The Complete Barrett-Crane Model and its Causal Structure

based on 2206.15442 (PRD) and 2112.00091 (JCAP)

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in collaboration with Daniele Oriti and Andreas Pithis

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Munich Center of Quantum Science and Technology
Friedrich-Schiller-Universität Jena

Motivation and Overview

Role of causal structure in QG

Malament 1977; Bianchi, Martin-Dussaud 2109.00986; Livine, Oriti gr-qc/0210064; Figure: Wikipedia

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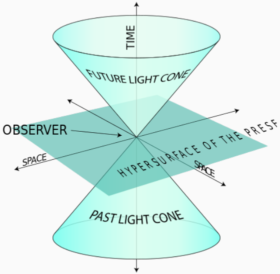
- Encodes all geometric information up to conformal factor
- Rich phenomenology: cosmological and black hole horizons



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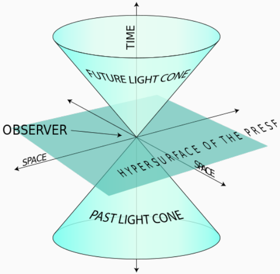

Expectation for QG theory: address the role of causality

- directly encode it into the quantum theory
- show how it arises in a classical and/or continuum limit

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A disambiguation of “causal structure” or “causality”

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Causal structure =	bare causality	+	time orientation
Locally	tangent vectors are timelike, lightlike or spacelike		timelike tangent vectors are future-pointing or past-pointing
Globally	two points have timelike, lightlike or spacelike separation		timelike separated points have a causal order

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$$K^\rho(\eta(X,Y)) = \frac{\sin \rho\eta}{\rho \sinh \eta}$$

$$K^\rho(T\cdot X, T\cdot Y) = K^\rho(-\eta(X,Y)) = K^\rho(X,Y)$$

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Time oriented BC model by Livine and Oriti

- Explicit realization of a quantum causal histories model

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Explicitly break invariance at the level of amplitudes

$$K^{\rho}(X,Y) = \sum_{\epsilon=\pm} \frac{\mathrm{e}^{i\epsilon\rho\eta}}{2i\rho\sinh(\epsilon\eta)}$$

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  - Exclusion of lightlike tetrahedra
  - No explicit GFT formulation

Perez, Rovelli gr-qc/0011037; Alexandrov, Kadar gr-qc/0501093; Speziale, Zhang 1311.3279; Conrady, Hnybida 1002.1959

Setting the scene

Jordan, Loll 1305.4582; Sorkin 1908.10022; Asante, Dittrich, Padua-Argüelles 2112.15387

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Objective

In the most democratic fashion, construct a GFT and spin foam model that includes spacelike, lightlike and timelike tetrahedra with all possible interactions

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Beyond the scope:

- (Space)time orientation
- Local causality conditions
  - Locally causal DT
  - Causality violations in Lorentzian Regge calculus

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The Barrett-Crane Model

The BC GFT model in an extended formulation

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BF quantization of first-order Palatini gravity with spacelike hypersurfaces

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Extended formulation:

$$\begin{array}{ccc} \mathrm{SL}(2,\mathbb{C})^4 & \longrightarrow & \mathrm{SL}(2,\mathbb{C})^4 \times \mathrm{H}^3 \\ \varphi(g_v) & \longrightarrow & \varphi(g_v; X) \\ \text{quadratic simplicity} & \longrightarrow & \text{linear simplicity} \end{array}$$

Barrett, Crane gr-qc/9904025; Baratin, Oriti 1108.1178; Oriti, Pithis, AJ 2112.00091

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$$S[\bar{\varphi}, \varphi] = K + V = \int [\mathrm{d}g]^4 \int \mathrm{d}X \, \bar{\varphi}(g_v; X)\varphi(g_v; X) +$$

$$+ \frac{\lambda}{5!} \int [\mathrm{d}g]^{10} \int [\mathrm{d}X]^5 \, \varphi_{1234}(X_1)\varphi_{4567}(X_2)\varphi_{7389}(X_3)\varphi_{962(10)}(X_4)\varphi_{(10)851}(X_5) + c.c.,$$

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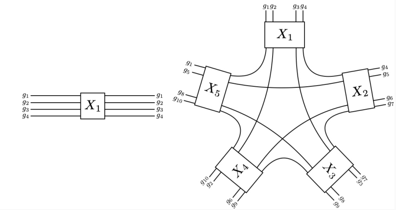
Right covariance : $\varphi(g_v;X) = \varphi(g_v h^{-1}; h\cdot X)$

Simplicity : $\varphi(g_v;X) = \varphi(g_v u_v;X)$



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  $\Rightarrow$ For $\gamma \to \infty$, simplicity is first class and should be imposed strongly

  (see Baratin, Oriti 1108.1178, 1111.5842)

  $\Rightarrow$ Unique formulation

- Constraints imposed covariantly
- Explicit form known in group representation
- Results of GFT condensate cosmology are recovered

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Geometric interpretation in bivector variables

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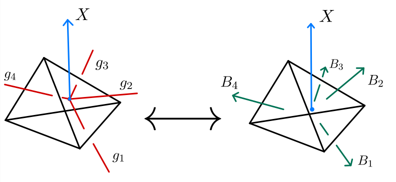
Geometric interpretation in bivector variables

Closure : $\sum_{t \in v} B_t = 0$

Simplicity : $X_A (*B)^{AB} = 0$



Completing the Barrett-Crane Model

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Idea 1

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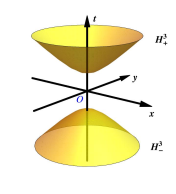
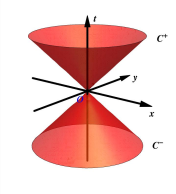
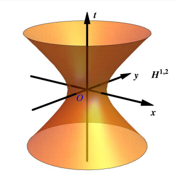
Allow for all normal vector signatures

α	$+$	0	$-$
	$X_+ = (1,0,0,0)$	$X_0 = (1,0,0,1)$	$X_- = (0,0,0,1)$
$\mathrm{U}^{(\alpha)}$	$\mathrm{SU}(2)$	$\mathrm{ISO}(2)$	$\mathrm{SU}(1,1)$
$\mathrm{SL}(2,\mathbb{C})/\mathrm{U}^{(\alpha)}$	H^3	C	$\mathrm{H}^{1,2}$







Speziale, Zhang 1311.3279; Conrady, Hnybida 1002.1959

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Allow for all possible simplicial interactions $\Rightarrow$ 21 vertices

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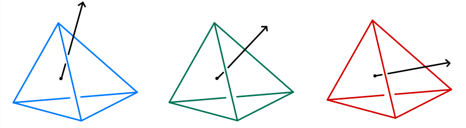
Allow for all possible simplicial interactions ⇒ 21 vertices

Kinetic term: $$K[\varphi,\bar{\varphi}] = \sum_\alpha \mu_\alpha \int [\mathrm{d}g]^4 \int \mathrm{d}X_\alpha\, \bar{\varphi}(g_v;X_\alpha)\varphi(g_v;X_\alpha)$$



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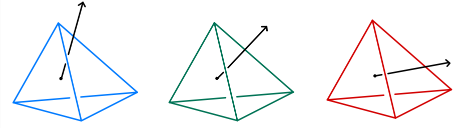

Interaction term: $$V[\varphi,\bar{\varphi}] = \sum_{(n_+,n_0,n_-)} \frac{\lambda^{(n_+,n_0,n_-)}}{5!}\mathcal{V}(n_+,n_0,n_-) + c.c.$$

$$\mathcal{V} = \int [\mathrm{d}g]^{10} \int [\mathrm{d}X_\alpha]^5\, \varphi_{1234}(X_{\alpha_1})\varphi_{4567}(X_{\alpha_2})\varphi_{7389}(X_{\alpha_3})\varphi_{962(10)}(X_{\alpha_4})\varphi_{(10)851}(X_{\alpha_5})$$

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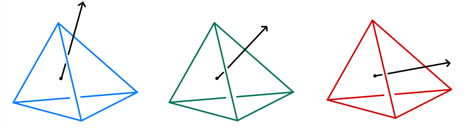

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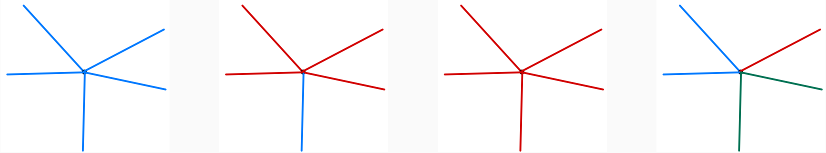

$(5,0,0)$ $(1,0,4)$ $(0,0,5)$ $(2,2,1)$

Spin representation

Spin representation

Canonical basis of unitary $\mathrm{SL}(2,\mathbb{C})$-irreps in the principal series

$$\mathcal{D}^{(\rho,\nu)}[\mathbb{C}^2] \cong \bigoplus_{j=|\nu|}^{\infty} \mathcal{Q}_j, \quad (\rho,\nu) \in \mathbb{R}\times\mathbb{Z}/2$$

Casimir operators of $\mathrm{SL}(2,\mathbb{C})$: $\quad \mathrm{Cas}_1 = -\rho^2+\nu^2-1, \qquad \mathrm{Cas}_2 = -\rho\nu$

Martin-Dussaud 1902.08439; Oriti, Pithis, AJ 2206.15442

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$$P^{(\rho,\nu),\alpha} = \int_{\mathrm{U}(\alpha)} \mathrm{d}u \, \boldsymbol{D}^{(\rho,\nu)}(u) = |\mathcal{I}^{(\rho,\nu),\alpha}\rangle \langle \mathcal{I}^{(\rho,\nu),\alpha}|$$

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$P^{(\rho,\nu),+} = \delta_{\nu,0} P^{(\rho,0),+}$	$\mathrm{Cas}_1 < 0$	s.l. triangles
$P^{(\rho,\nu),0} = \delta_{\nu,0} P^{(\rho,0),0}$	$\mathrm{Cas}_1 < 0$	s.l. triangles
$P^{(\rho,\nu),-} = \delta_{\nu,0} P^{(\rho,0),-} + \delta(\rho)\chi_{\nu\in 2\mathbb{N}+} P^{(0,\nu),-}$	$\mathrm{Cas}_1 \lessgtr 0$	s.l. or t.l. triangles

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Spin representation of group field

$$\varphi(g_v;X_\alpha) = \left[\prod_{i=1}^{4}\sum_{\nu_i}\int \mathrm{d}\rho_i\, 4\left(\rho_i^2+\nu_i^2\right)\right]\varphi^{\rho_v\nu_v,\alpha}_{j_v m_v}\prod_i D^{(\rho_i,\nu_i)}_{j_i m_i l_i n_i}(g_i g_{X_\alpha})\bar{\mathcal{I}}^{(\rho_i,\nu_i),\alpha}_{l_i n_i}$$

$$\int \mathrm{d}X_\alpha\, \varphi(g_v;X_\alpha) = \left[\prod_{i=1}^{4}\sum_{\nu_i}\int \mathrm{d}\rho_i\, 4\left(\rho_i^2+\nu_i^2\right)\right]\varphi^{\rho_v\nu_v,\alpha}_{j_v m_v} B^{\rho_v\nu_v,\alpha}_{l_v n_v}\prod_i D^{(\rho_i,\nu_i)}_{j_i m_i l_i n_i}(g_i)$$

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Spin representation of group field

$$\varphi(g_v;X_\alpha) = \left[\prod_{i=1}^{4}\sum_{\nu_i}\int \mathrm{d}\rho_i\, 4\left(\rho_i^2+\nu_i^2\right)\right]\varphi^{\rho_v\nu_v,\alpha}_{j_v m_v}\prod_i D^{(\rho_i,\nu_i)}_{j_i m_i l_i n_i}(g_i g_{X_\alpha})\bar{\mathcal{I}}^{(\rho_i,\nu_i),\alpha}_{l_i n_i}$$

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Generalized Barrett-Crane intertwiner

$$B^{\rho_v\nu_v,\alpha}_{j_v m_v} = \int_{\mathrm{SL}(2,\mathbb{C})/\mathrm{U}^{(\alpha)}} \mathrm{d}X_\alpha\prod_{i=1}^{4} D^{(\rho_i,\nu_i)}_{j_i m_i l_i n_i}(g_{X_\alpha})\bar{\mathcal{I}}^{(\rho_i,\nu_i),\alpha}_{l_i n_i}$$

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- Right covariance: $(g_v;X)\longrightarrow(g_v h^{-1};h\cdot X)$
- Simplicity: $(g_v;X)\longrightarrow(g_v u_v;X)$
- Change of representative: $g_X\longrightarrow g_X u$

Spin foam formulation

Spin foam formulation

Define kernels

$$D^{(\rho,\nu)}_{\alpha_1\alpha_2}(g^{-1}_{X_1}g_{X_2}) \equiv K^{(\rho,\nu)}_{\alpha_1\alpha_2}(X_1,X_2) := \sum_{jmln} \mathcal{I}^{(\rho,\nu),\alpha_1}_{jm} D^{(\rho,\nu)}_{jmln}(g^{-1}_{X_1}g_{X_2})\bar{\mathcal{I}}^{(\rho,\nu),\alpha_2}_{ln}$$

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Spin foam amplitude for a Lorentzian 2-complex Γ

$$\mathcal{A}_\Gamma = \prod_f \sum_{\nu_f} \int \mathrm{d}\rho_f \, \mathcal{A}_f(\rho_f,\nu_f) \prod_e \mathcal{A}^{\alpha_e}_e(\rho_{i_e},\nu_{i_e}) \prod_v \mathcal{A}^{\alpha_1,\dots,\alpha_5}_v(\rho_{v_a},\nu_{v_a})$$

$$\mathcal{A}_f(\rho_f,\nu_f) = 4(\rho_f^2+\nu_f^2)$$

$$\mathcal{A}^{\alpha_e}_e(\rho_{i_e},\nu_{i_e}) = 1$$

$$\mathcal{A}^{\alpha_1,\dots,\alpha_5}_v(\rho_{v_a},\nu_{v_a}) = \int [\mathrm{d}X_\alpha]^5 \prod_{a<b} K^{(\rho_{ab},\nu_{ab})}_{\alpha_a\alpha_b}(X_a,X_b).$$

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Spacelike triangles are labelled by $(\rho_f,0)$: continuous spectrum

Timelike triangles are labelled by $(0,\nu_f)$: discrete spectrum

Properties of the kernels

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Explicit expressions for the kernels via integral geometry

Gel'fand, Graev, Vilenkin 1966; Perez, Rovelli gr-qc/0011037; Oriti, Pithis, AJ 2206.15442

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$$K_{+0}^{(\rho,\nu)}(X_+,Y_0) = \delta_{\nu,0}\left(X_+^\mu Y_{0\mu}\right)^{i\rho-1}$$

$$K_{+-}^{(\rho,\nu)}(\eta,\hat{\boldsymbol{r}}) = \delta_{\nu,0}K_{+-}^{(\rho,0)}(\eta,\hat{\boldsymbol{r}})$$

$$K_{-0}^{(\rho,\nu)}(X_-,Y_0) = \delta_{\nu,0}\left|X_-^\mu Y_{0\mu}\right|^{i\rho-1}$$

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Spacetime orientation

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Spacetime orientation

Observation

Completion of BC model is unoriented

Extended symmetry under $\mathrm{O}(1,3) = \mathrm{SO}(1,3)^+ \rtimes \{1,T,P,TP\}$

$$K^{(\rho,\nu)}_{\alpha_1\alpha_2}(h\cdot X_1, h\cdot X_2) = K^{(\rho,\nu)}_{\alpha_1\alpha_2}(X_1,X_2), \qquad \forall h\in\mathrm{O}(1,3)$$

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Context and Applications

*Restricting the complete BC model

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Timelike normal vector BC model: $\alpha = +$

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$$K^{(\rho,\nu)}(\eta(X,Y)) = \frac{\sin\rho\eta}{\rho\sinh\eta}$$

Barrett, Crane gr-qc/9904025; Baratin, Oriti 1108.1178; Perez, Rovelli gr-qc/0011037

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- Only spacelike triangles

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Spacelike normal vector BC model (Perez-Rovelli): $\alpha = -$

- Single interaction with 5 timelike tetrahedra
- All combinations of spacelike and timelike triangles
- Extended formulation

*Relation to Conrady-Hnybida extension of EPRL

EPRL-CH	cBC

Conrady, Hnybida 1002.1959; Alexandrov, Kadar gr-qc/0501093

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$X_A((*B)^{AB} + \frac{1}{\gamma} B^{AB}) = 0$	$X_A(*B)^{AB} = 0$

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spatial discreteness $j \in \mathbb{N}/2$	spatial continuity $\rho \in \mathbb{R}$
temporal continuity $s \in \mathbb{R}$	temporal discreteness $\nu \in \mathbb{Z}/2$

Conrady, Hnybida 1002.1959; Alexandrov, Kadar gr-qc/0501093

CDT-like GFT model as a causal tensor model

Ambjorn, Goerlich, Jurkiewicz, Loll, 1203.3591

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Geometric assumptions in CDT:

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Geometric assumptions in CDT:

- Fixed edge lengths
- Timelike and spacelike edge lengths related by $\alpha > 0$: $\quad l_+^2 = \alpha a^2, \quad l_-^2 = -a^2$
- Two simplicial building blocks

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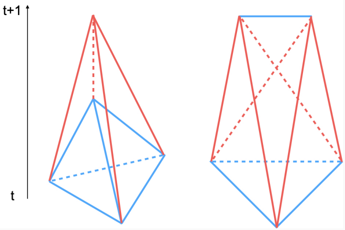
	$(4,1)$	$(3,2)$
tetrahedra	1 spacelike	0 spacelike
	4 timelike	5 timelike
triangles	4 spacelike	1 spacelike
	6 timelike	9 timelike
edges	6 spacelike	3 spacelike
	4 timelike	7 timelike



- No topological singularities
- Global foliation structure

CDT-like GFT model as a causal tensor model

Dittrich, Kogios 2203.02409; Gurau 1109.4812; Benedetti, Henson 0812.4261

CDT-like GFT model as a causal tensor model

▶ Fix representation labels $(\rho_v, \nu_v) = (\rho^*, \nu^*)$	fix areas
▶ Relate ρ^* and ν^* via α: $\rho^2 = 3\dfrac{1-\nu^2}{1+4\alpha} - 1$	relate s.l. and t.l. areas
▶ Allow only for two interactions	restrict to $(4,1)$ and $(3,2)$
▶ Introduce coloring	exlucde top. singularities
▶ Introduce dual-weighting	enforce foliation constraint

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$$Z = \sum_{\Gamma} \frac{1}{\mathrm{sym}(\Gamma)} \mathrm{e}^{S_{\mathrm{eff}}}$$

$$S_{\mathrm{eff}}(\Gamma) = -F_s \ln\left(\rho^{*2}\right) - F_t \ln\left(\nu^{*2}\right) - V_{(4,1)} \ln\left(\lambda_{(4,1)} \mathcal{A}^{(4,1)}\right) - V_{(3,2)} \ln\left(\lambda_{(3,2)} \mathcal{A}^{(3,2)}\right)$$

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Result

CDT-like GFT model $\Leftrightarrow$ causal tensor model

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Applications: Condensate cosmology and Landau-Ginzburg

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Physical reference frames in GFT condensate cosmology:

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- Background independence requires relational description
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Marchetti, Oriti 2008.02774;

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- How to properly distinguish clocks from rods?
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- What are the implications on cosmology, in particular perturbations?

Marchetti, Oriti 2008.02774; Marchetti, Oriti 2112.12677;

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Marchetti, Oriti 2008.02774; Marchetti, Oriti 2112.12677; Marchetti, Oriti, Pithis, Thürigen 2209.04297

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- What is the phase structure of the cBC model?

Marchetti, Oriti 2008.02774; Marchetti, Oriti 2112.12677; Marchetti, Oriti, Pithis, Thürigen 2209.04297

Summary, Open Problems and Perspectives

Summary, open problems and perspectives

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Open problems and perspectives

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- Examine phase structure via LG analysis or FRG

Back-Up

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