Plebanski sectors of the new spin-foam models

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Spin-foams:

Original goal: define *path integral dynamics for loop quantum gravity.* [Rovelli-Reisenberger 1996]

`New' spin-foam models:

[Freidel, Krasnov, 2007; E., Pereira, Livine, Rovelli 2007; Kaminski, Kisielowski, Lewandowski 2009; Conrady, Hnybida 2010; Han,Thiemann, 2010]

	Euclidean	Lorentzian	State space summed over
"Master constraint" approach to 2nd class constraints	EPRL, KKL (arbitrary cells)	EPRL, CH (time-like tetrahedra)	=LQG states
Coh. State approach to 2nd class constraints	FK (= EPRL for $\gamma<1$)		For $\gamma>1$, >LQG
"Commuting B fields"	HT(=FK for large j)		Seems >LQG (?)

New basic element common to all of these models:

The use of **linear simplicity constraints** [E., Pereira, Rovelli 2007]

(will define later)

Spin-foam models are arrived at via the *Plebanski formulation* of gravity:

Gravity = BF theory + simplicity constraints

BF theory: $B^{IJ}_{\mu\nu}, \omega^{IJ}_{\mu}$, $\quad S = \int \left(B + \frac{1}{\gamma}{}^{\star}B\right) \wedge F.$

Plebanski constraint: $\quad \epsilon_{IJKL}(B^{IJ}_{\mu\nu}B^{KL}_{\rho\sigma} - \frac{1}{4!}\eta_{\mu\nu\rho\sigma}\eta^{\alpha\beta\gamma\delta}B^{IJ}_{\alpha\beta}B^{KL}_{\gamma\delta}) = 0$

Solutions are in 5 sectors:

(I±) $B^{IJ} = \pm e^I \wedge e^J$ for some e^I_μ

(II±) $B^{IJ} = \pm\frac{1}{2}\epsilon^{IJ}{}_{KL}e^K \wedge e^L$ for some e^I_μ

(deg) $\epsilon_{IJKL}\eta^{\mu\nu\rho\sigma}B^{IJ}_{\mu\nu}B^{KL}_{\rho\sigma} = 0$ (degenerate case).

In sector (II+), BF action reduces to *Holst action of gravity*:

$$S_{Holst} = \frac{1}{4\kappa}\int \left(\epsilon_{IJKL}e^K \wedge e^L + \frac{2}{\gamma}e_I \wedge e_J\right) \wedge F^{IJ}$$

the Legendre transform of which is the basis of *LQG*.

What we will show, #1: **Linear simplicity** imposes restriction to **(II±)**, **(deg).**

That is: its better than original Plebanski constraint, but *the theory still includes more than grav. sector.*

Many of us were aware of these different sectors at some level, but it was never made **precise**.

Basic variables: ***Discrete***	**Tension** ◄——————►	Definition of Plebanski sectors: ***Continuum***

Most precise is probably in **Conrady and Freidel** 2008 “On the semiclassical limit of 4d spin-foam models”, but even there, one has only a discrete **analogue** of the Plebanski sectors. **There is still a gap.**

In this talk we will bridge this gap.

Furthermore:

The asymptotics of the EPRL model [Barrett et al., 2009] :

$$Ae^{iS_{grav}} + Ae^{-iS_{grav}} + Be^{\frac{i}{\gamma}S_{grav}} + Ce^{-\frac{i}{\gamma}S_{grav}}$$

But $e^{iS_{grav}}$ by itself is required for the correct classical limit! Whence come the other terms?

What we will show, #2: These extra terms **are caused precisely** by the non-(II+) sectors **allowed by linear simplicity.**

That is: We will show that the simplicity constraints **being too weak is the reason** for the wrong semiclassical limit.

What we will show, #3: How to modify EPRL vertex amplitude to restrict to only (II+).

Resulting vertex amplitude still has ***all the properties we like about EPRL*** (LQG boundary states, linear in the boundary state, SU(2) gauge-invariant), ***plus***

- Only a ***single*** term $\mathbf{e}^{\mathbf{iS}_{\mathrm{Regge}}}$ in the asymptotics
- **Degenerate** configurations are **exponentially suppressed**

I.
Discrete Plebanski Sectors Defined

Consider oriented **4-simplex S**. Number **tetrahedra a=0, … 4**.

t_a with orientation as part of S.

Label **triangles by unordered pair (ab)** of tetrahedra on either side:

Δ_{ab}, with orientation as part of ∂t_a.

S and each tetrahedron has its own `frame'.

Variables:

$Spin(4) \ni G_a$: Parallel transport from 'a'-frame to 'S'-frame.

$\mathfrak{so}(1,3) \ni B_{ab} = (\mathbf{B}^-_{ab}, \mathbf{B}^+_{ab})$ '$= \int_{\Delta_{ab}} B^{IJ}$ in the 'a'-frame.'

$$B_{ab}(S) := G_a \triangleright B_{ab} \; : \; \text{Transported to the 'S'-frame.}$$

Constraint (Part of def. of var.s): $|\mathbf{B}^\pm_{ab}|^2 = |\mathbf{B}^\pm_{ba}|^2$

Constraints (consistency):

Orientation: $B^{IJ}_{ab} = -B^{IJ}_{ba}$
Closure: $\sum_{b \neq a} B^{IJ}_{ab} = 0$, $a = 0, \ldots 4$.

Necessary b/c we reconstruct $B^{IJ}_{\mu\nu}$ as ***constant*** in the 4-simplex:
$\sum_{b \neq a} B^{IJ}_{ab} = \oint_{\partial t_a} B^{IJ} = \int_{t_a} dB^{IJ} = 0$

Definition. A *discrete Plebanski field* is a set of bivectors $\{B^{IJ}_{ab}\}$, $a, b = 0, \ldots 4$, such that

(i.) $B^{IJ}_{ab} = -B^{IJ}_{ba}$

(ii.) $\sum_{b \neq a} B^{IJ}_{ab} = 0$

- Let $M := \mathbb{R}^4$, as *oriented affine manifold* with standard ϵ, ∂_a
- ∂_a defines notion of straight line segments
- ∂_a lets us identify T_pM for all p, and lets us identify each T_pM with space of constant vector fields.

Definition. A *geometrical 4-simplex* in M is the convex hull of five points in M, not all of which lie in same 3-plane.

Definition. An *ordered 4-simplex* σ is

- a geometrical 4-simplex in M with an assignment of $0, \ldots 4$ to each of its 5 vertices ordered such that $(\vec{01}, \vec{02}, \vec{03}, \vec{04})$ has positive orientation.
- Each tetrahedron is labeled by the number of the vertex it does not contain.

Definition. Let $\Delta_{ab} = \Delta_{ab}(\sigma)$ denote triangle between tetrahedra a and b, with orientation

$$\epsilon^{\alpha\beta}_{[ab]} := \epsilon^{\gamma\delta\alpha\beta}(N_a)_\gamma(N_b)_\delta$$

where $(N_a)_\alpha$, $(N_b)_\alpha$ are any covariant outward normals to tetrahedra a and b, respectively.

(*same as* orientation as part of ∂t_a, where t_a has orientation as part of $\partial\sigma$.)

Lemma 1. Given a discrete Plebanski field $\{B_{ab}^{IJ}\}$ and any choice of ordered 4-simplex σ in M, **there exists a unique** constant $\mathfrak{so}(1,3)$-valued 2-form $B_{\mu\nu}^{IJ}$ such that [Barrett, Fairbairn, Hellmann, 2009]

$$B_{ab}^{IJ} = \int_{\Delta_{ab}} B^{IJ}$$

Heuristic counting argument:

→ Each B_{ab}^{IJ} is skew 4×4: *6 vars*

→ B/c $B_{ab} = -B_{ba}$, only one indep. B per triangle

→ 10 triangles $\times 6 = 60$

→ Closure: $\sum_{b \neq a} B_{ab}^{IJ} = 0$, $\forall a$: Each eq'n has 6 compts. 5 eq'ns – but only 4 of them are indep. ∴ # of indep. closure constraint compts $= 4 \times 6 = 24$

→ $60 - 24 = 36$ indep. compts.

But $B_{\mu\nu}^{IJ}$ has *also* $6 \times 6 = 36$ compts. '■'

Definition. We call $B^{IJ}_{\mu\nu}$ so-det. the '*2-form of* $\{B^{IJ}_{ab}\}$ *adapted to* σ'.

Recall: **(I±)** $B^{IJ} = \pm e^I \wedge e^J$ for some e^I_μ

(II±) $B^{IJ} = \pm\frac{1}{2}\epsilon^{IJ}{}_{KL} e^K \wedge e^L$ for some e^I_μ

(deg) $\epsilon_{IJKL}\eta^{\mu\nu\rho\sigma} B^{IJ}_{\mu\nu} B^{KL}_{\rho\sigma} = 0$ (degenerate case).

If $\{B^{IJ}_{ab}\}$ has 2-form adapted to σ in Pleb. sector (I±), (II±), or (deg), ***we say that $\{B^{IJ}_{ab}\}$ is in Plebanski sector (I±), (II±), or (deg) relative to σ.***

Lemma 2. If a discrete Plebanski field B^{IJ}_{ab} is in a given Plebanski sector relative to a given ordered 4-simplex σ, then it is in the *same* Plebanski sector relative to *any* ordered 4-simplex.

Sketch of proof:

- Let a disc. Pleb. field $\{B^{IJ}_{ab}\}$, and two ordered 4-simplices σ, σ' be given, and let ${}^{\sigma}B^{IJ}_{\mu\nu}$, ${}^{\sigma'}B^{IJ}_{\mu\nu}$ denote corresp. 2-forms.

- $\exists! G \in IGL(4)$ mapping each vertex of σ into the corresp. vertex of σ', in order.

- Because of the positive orientation condition on the vertices of σ and σ', $G \in IGL(4)^{+}$ (preserves orientation of M).

- Only structure used in constructing ${}^{\sigma}B^{IJ}_{\mu\nu}$ from σ and $\{B^{IJ}_{ab}\}$: orient. of M and ∂_a. $\therefore$ construction is $IGL(4)^{+}$ *covariant*.

- As $G \cdot \sigma = \sigma'$, it follows $G \triangleright {}^{\sigma}B^{IJ}_{\mu\nu} = {}^{\sigma'}B^{IJ}_{\mu\nu}$.

- But action of G is via a specific orient. preserving diffeo. Plebanski sectors are invariant under orient. preserving diffeos.

- Hence ${}^{\sigma}B^{IJ}_{\mu\nu}$ and ${}^{\sigma'}B^{IJ}_{\mu\nu}$ are in the same Plebanski sector. ■

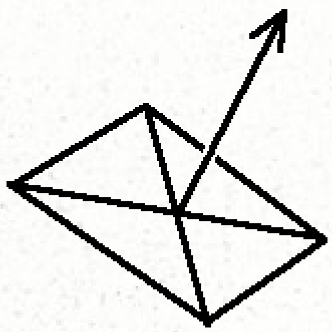
Thus, notion of discrete Pleb. field $\{B_{ab}^{IJ}\}$
being in a given Plebanski sector
is *independent* of 4-simplex used.

II.
Sectors of Linear Simplicity

Linear simplicity

(1.) For each a, $\exists N_a^I$ such that $({}^\star B_{ab}(S))^{IJ}(N_a)_J = 0$ for all b.

(One condition per tetrahedron.)



(1.) will be applied in the ***4-simplex frame*** S. In the ***frame of each tetrahedron*** a, we will impose a ***gauge-fixed*** version of **(1.)** with $N^I = \mathcal{N}^I := (1,0,0,0)$:

$$C^i_{ab} := \frac{1}{2}\epsilon^i_{jk} B^{jk}_{ab} \approx 0 \text{ for all } a \neq b$$

Solution parameterized by 'reduced boundary data' $(\mathbf{n}_{ab}, A_{ab}, A_{ba})$:

$$B_{ab} = (\mathbf{B}^-_{ab}, \mathbf{B}^+_{ab}) = \frac{1}{2}A_{ab}(-\mathbf{n}_{ab}, \mathbf{n}_{ab})$$

In 4-simplex frame *S*:

$$B_{ab}(S) := G_a \triangleright B_{ab}$$
$$= \boxed{\frac{1}{2}A_{ab}(-X_a^- \triangleright \mathbf{n}_{ab}, X_a^+ \triangleright \mathbf{n}_{ab}) =: B_{ab}^{\text{phys}}(A_{ab}, \mathbf{n}_{ab}, X_a^{\pm}).}$$

We have defined

- Meaning of Plebanski sectors for $\{B_{ab}(S)\}$.
- Consequences of linear simplicity for $\{B_{ab}(S)\}$.

The question is ***now well-defined***:

1. Does linear simplicity imply $\{B_{ab}(S)\}$ is in a Plebanski sector?

2. Which Plebanski sectors does linear simplicity isolate?

We show the answers are
(1.) **Yes** and (2.) **(II+), (II-), (deg)**

Sketch of Proof:

Definition: $(B_{ab}^{\text{geom}})(\sigma)^{IJ} := A(\Delta_{ab})\frac{(N_a \wedge N_b)^{IJ}}{|N_a \wedge N_b|}$
where N_a is outward pointing normal to tetrahedron a in σ.

Lemma: $\{(B_{ab})(\sigma)^{IJ}\}$ is always in Plebanski sector (II+)!

Definition: If $\exists Y \in SU(2)$ and five signs ϵ_a such that

$$U_a^2 = \epsilon_a Y U_a^1$$

we write $\{U_a^2\} \sim \{U_a^1\}$.

Lemma: If $\{X_a^-\} \sim \{X_a^+\}$ then $\{B_{ab}^{\text{phys}}(A_{ab}, \mathbf{n}_{ab}, X_a^{\pm})\}$ is in (deg).

Reconstruction Theorem [Barrett et al., 2009]**:**

If
- If linear simplicity is satisfied
- $\{A_{ab}, \mathbf{n}_{ab}\}$ is non-degenerate satisfying closure.
- $\{X_a^{\pm}\}$ satisfies orientation constraint
- $\{X_a^{+}\} \not\sim \{X_a^{-}\}$

Then $\exists\sigma$ such that

$$B_{ab}^{\text{phys}}(A_{ab}, \mathbf{n}_{ab}, X^{\pm}) = \mu B_{ab}^{\text{geom}}(\sigma)$$

where $\mu = \pm 1$ is indep. of ambig. in σ.

Theorem: If $\{A_{ab}, \mathbf{n}_{ab}\}$ is non-deg. and satisfies closure and $\{X_a^{\pm}\}$ satisfies orientation,

(i.) if $\{X_a^{-}\} \not\sim \{X_a^{+}\}$, then $\{B_{ab}^{\text{phys}}\}$ is in **(II+)** or **(II-)**, dep. on μ.

(ii.) if $\{X_a^{-}\} \sim \{X_a^{+}\}$, then B_{ab}^{phys} is in **(deg)**.

Summary:

If:

	Condition		*Way imposed in EPRL vertex*
•	Tetrahedron non-degeneracy		**Not** imposed !
•	Closure	}	Imposed by asymptotics of EPRL
•	Orientation		
•	Linear simplicity	}	Directly imposed

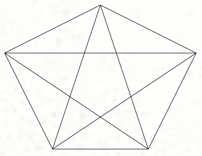
Then: (II+), (II-), (deg)

Possible cleaner result:

•Can we prove **tetrahedron degeneracy** implies **(deg)**?
(In fact this is what happens in EPRL vertex due to asymptotics!)
•If so, then first condition above can be dropped.

III.
Vertex amplitude and interpretation of the asymptotics

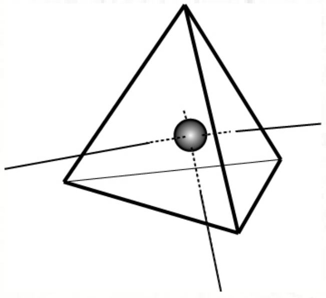
$SU(2)$ ***spin-nets on*** ∂S:



- V_k — spin-k $SU(2)$ rep.
- one k_{ab} and two $\psi_{ab}, \psi_{ba} \in V_{k_{ab}}$ per triangle (ab)
- one $SU(2)$ argument per triangle (ab)
- $\epsilon : V_k \times V_k \to \mathbb{C}$ — bilinear, skew symm. 'ϵ inner product' constructed from ϵ_{AB}.

$$\Psi_{\{k_{ab},\psi_{ab}\}}(\{g_{ab}\}) := \prod_{a<b} \epsilon(\psi_{ab}, \rho_{k_{ab}}(g_{ab})\psi_{ba})$$

$$\iota : \mathcal{H}^{LQG}_{\partial S} \to \mathcal{H}^{Spin(4)}_{\partial S}$$



(Triangulation)	*(Spinnet graph)*
triangle	link
tetrahedron	node

$$
\begin{aligned}
A_v(\{k_{ab}, \psi_{ab}\}) &:= A_v(\Psi_{\{k_{ab},\psi_{ab}\}}) \\
&= \int_{Spin(4)^5} \prod_a dG_a \left(\iota\Psi_{\{k_{ab},\psi_{ab}\}}\right)(G_{ab})
\end{aligned}
$$

where $G_{ab} := G_a^{-1} G_b$.

- G_a integral can be viewed as **integration over discrete connection** leading to the initial BF spin-foam sum.
- In the **asymptotic analysis** [Barrett, et. al. 2009], the G_a behave ***exactly*** like the parallel transports from tetrahedra to the 4-simplex frame.

Perelomov coherent states: $|k,\mathbf{n}\rangle \in V_k,\ \mathbf{n}\cdot\hat{L}|k,\mathbf{n}\rangle = k|k,\mathbf{n}\rangle$

Boundary coherent state:

$$\Psi_{\{k_{ab},\mathbf{n}_{ab}\}} := \Psi_{\{k_{ab},\psi_{ab}\}} \quad \text{with } |\psi_{ab}\rangle = |k_{ab},\mathbf{n}_{ab}\rangle$$

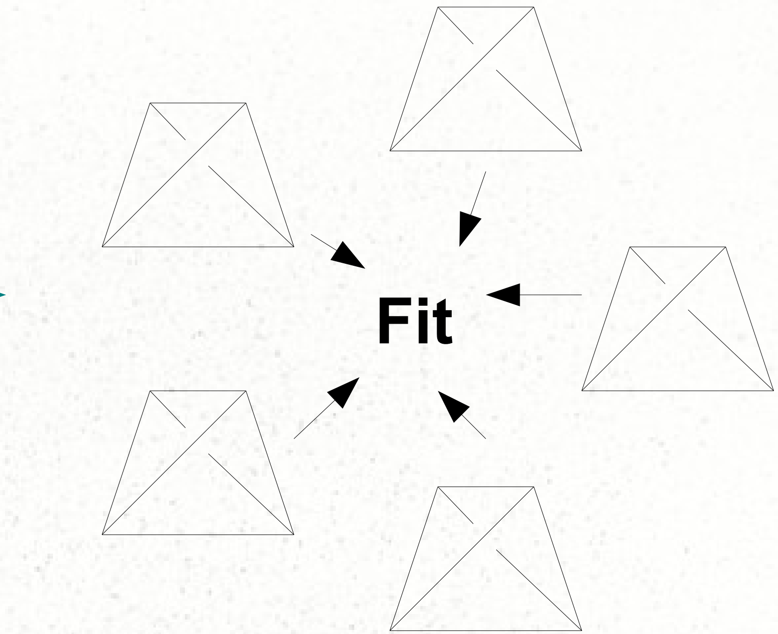
Is coherent boundary state corresponding to *classical* reduced boundary data:

$$A_{ab} = A(k_{ab}) := 8\pi G\gamma k_{ab}, \quad \mathbf{n}_{ab}$$

(Note overall phase ambiguity in $\Psi_{\{k_{ab},\mathbf{n}_{ab}\}}$)

If $\{k_{ab}, \mathbf{n}_{ab}\}$ satisfies

- ➔Tetrahedron non-degeneracy
- ➔Closure
- ➔Gluing constraint



then $\{k_{ab}, \mathbf{n}_{ab}\}$ is ***Regge-like***

For Regge-like boundary data $\{k_{ab}, \mathbf{n}_{ab}\}$, global phase ambiguity in $\Psi_{\{k_{ab}, \mathbf{n}_{ab}\}}$ can be *fixed* [Barrett et al., 2009] — yields the **Regge state** $\Psi^{\text{Regge}}_{\{k_{ab}, \mathbf{n}_{ab}\}}$.

$\{k_{ab}, \mathbf{n}_{ab}\}$ is a **vector geometry** if it satisfies ***closure***, and (roughly speaking) **data from different tetrahedra still** ***'fit'***, but ***data in each tetrahedron can be be degenerate***.

Strategy:

- Write the vertex as integral over the G_a's

$$A_v(\{k_{ab}, \mathbf{n}_{ab}\}) = \int_{Spin(4)} \prod_a dG_a e^{S[G_a]}$$

- Critical points: stationary S and maximal ReS

Critical point equations:

Restriction on boundary data for critical points to *exist*.

- Closure ($\sum_{b \neq a} k_{ab} \mathbf{n}_{ab} = 0$)
- Orientation ($X_a^{\pm} \triangleright \mathbf{n}_{ab} = -X_b^{\pm} \triangleright \mathbf{n}_{ba}$).

Just as in classical discrete theory!
Is consistent with interp. of $G_a = (X_a^-, X_a^+)$ as parallel transports to 4-simplex frame .

Theorem (EPRL asymptotics) (Barrett et al., 2009). Suppose $\mathcal{B} = \{k_{ab}, \mathbf{n}_{ab}\}$ satisfies closure. Then in the limit $\lambda \to \infty$,

1. If $\mathcal{B}$ is Regge-like, then

$$A_v(\Psi^{\text{Regge}}_{\{\lambda k_{ab}, n_{ab}\}}) \sim \lambda^{-12}\left[Ae^{iS_{\text{Regge}}} + Ae^{-iS_{\text{Regge}}} + Be^{\frac{i}{\gamma}S_{\text{Regge}}} + Ce^{-\frac{i}{\gamma}S_{\text{Regge}}}\right] \qquad (1)$$

2. If $\mathcal{B}$ is not Regge-like, but is a vector geometry, then

$$A_v(\Psi_{\{\lambda k_{ab}, n_{ab}\}}) \sim \lambda^{-12} N \qquad (2)$$

3. If $\mathcal{B}$ is not a vector geometry, then $A_v(\Psi_{\{\lambda k_{ab}, n_{ab}\}})$ decays exponentially with λ.

- **Each term** corresponds to a critical point in the G_a integral of the vertex, and therefore a particular value of the $\{G_a\}$'s.

- Can now ask: At each critical point, ***to what Plebanski sector does*** $\{B^{phys}_{ab}(A(k_{ab}), \mathbf{n}_{ab}, G_a)\}$ ***belong?***

1st two terms for Regge-like $\mathcal{B}$	:	(II+), (II-), respectively
All other non-exponentially suppressed terms in all cases	:	(deg)

Proper EPRL vertex amplitude

Can give an **explicit** expression for the **geometrical bivectors** of σ gauranteed by Reconstruction Thm!:

$$B_{ab}^{\text{phys}}(\sigma) = A(k_{ab})\beta_{ab}(\{G_{a'b'}\})(G_a \cdot \mathcal{N}) \wedge (G_b \cdot \mathcal{N})$$

where $\mathcal{N}^I := (1,0,0,0)$ and

$$\beta_{ab}(\{G_{a'b'}\}) := -\text{sgn}\big[\epsilon_{ijk}(G_{ac}\cdot\mathcal{N})^i(G_{ad}\cdot\mathcal{N})^j(G_{ae}\cdot\mathcal{N})^k \cdot \\ \cdot\epsilon_{lmn}(G_{bc}\cdot\mathcal{N})^l(G_{bd}\cdot\mathcal{N})^m(G_{be}\cdot\mathcal{N})^n\big]$$

where $\{c,d,e\} = \{0,\ldots 4\} \setminus \{a,b\}$ in any order, and $\text{sgn}(0) := 0$.

In terms of $G_{ab} = (X^-_{ab}, X^+_{ab})$: $\quad (G_{ab}\cdot\mathcal{N})^i = \text{tr}(\tau^i X^-_{ab} X^+_{ba})$

where $\tau^i := \frac{-i}{2}\sigma^i$

4-dim. closure of $B_{ab}^{\text{geom}}(\sigma)$ was key in deriving signs! $\qquad \sum_a V_a N_a^I = 0$

Recall $B^{\text{phys}}_{ab}(k_{ab}, \mathbf{n}_{ab}, G_a) = \mu B^{\text{geom}}_{ab}(\sigma)$

We now have an explicit expression for both sides and hence for μ!

$$\begin{aligned} \mu &= B^{\text{geom}}_{ab}(\sigma)_{IJ} B^{\text{phys}}_{ab}(k_{ab}, \mathbf{n}_{ab}, G_a)^{IJ} \\ &= (\text{pos. const.})\beta_{ab}(\{G_{a'b'}\})\text{tr}(\tau_i X^-_{ab} X^+_{ba}) n^i_{ab} \end{aligned}$$

Thus, condition for (II+):

$$\beta_{ab}(\{G_{a'b'}\})\text{tr}(\tau_i X^-_{ab} X^+_{ba}) n^i_{ab} > 0$$

Is sufficient to also impose non-degeneracy!

We will partially quantize this condition and insert it into the vertex amplitude.

Rewrite in terms of spatial rotation generators.

Spin(4) generators:

$$J_{ab} := \frac{1}{8\pi G}\left(B_{ab} + \frac{1}{\gamma}{}^*B_{ab}\right) = \frac{1}{8\pi G\gamma}((\gamma-1)\mathbf{B}^-_{ab}, (\gamma+1)\mathbf{B}^+_{ab})$$

$$= \frac{A_{ab}}{16\pi G\gamma}((1-\gamma)\mathbf{n}_{ab}, (1+\gamma)\mathbf{n}_{ab}) = \frac{k_{ab}}{2}((1-\gamma)\mathbf{n}_{ab}, (1+\gamma)\mathbf{n}_{ab})$$

SU(2) spatial rotation generators:

$$L^i_{ab} := (J^-_{ab})^i + (J^+_{ab})^i = k_{ab} n^i_{ab}$$

$$\boxed{\beta_{ab}(\{G_{a'b'}\})\mathrm{tr}(\tau_i X^-_{ab} X^+_{ba}) L^i_{ab} > 0}$$

$$\iota_k : \quad V_k \to \quad V_{j^-,j^+} \equiv V_{j^-} \otimes V_{j^+}$$

$$|k,m\rangle \mapsto \quad |j^+, j^-, k, m\rangle \qquad \text{where } s^\pm := \tfrac{1}{2}(1 \pm \gamma)k \ .$$

$$\epsilon : V_{j^-,j^+} \times V_{j^-,j^+} \to \mathbb{C}, \quad \epsilon(\alpha^- \otimes \alpha^+, \beta^- \otimes \beta^+) := \epsilon(\alpha^-, \beta^-)\epsilon(\alpha^+, \beta^+).$$

$$\left(\iota\Psi_{\{k_{ab},\psi_{ab}\}}\right)(G_{ab}) = \prod_{a<b} \epsilon\left(\iota_{k_{ab}}\psi_{ab}, \rho(G_{ab})\iota_{k_{ab}}\psi_{ba}\right)$$

$$\left(\hat{L}^i_{ab}\iota\Psi_{\{k_{cd},\psi_{cd}\}}\right)(G_{cd}) \quad = \quad \epsilon\left(\iota_{k_{ab}}\hat{L}^i\psi_{ab}, \rho(G_{ab})\iota_{k_{ab}}\psi_{ba}\right) \prod_{c<d,(cd)\neq(ab)} \epsilon\left(\iota_{k_{cd}}\psi_{cd}, \rho(G_{cd})\iota_{k_{cd}}\psi_{dc}\right)$$

$$\left(\hat{L}^i_{ba}\iota\Psi_{\{k_{cd},\psi_{cd}\}}\right)(G_{cd}) \quad = \quad \epsilon\left(\iota_{k_{ab}}\psi_{ab}, \rho(G_{ab})\iota_{k_{ab}}\hat{L}^i\psi_{ba}\right) \prod_{c<d,(cd)\neq(ab)} \epsilon\left(\iota_{k_{cd}}\psi_{cd}, \rho(G_{cd})\iota_{k_{cd}}\psi_{dc}\right)$$

Original vertex expression

$$\begin{aligned} A_v(\{k_{ab},\psi_{ab}\}) &:= A_v(\Psi_{\{k_{ab},\psi_{ab}\}}) \\ &= \int_{Spin(4)^5} \prod_a dG_a \left(\iota\Psi_{\{k_{ab},\psi_{ab}\}}\right)(G_{ab}) \\ &= \int_{Spin(4)^5} \prod_a dG_a \prod_{a<b} \epsilon(\iota_{k_{ab}}\psi_{ab}, \rho(G_{ab})\iota_{k_{ab}}\psi_{ba}) \end{aligned}$$

$(G_{ab} := G_a^{-1} G_b)$

Modification

- Would like to insert

$$\Theta\left(\beta_{ab}(\{G_{a'b'}\})\mathrm{tr}(\tau_i X^-_{ab} X^+_{ba}) L^i_{ab}\right)$$

- But L^i_{ab} is not in path integral! Instead ψ_{ab}, a vector in an irrep of $\hat{L}^i_{ab}$: **Quantize** L^i_{ab}.

- $\hat{L}^i_{ab}$ acts on ψ_{ab} via the $SU(2)$ generators $\hat{L}^i$. $\therefore$ we insert into **each face factor**

$$\boxed{\hat{P}_{ab}(\{G_{a'b'}\}) := P_{(0,\infty)}\left(\beta_{ab}(\{G_{a'b'}\})\mathrm{tr}(\tau_i X^-_{ab} X^+_{ba})\hat{L}^i\right)}$$

Yields `Proper EPRL vertex amplitude'

$$A_v^{II+}(\{k_{ab},\psi_{ab}\}) := \int_{Spin(4)^5} \prod_a dG_a \prod_{a<b} \epsilon(\iota_{k_{ab}}\psi_{ab}, \rho(G_{ab})\iota_{k_{ab}} P_{ba}(\{G_{a'b'}\})\psi_{ba})$$

Note: Projector can be put anywhere in each face factor (P_{ab} if on left of $\rho(G_{ab})$, P_{ba} if on right), and vertex is the *same*.

Properties:

➔SU(2) Spin-net data on boundary: can use to define SF dynamics **for LQG**

➔**Linear in the boundary state** --- needed for final transition amplitude to be **linear in initial and antilinear in final state**

$$\langle P_{\text{phys}}\Psi_f, P_{\text{phys}}\Psi_i\rangle_{\text{phys}}$$

➔One can show: **SU(2) gauge-invariant**

➔One can show: In **asymptotics**

- *Only* **single term $e^{iS_{\text{Regge}}}$**.
- All **deg. configurations exp. suppressed**.

Final Message

By defining the continuum 2-form assoc. to $\{B_{ab}\}$ via $B_{ab} = \int_{\Delta_{ab}} B$, we have proven that

- Linear simplicity allows Plebanski sectors $(II+), (II-)$, and (deg). Only $(II+)$ is usual GR.

- In the asymptotics of EPRL: The config. at the critical point for the term $e^{iS_{\text{Regge}}}$ is in (II+). ***For all other terms they are in (II-) or (deg).***

That is, non-GR asymptotics of EPRL is due to linear simplicity being too weak!

By supplementing Linear Simplicity ***precisely*** by imposing (II+), we obtain a **new vertex amplitude** not only with

- ***Same desireable properties of the EPRL model*** (LQG boundary states, *SU(2)* inv., linearity in the boundary state), **but also**
- ***Correct asymptotics*** $(\exp(iS_{Regge}))$***!!***

Outlook

➔ ***Lorentzian case:*** expected to be straightforward (if anyone is motivated to work with me on this, let me know.)

➔ ***General 4-cell (a la KKL):*** Basic ideas need to be rethought.

- $B^{IJ}_{\mu\nu}$ is over-constrained: More free components of $\{B^{IJ}_{ab}\}$ than of $B^{IJ}_{\mu\nu}$.
- In existing definition of $\beta_{ab}(\{G_{a'b'}\})$, that there are 5 tetrahedra per 4-simplex is used in key way.

Sum over orientations of the 4-simplex

Before this work, it was sometimes said that (II+) and (II-) terms in asymptotics correspond to sum over orientations of 4-simplex.

The above arguments lead to a different interpretation.

Difficulties I have regarding `sum over orientation interp.':

(1.) •Orientation is usually fixed once and for all just to be able to integrate
•In a path integral, one sums over dynamical variables
•Orientation is a dynamical variable in neither GR nor BF theory
$\therefore$ why does it make physical sense for there to be a sum over orientations in the path integral?

(2.) What about (deg) sector terms? Does the 4-simplex have no orientation in this sector? We can still integrate forms, so it seems it must have an orientation.