Why study boundaries?

Important physics happens at boundaries

- Generic presence of edge/surface/boundary/corner modes/charges/states/waves
- In gauge theories (e.g. gravity) boundaries support charges and symmetry algebras
- Boundaries may be at ∞ (e.g. $\mathcal{I}^+$), finite distance (BH), "fake" (entanglement), ...

Fits the logic of LQG

- Take seriously the **classical structure of general relativity**
- Build a quantization of GR which puts **symmetries at the center** (Diff, SU(2) so far)
- Start with **quasi-local** data and **build spacetime from gluing and coarse-graning**

New opportunities for LQG

- Can reconcile LQG (bulk-based) with "holographic" (boundary-based) approaches
- Possible tests of the classical underpinnings (Barbero–Immirzi parameter and tetrads)
- LQG tools are well-suited for the quantization of the boundary symmetries

Many tools already available

- Tensors networks and holography, boundary states, boundary dual theories, ...
- [Bianchi, Bodendorfer, Campiglia, Carrozza, Chirco, Colafranceschi, Delcamp, Dittrich, Livine, Goeller, Han, Hoehn, Hung, Husain, Oriti, Perez, Riello, Sengupta, Speziale, Steinhaus, Thiemann, Varadarajan, Wen, Zhang, ...]

What classical GR tells us

Charges and symmetries

- In gauge theories, Noether's theorems assign codimension-2 charges to symmetries

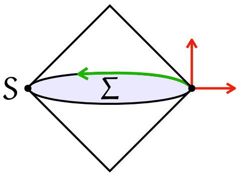
$$\not\delta Q_\xi = \delta_\xi \cdot \Omega = \delta Q_{\text{Noether}} + Q_{\text{flux}}$$

- Evolution of a charge

$$\begin{array}{ccccc} \delta_\xi Q_\zeta & = & Q_{[\xi,\zeta]} & + & \delta_\zeta \cdot Q_{\text{flux}} \\ \text{evolution} & = & \text{rotation} & + & \text{dissipation} \end{array}$$

- Corner symmetry group

$$\begin{array}{ccccc} G_S & = & (\text{Diff}(S) \ltimes H) & \ltimes & \mathbb{R}^2 \\ \text{group} & = & \text{kinematical} & \ltimes & \text{dynamical} \end{array}$$



Formulation-dependent

- For a given formulation F of gravity, the symplectic structure is

$$\Omega_F = \Omega_{\text{ADM}} + d\Omega_{F/\text{ADM}}$$

- Different formulations have different symmetry groups → **inequivalent quantizations**

ADM	H	$=$	$\emptyset$
Einstein–Hilbert	H	$=$	$SL(2,\mathbb{R})_\perp$
Einstein–Cartan	H	$=$	$SL(2,\mathbb{R})_\parallel \times (\text{Boosts})$
Einstein–Cartan–Holst	H	$=$	$SL(2,\mathbb{R})_\parallel \times SL(2,\mathbb{C})$

Some implications

Discreteness at the classical and continuum level

- LQG symplectic structure

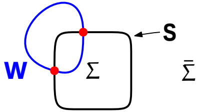
$$\Omega_{\text{LQG}} = \Omega_{\text{ADM}} + \mathrm{d}\big(\,\delta E_I\,\delta n^I - \gamma\,\delta e_I \wedge \delta e^I\,\big)$$

- Fluxes E_I form the familiar $\mathfrak{su}(2)$ algebra of LQG
- Tangential metric $q_{ab} = e^I_a e^J_b \eta_{IJ}$ on S forms an $\mathfrak{sl}(2,\mathbb{R})$ algebra

$$\big\{q_{ab}(x), q_{cd}(y)\big\} = -\gamma\big(q_{ac}\epsilon_{bd} + q_{bc}\epsilon_{ad} + q_{ad}\epsilon_{bc} + q_{bd}\epsilon_{ac}\big)(x)\delta^2(x-y)$$

- Casimirs related by $C_{\mathrm{SL}(2,\mathbb{R})} = -\big(\gamma^{-1}\sqrt{q}\big)^2 = C_{\mathrm{SU}(2)} \to$ **quantization of area element**

$$\sqrt{q}(x) = \gamma \ell_{\text{Pl}}^2 \sum_i \sqrt{j_i(j_i+1)}\,\delta^2(x-x_i)$$



Symmetries control the behavior of subsystems

- Subsystem can be subregion (how to cut / glue ?) or whole spacetime (“holography”)
- The logic of LQG tells us to label states with reps. of the boundary symmetry group