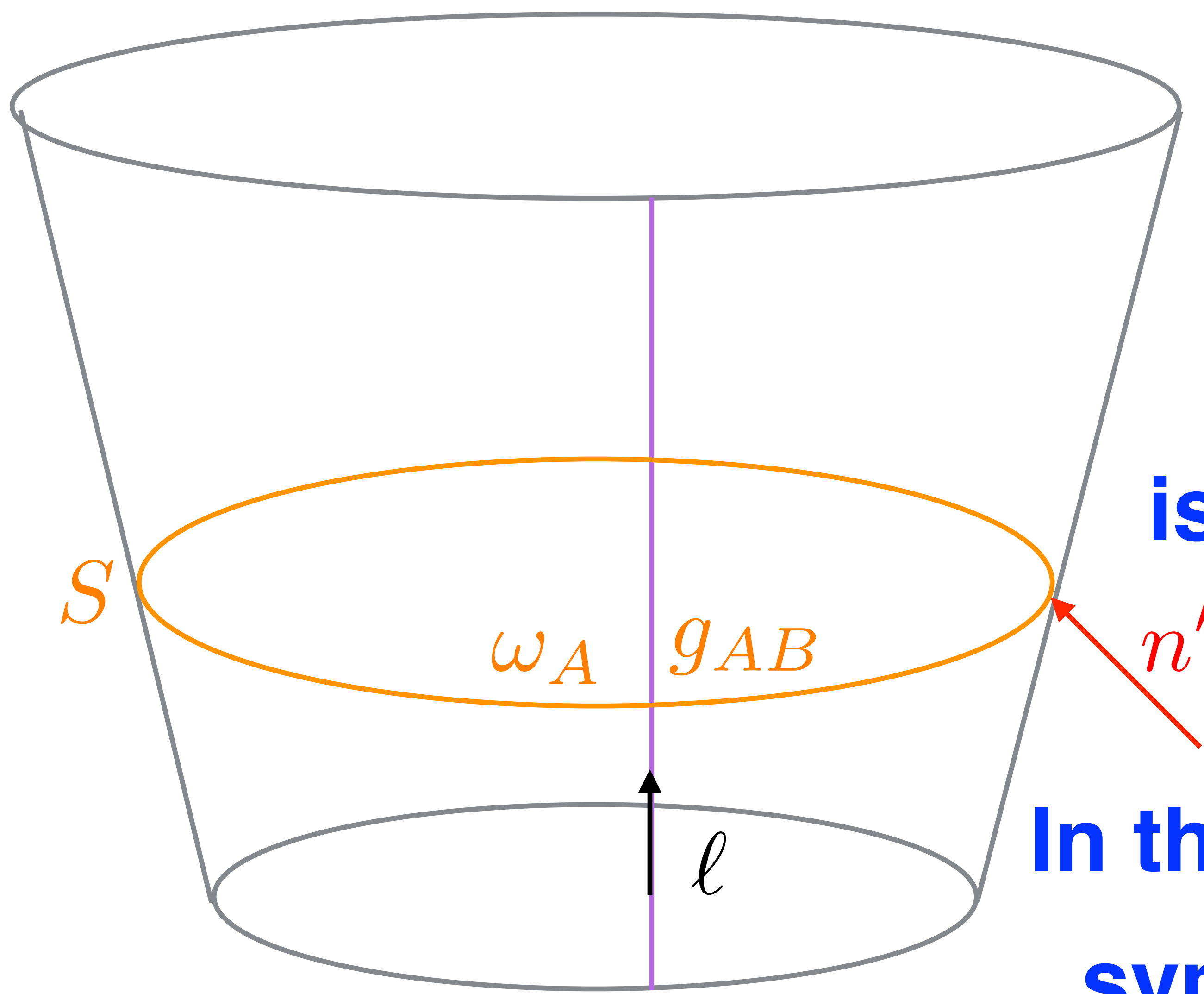
$$\bar{m}^A\bar{m}^B\nabla_A\nabla_B(\Psi + \frac{1}{6}\Lambda)^{-\frac{1}{3}} = 0$$

Non-twisting
of the second principal null direction of the Weyl tensor

Theorem:

Suppose (g_{AB}, ω_A) **satisfy the NHG equation, namely**

$$\nabla_{(A}\omega_{B)} + \omega_A\omega_B + \frac{1}{2}(\Lambda - K)g_{AB} = 0$$



Then the null vector n' **orthogonal to the corresponding slice** S **is a double principal direction of the spacetime Weyl tensor at *H*.**

In that case there exists another symmetry *t'* that is extremal

No-hair theorem for axisymmetric solutions to the Petrov type D equation.

Theorem

The family of axisymmetric solutions of the Petrov type D equation with (or without) cosmological constant defined on a topological sphere can be parametrized by two numbers *(A,J)*: the area and angular momentum, respectively. They can take the following values:

$$\text{for } \Lambda > 0: J \in \Big(-\infty, \infty\Big) \text{ for } A \in \Big(0, \frac{12\pi}{\Lambda}\Big) \text{ and } |J| \in \Big[0, \frac{A}{16\pi}\sqrt{\frac{\Lambda\ A}{12\pi} - 1}\Big) \text{ for } A \in \Big(\frac{12\pi}{\Lambda}, \infty\Big)$$

$$\text{for } \Lambda < 0: J \in \Big(-\infty, \infty\Big) \text{ and } A \in \Big(0, \infty\Big)$$

Lewandowski, Pawlowski (2003) for $\Lambda = 0$

Embeddability of the axisymmetric solutions

Every solution defines a type D isolated horizon whose intrinsic geometry coincides with the intrinsic geometry of a non-extremal Killing horizon contained in one of the following spacetimes:

1). Kerr - (anti) de Sitter;

2). Schwarzschild - (anti) de Sitter ;

3). Near horizon limit spacetime near an extremal horizon contained either in the K(a)dS or S(a)dS spacetime;

The Petrov type D equation on S of genus > 0

$$g_{AB}dx^A dx^B = \frac{2}{P^2} dz d\bar{z} \qquad m^A \partial_A = P\partial_z$$

The Petrov type D equation: $\partial_{\bar{z}}\left(P^2 \partial_{\bar{z}}\left(\Psi + \frac{\Lambda}{6}\right)^{-\frac{1}{3}}\right) = 0$

$$\Rightarrow \quad \partial_{\bar{z}}\left(\Psi + \frac{\Lambda}{6}\right)^{-\frac{1}{3}} = \frac{F(z)}{P^2}$$

$$\Rightarrow \quad F(z)\partial_z$$

is a globally defined holomorphic vector field

$\Rightarrow$

$F(z) = \mathrm{const}$ if genus =1

$F(z) = 0$ if genus >1

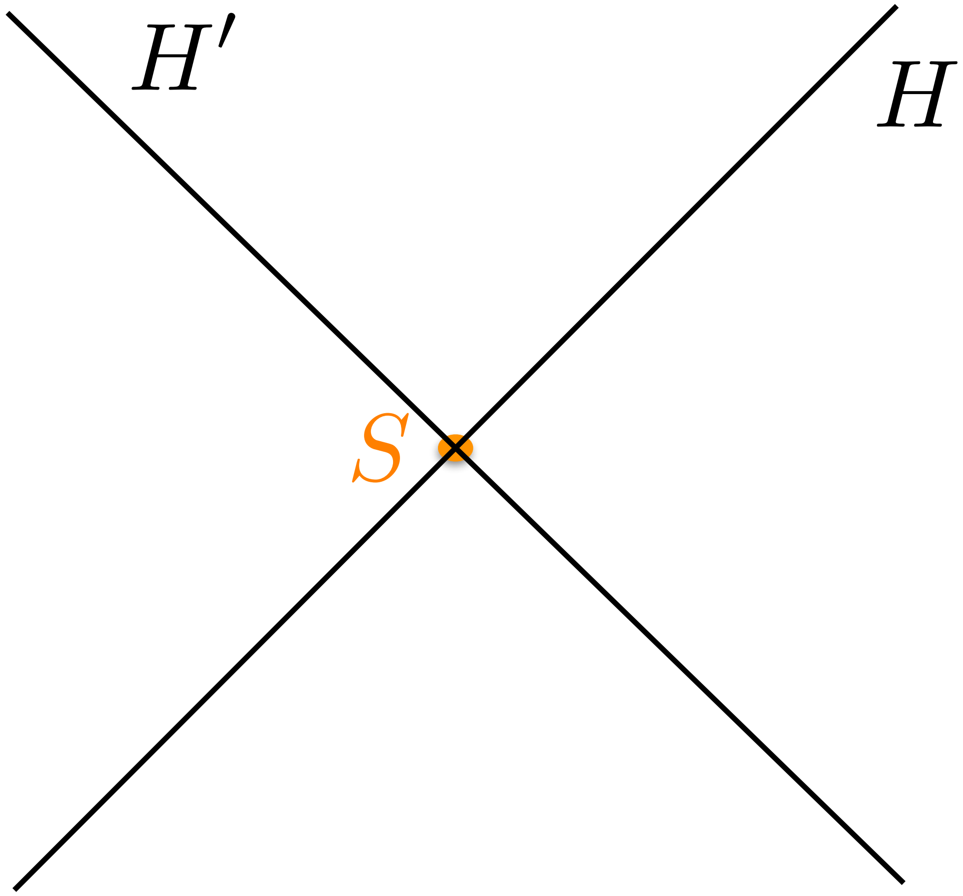
The Petrov type D equation on S of genus > 0

Theorem. Suppose S is a compact 2-surface of genus >0. The only solutions to the Petrov type D equation with a cosmological constant Λ are (g, ω) such that

$$d\omega = 0 \qquad \textbf{and} \qquad K = \text{const} \neq \frac{\Lambda}{3}$$

Remark: There are no rotating solutions

A bifurcated Petrov type D horizon: data



$$g'_{AB} = g_{AB}$$

$$\omega'_A = -\omega_A$$

$$\Psi' = \bar{\Psi}$$

M. J. Cole, I. Racz, J. A. Valiente Kroon, 2018

J. Lewandowski, A. Szereszewski, 2018

A bifurcated Petrov type D horizon: equations

The Petrov type D equations

for H:

$$\bar{m}^A\bar{m}^B\nabla_A\nabla_B(\Psi + \frac{1}{6}\Lambda)^{-\frac{1}{3}} = 0$$

and for *H':*

$$m^A m^B\nabla_A\nabla_B(\Psi + \frac{1}{6}\Lambda)^{-\frac{1}{3}} = 0$$

hold simultaneously on S

In local conformally flat coordinates

$$g_{AB}dx^A dx^B = \frac{2}{P^2}dz d\bar{z} \qquad m^A\partial_A = P\partial_z$$

$$\partial_{\bar{z}}(P^2\partial_{\bar{z}}(\Psi + \frac{\Lambda}{6})^{-\frac{1}{3}}) = 0 \qquad \Rightarrow \qquad \partial_{\bar{z}}(\Psi + \frac{\Lambda}{6})^{-\frac{1}{3}} = \frac{F(z)}{P^2}$$

$$\partial_z(P^2\partial_z(\Psi + \frac{\Lambda}{6})^{-\frac{1}{3}}) = 0 \qquad \Rightarrow \qquad \partial_z(\Psi + \frac{\Lambda}{6})^{-\frac{1}{3}} = \frac{\bar{G}(\bar{z})}{P^2}$$

$$\Rightarrow \qquad \partial_z\left(\frac{F(z)}{P^2}\right) = \partial_{\bar{z}}\left(\frac{\bar{G}(\bar{z})}{P^2}\right)$$

$$\Rightarrow \qquad \mathcal{L}_\Phi g_{AB} = 0 \qquad \Phi := F(z)\partial_z - \bar{G}(\bar{z})\partial_{\bar{z}}$$

$$\Rightarrow \qquad \mathcal{L}_\Phi\, d\omega = 0$$

The axial symmetry without the rigidity theorem

Theorem:

Suppose (g_{AB}, ω_A) **defined on** S **satisfy the Petrov type D equation**

$$\bar{m}^A \bar{m}^B \nabla_A \nabla_B (\Psi + \frac{1}{6}\Lambda)^{-\frac{1}{3}} = 0$$

and the conjugate one

$$m^A m^B \nabla_A \nabla_B (\Psi + \frac{1}{6}\Lambda)^{-\frac{1}{3}} = 0$$

Then, there is a vector field Φ **at** S **such that**

$\mathcal{L}_\Phi g_{AB} = 0$ **and** $\mathcal{L}_\Phi d\omega = 0$

$$\Phi^A = \mathrm{Re}/\mathrm{Im}\left(d\mathrm{Area}^{AB} \partial_A (\Psi + \frac{\Lambda}{6})^{-\frac{1}{3}} \right)$$

Summary

- The type D equation:

$$\bar{m}^A \bar{m}^B \nabla_A \nabla_B \left(-\frac{1}{2}K - \frac{1}{2}i\mathcal{O} + \frac{\Lambda}{6} \right)^{-\frac{1}{3}} = 0$$

- Non-twisting of the second double principal vector if:

$$\nabla_{(A}\omega_{B)} + \omega_A \omega_B + \frac{1}{2}(\Lambda - K) g_{AB} = 0$$

- All the axisymmetric solutions of the type D eq. on topological sphere parametrized by (A, J);

- All solutions on genus>0 derived (non-rotating);

- The extra (axial) symmetry in the case of bifurcated horizon;

- Open problems: existence of non-axisymmetric solutions on topological sphere

Thank You