Black Holes and LQG: recent developments

Alejandro Perez
Centre de Physique Théorique, Marseille, France

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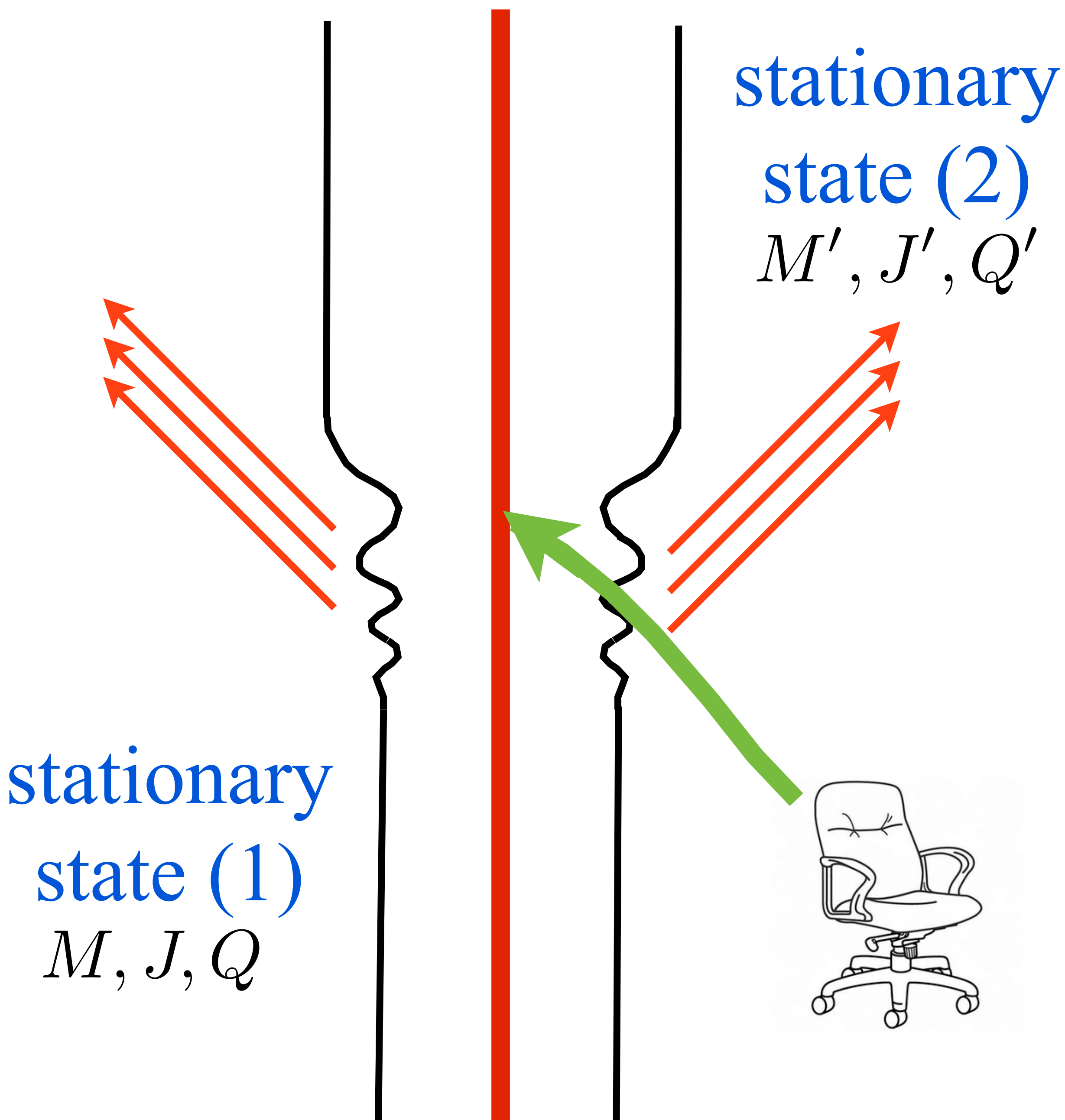
Based on resent results obtained in collaboration with
Amit Ghosh and Ernesto Frodden

Black Hole Thermodynamics

The 0th, 1st, 2nd and 3rd laws of BH

Some definitions

- $\Omega \equiv$ horizon angular velocity
- $\kappa \equiv$surface gravity ('grav. force' at horizon)
  If ℓ^a =killing generator, then $\ell^a \nabla_a \ell^b = \kappa \ell^b$.
- $\Phi \equiv$electromagnetic potential.



0th law: the surface gravity κ is constant on the horizon.

1st law:

$$\delta M = \frac{\kappa}{8\pi}\delta A + \underbrace{\Omega \delta J + \Phi \delta Q}_{\text{work terms}}$$

2nd law:

$\delta A \geq 0$

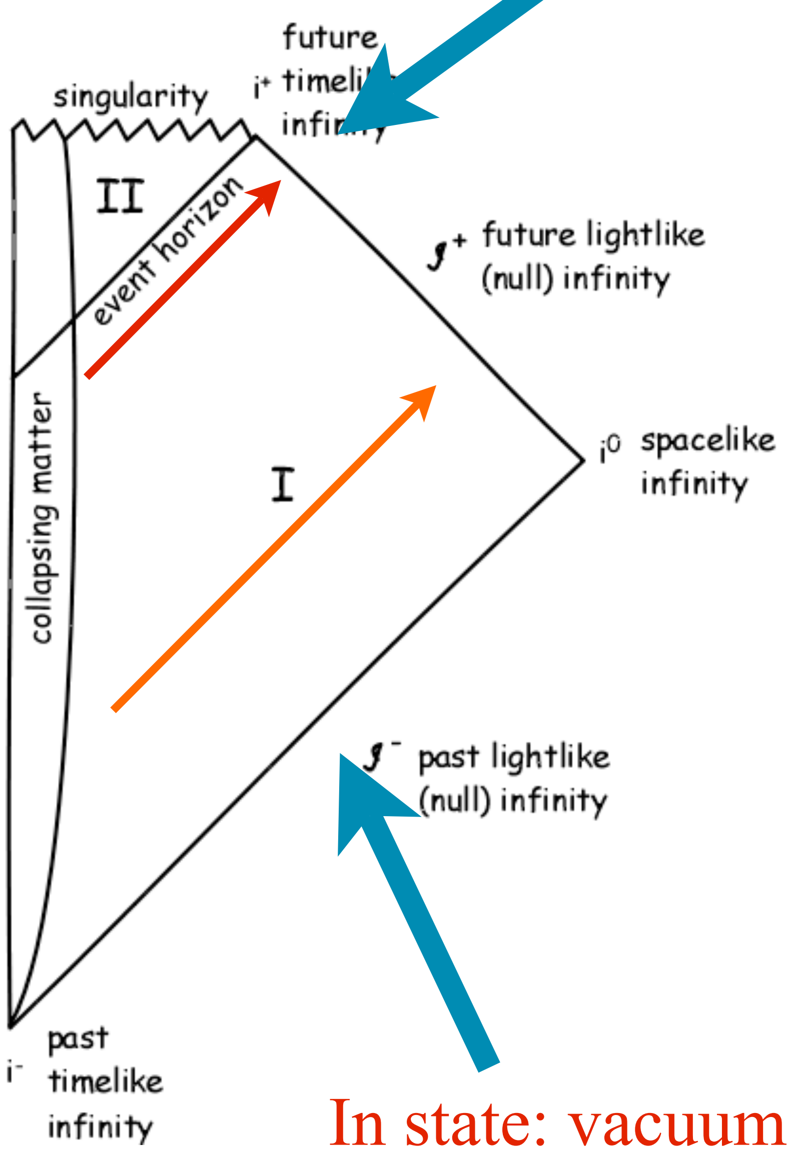
3rd law: the surface gravity value $\kappa = 0$ (extremal BH) cannot be reached by any physical process.

Black Hole Thermodynamics

Hawking Radiation: QFT on a BH background

Out state: thermal flux of particles as we approach the point **i+**

$$\langle \mathcal{N} \rangle = \frac{\Gamma}{\exp\left(\frac{2\pi}{\kappa}(\omega - \Omega m - q\Phi)\right) \pm 1},$$



Temperature at infinity

$$T_\infty = \frac{\kappa}{2\pi}$$

From the first law

$$\delta M = \frac{\kappa}{8\pi}\delta A + \Omega\delta J + \Phi\delta Q$$

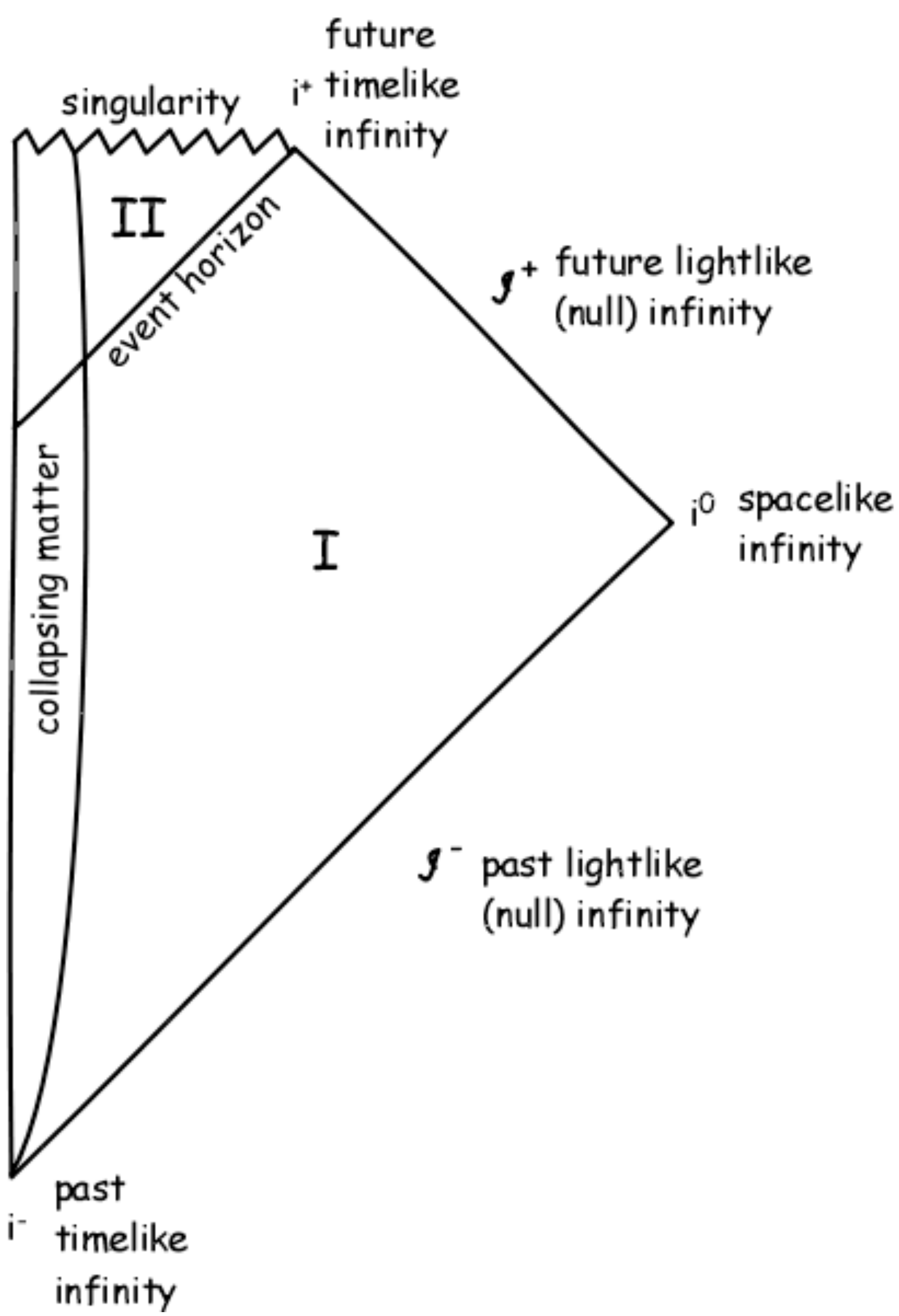
In state: vacuum far from **i-**

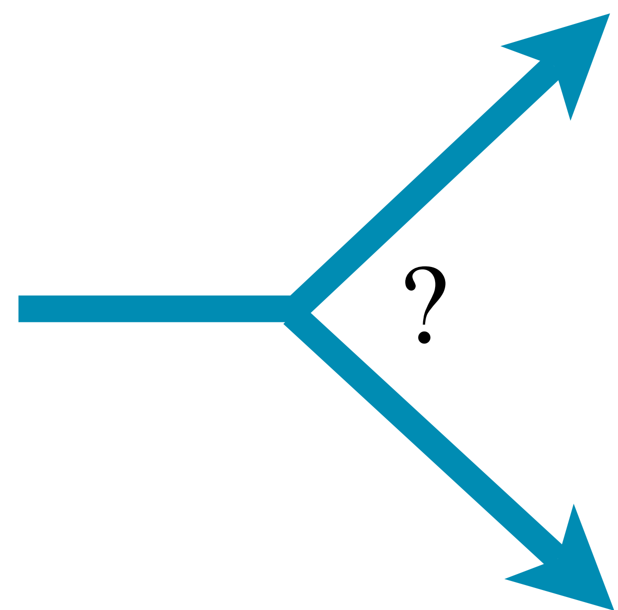
One gets the ENTROPY

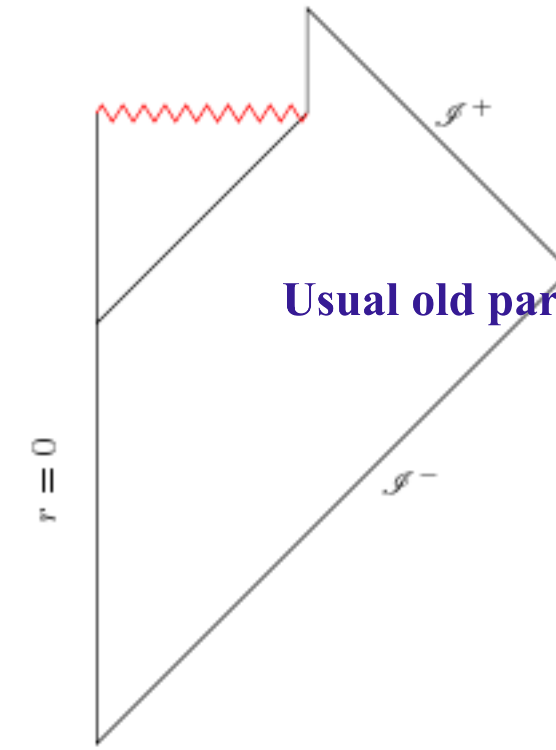
$$S = \frac{A}{4\ell_p^2}$$

Black Hole entropy in LQG

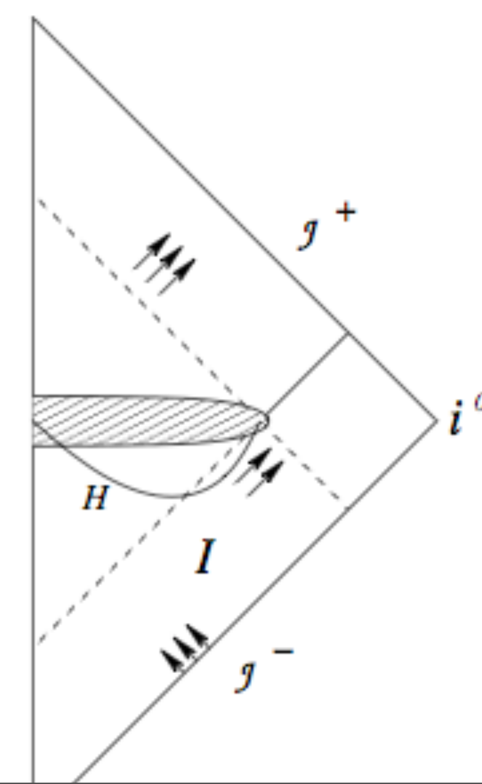
The standard definition of BH is GLOBAL (need a quasi-local definition)







Usual old paradigm

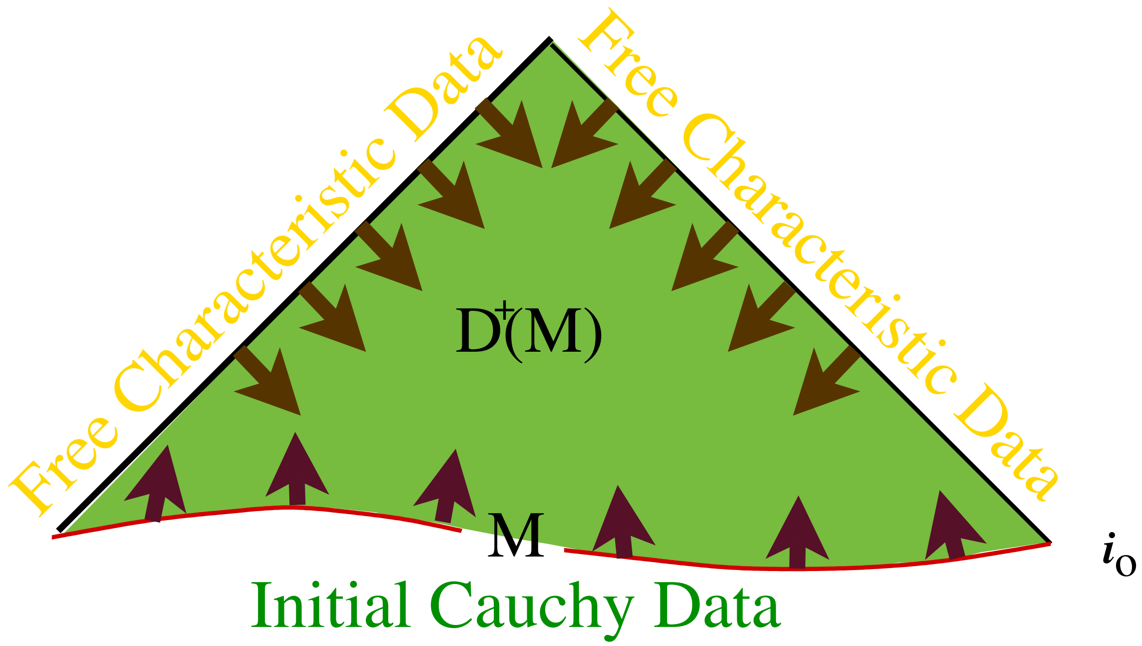


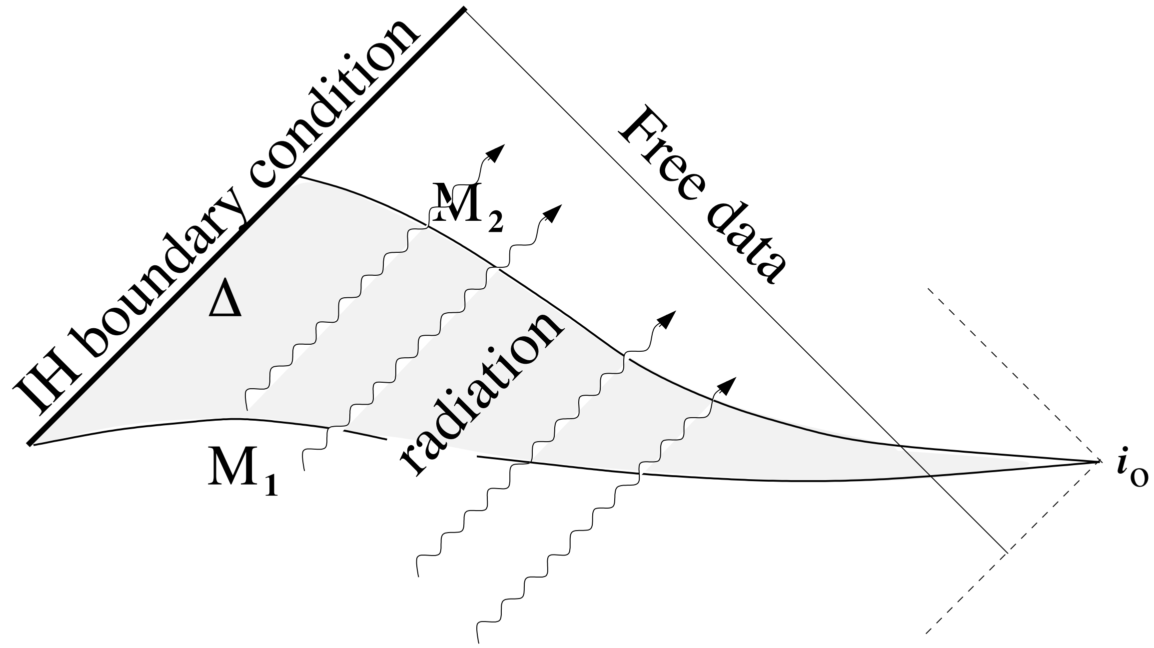
LQG Paradigm:
Ashtekar-Bojowald (2005),
Ashtekar-Taveras-Varadarajan (2008),
Ashtekar-Pretorius-Ramazanoglu (2011).

Isolated Horizons

classical foundation of the problem

[Ashtekar, Beetle, Corichi, Dreyer, Fairhurst, Krishnan, Lewandowski, Wisniewski]





• *Manifold conditions:* $\Delta \approx S^2 \times R$, foliated by a (preferred) family of 2-spheres S and equipped with an foliation preserving equivalence class $[\ell^a]$ of transversal future pointing vector fields.

• *Dynamical conditions:* All field equations hold at Δ.

• *Matter conditions:* On Δ the stress-energy tensor T_{ab} of matter is such that $-T^a{}_b\ell^b$ is causal and future directed.

• *Conditions on the metric g determined by e, and on its levi-Civita derivative operator ∇:* (iv.a) The expansion of ℓ^a within Δ is zero. (iv.b) $[\mathscr{L}_\ell, D] = 0$.

• *Restriction to 'good cuts.'* One has $D_a\ell^b = \omega_a\ell^b$ for some ω_a intrinsic to Δ. A 2-sphere cross-section S of Δ is called a 'good cut' if the pull-back of ω_a to S is divergence free with respect to the pull-back of g_{ab} to S.

The plan

BH thermodynamics from a local perspective

1. (Classical) There is a well defined notion of quasilocal energy for a black hole horizon close to equilibrium and an associated local first law of BH mechanics. This provides a dynamical process version of first law for *isolated horizons.*

2. (Quantum) The statistical mechanical treatment of the quantized horizon degrees of freedom reproduce the expected results known from Hawking semiclassical analysis (QFT on curved background calculation) for all values of the Immirzi parameter.

3. (Speculative for the moment) There might be potential observable effects (damping of Hawing radiation for Fermionic fields).

Local laws of BH mechanics

BH thermodynamics from a local perspective

Ghosh-Frodden-AP available at
http://inspirehep.net/record/940357

Black Hole Thermodynamics

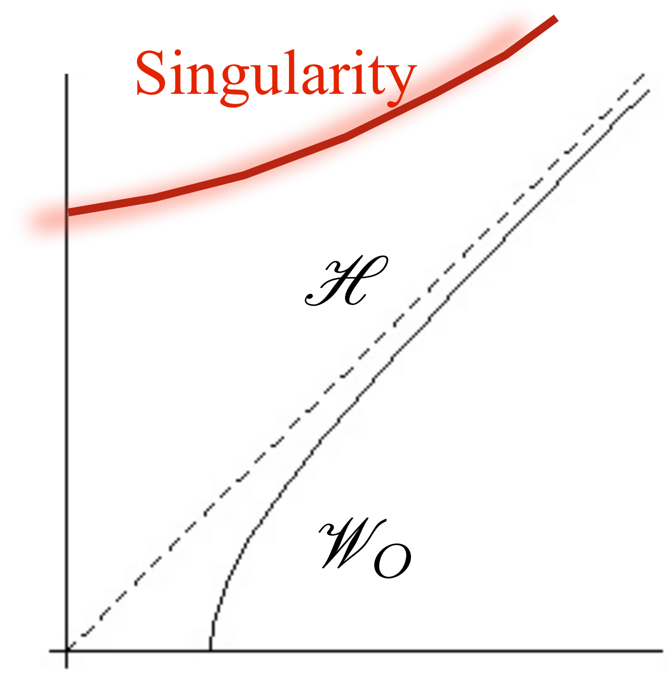
Stationary BHs from a local perspective

$\ell^2 << A$

Introduce a family of
local stationary observers
~ZAMOS

$$\chi = \xi + \Omega\,\psi = \partial_t + \Omega\,\partial_\phi$$

$$u^a = \frac{\chi^a}{\|\chi\|}$$

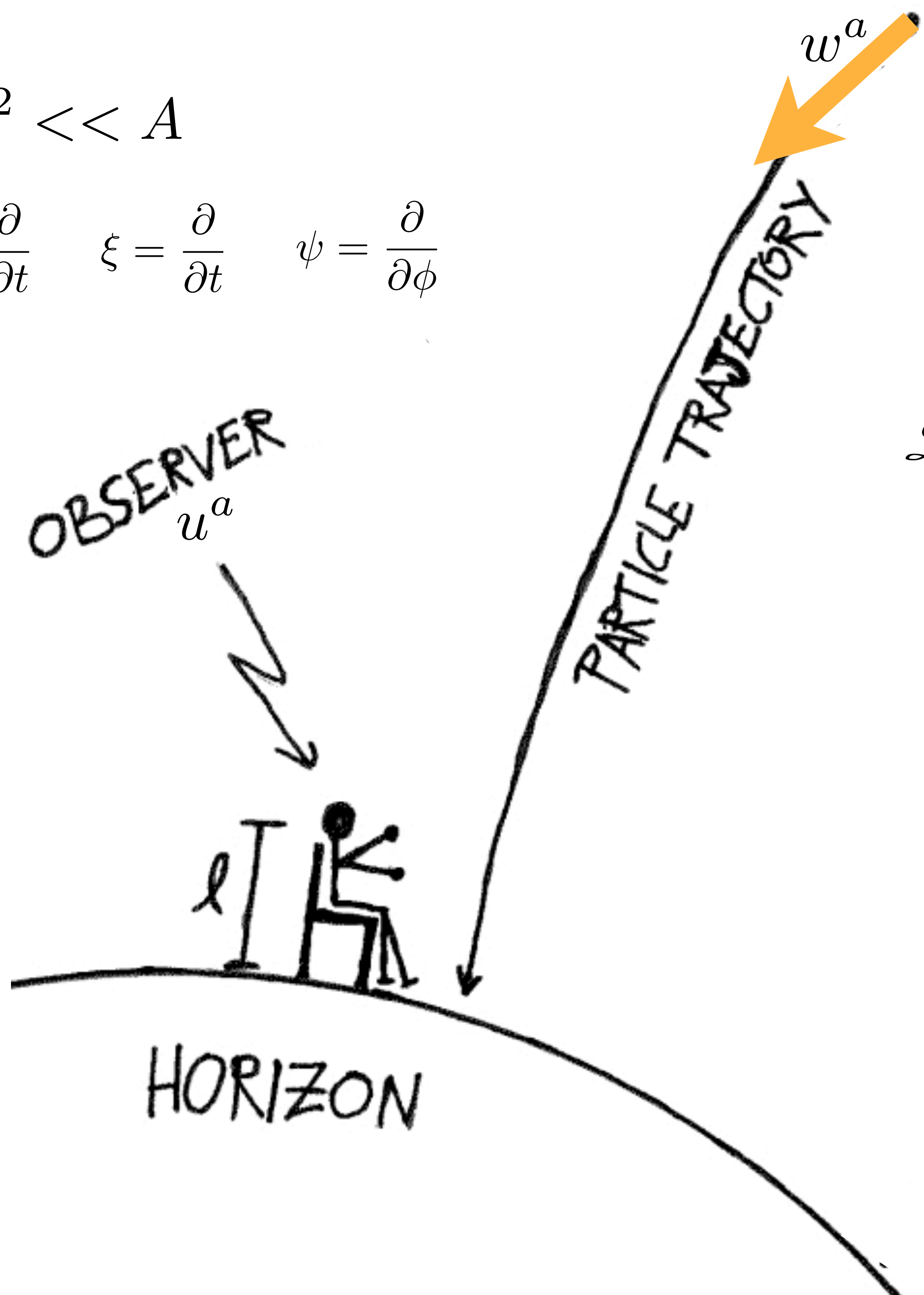


A thought experiment

throwing a test particle from infinity

$$\ell^2 << A$$

$$\chi = \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial t} \qquad \xi = \frac{\partial}{\partial t} \qquad \psi = \frac{\partial}{\partial \phi}$$



Particle's equation of motion

$$w^a \nabla_a w_b = q\, F_{bc} w^c$$

Symmetries of the background

$$\mathscr{L}_\xi g_{ab} = \mathscr{L}_\psi g_{ab} = \mathscr{L}_\xi A_a = \mathscr{L}_\psi A_a = 0$$

Conserved quantities

$$\mathcal{E} \equiv -w^a \xi_a - q A^a \xi_a$$

$$L \equiv w^a \psi_a + q A^a \psi_a$$

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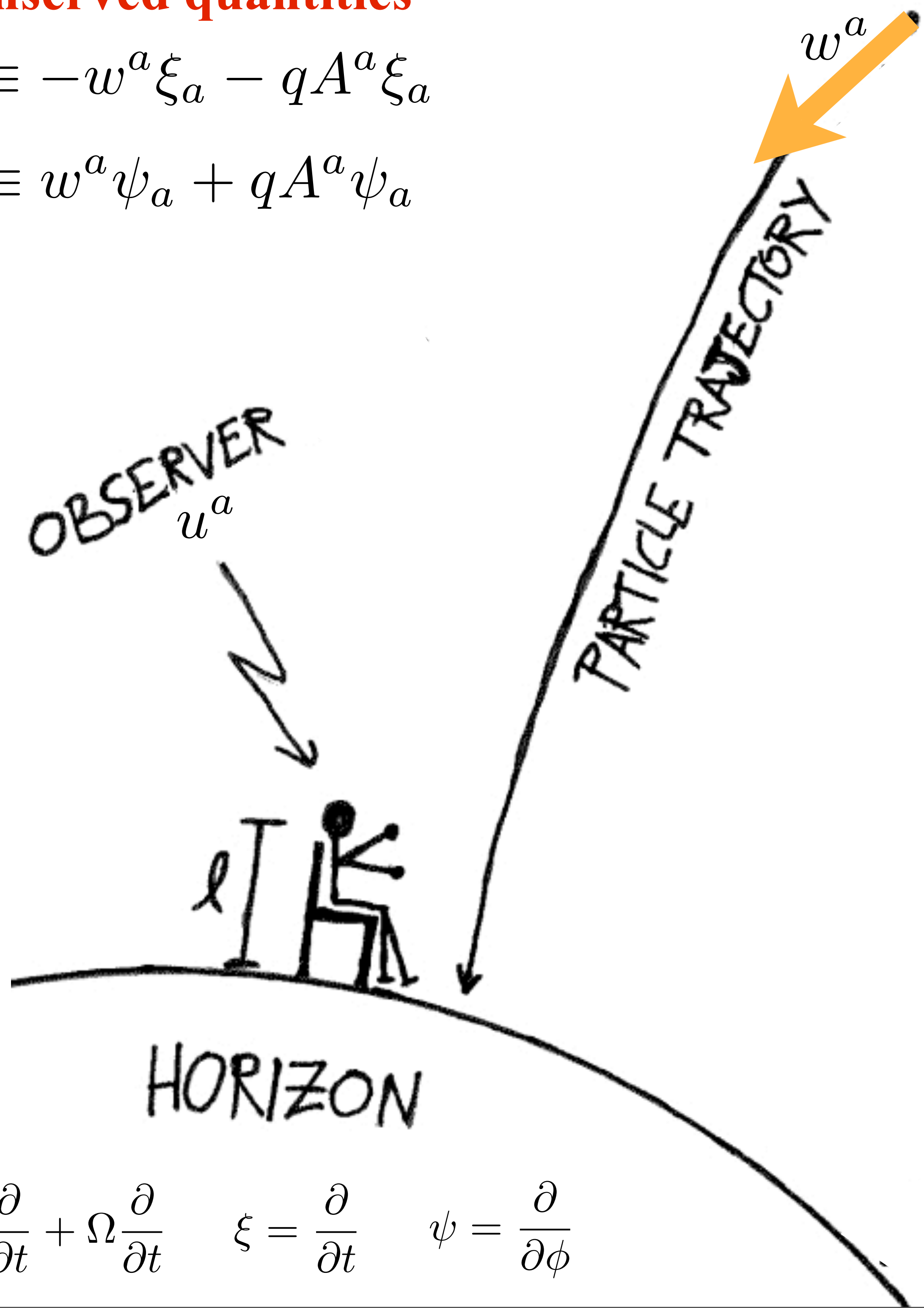

Particle at infinity

$$\mathcal{E} = -w^a \xi_a|_\infty \equiv \text{energy}$$

$$L = w^a \Psi_a|_\infty \equiv \text{angular momentum}$$

$$\chi = \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial t} \qquad \xi = \frac{\partial}{\partial t} \qquad \psi = \frac{\partial}{\partial \phi}$$

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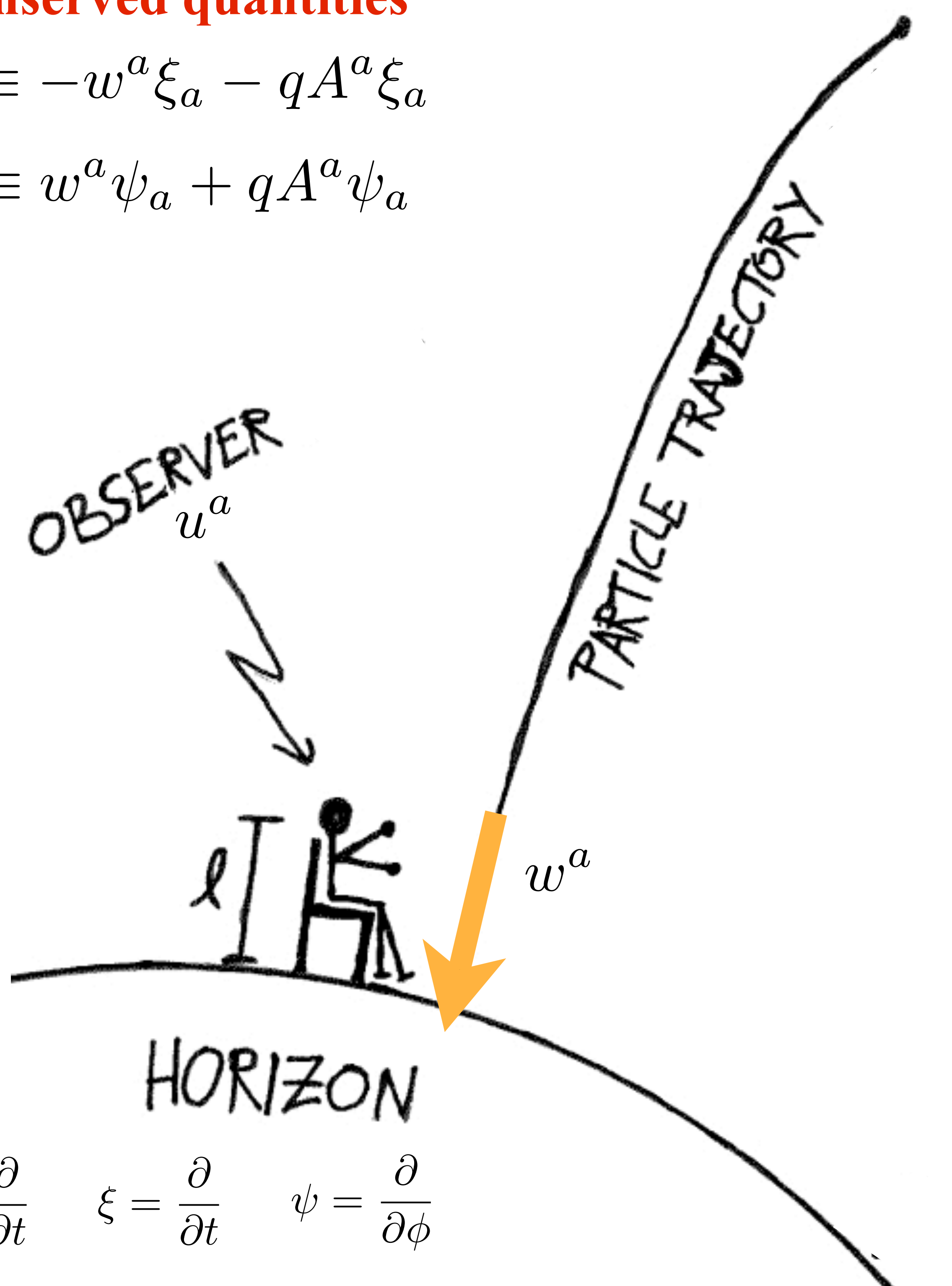

At the local observer

$$\mathcal{E}_{\ell oc} \equiv -w^a u_a \equiv \text{local energy}$$

$$\chi = \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial t} \qquad \xi = \frac{\partial}{\partial t} \qquad \psi = \frac{\partial}{\partial \phi}$$

After absorption
seen from infinity

$$\chi = \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial t} \qquad \xi = \frac{\partial}{\partial t} \qquad \psi = \frac{\partial}{\partial \phi}$$

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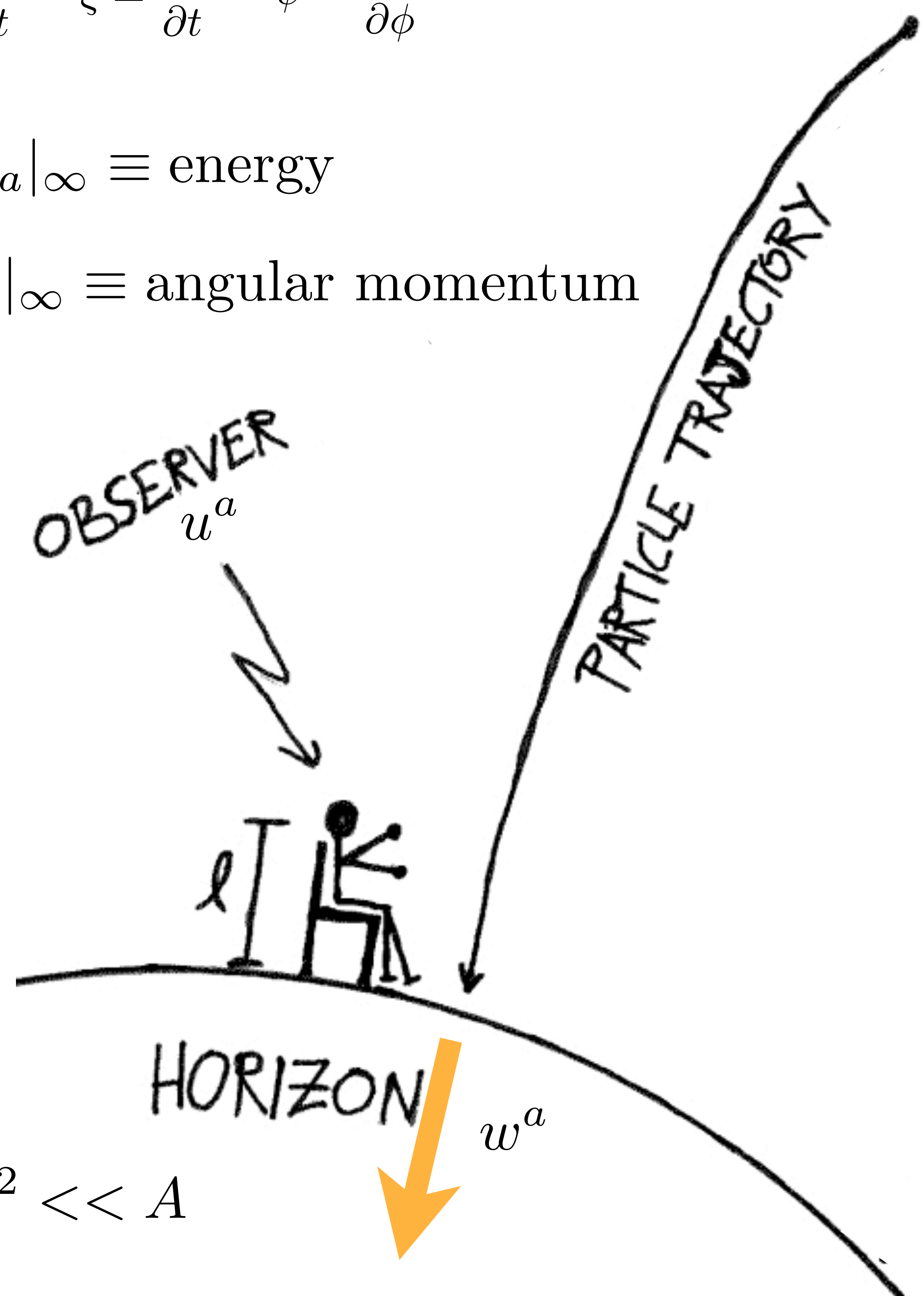


The BH readjusts parameters

$$\delta M = \mathcal{E} \qquad \delta J = L$$

$$\delta Q = q$$

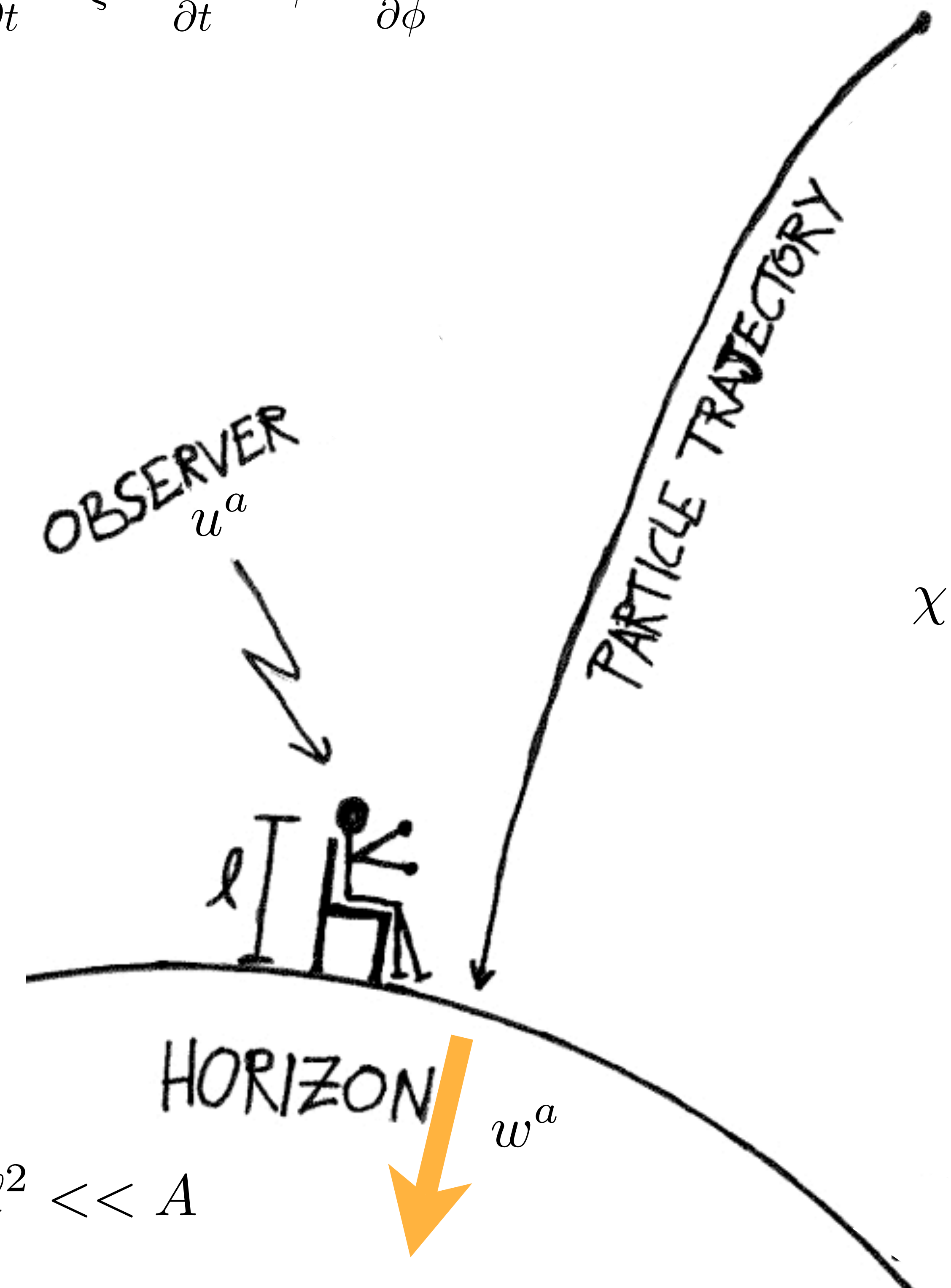
The area change from 1st law

$$\delta M = \frac{\kappa}{8\pi}\delta A + \Omega \delta J + \Phi \delta Q$$

$$\frac{\kappa}{8\pi}\delta A = \mathcal{E} - \Omega L - \Phi q$$

After absorption
seen by a local observer

$$\chi = \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial t} \qquad \xi = \frac{\partial}{\partial t} \qquad \psi = \frac{\partial}{\partial \phi}$$



$\ell^2 << A$

$$\frac{\kappa}{8\pi}\delta A = \mathcal{E} - \Omega L - \Phi q$$

At the local observer

$$\mathcal{E}_{\ell oc} \equiv -w^a u_a \equiv \text{local energy}$$

$$\chi = \xi + \Omega\,\psi = \partial_t + \Omega\,\partial_\phi \qquad u^a = \frac{\chi^a}{\|\chi\|}$$

$$\mathcal{E}_{\ell oc} = -\frac{w^a \xi_a + \Omega w^a \psi_a}{\|\chi\|}$$

$$\mathcal{E}_{\ell oc} = \frac{\mathcal{E} - \Omega L - q\Phi}{\|\chi\|}$$

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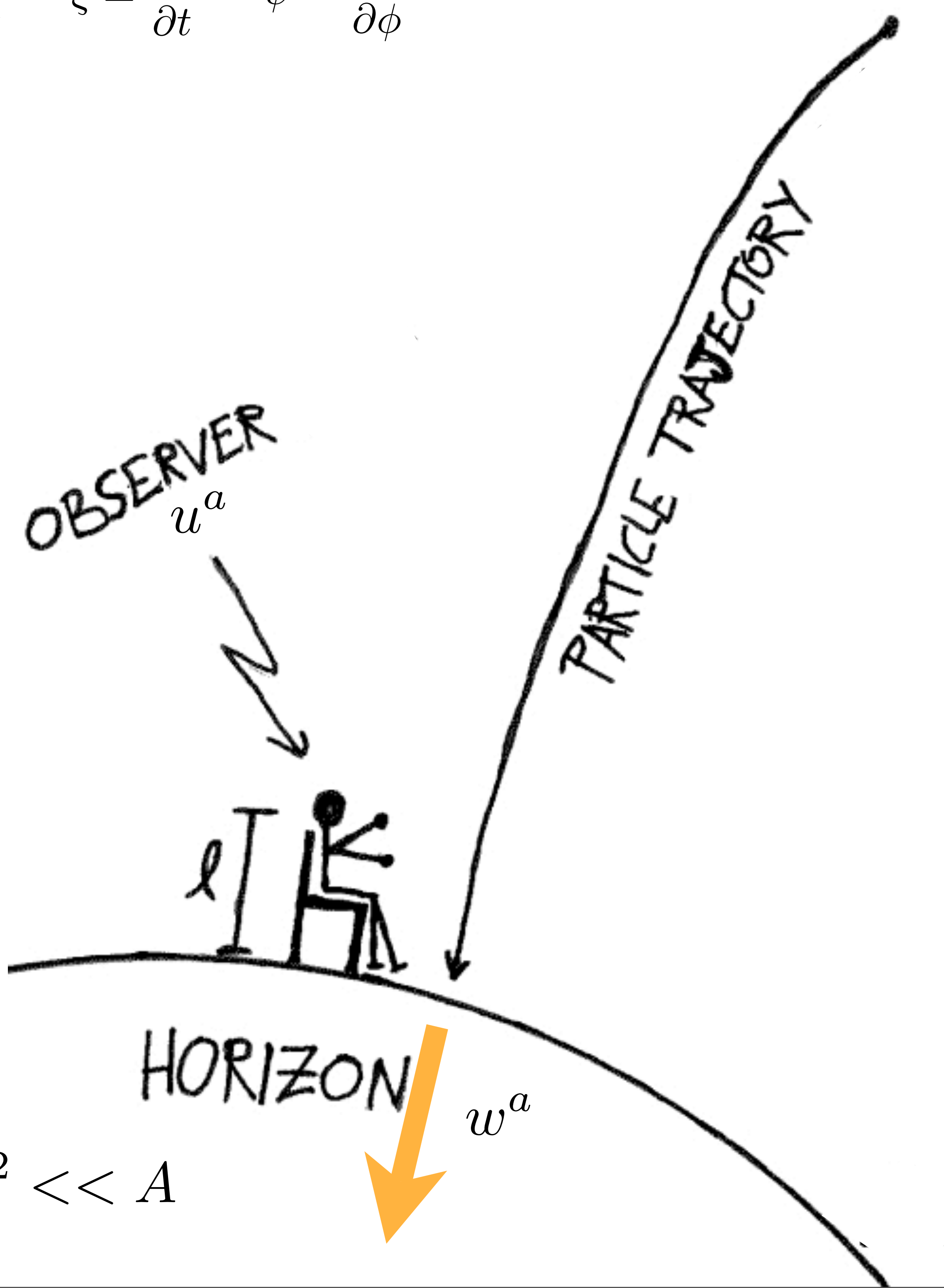


$\ell^2 << A$

$$\frac{\kappa}{8\pi}\delta A = \mathcal{E} - \Omega L - \Phi q$$

$$\mathcal{E}_{\ell oc} = \frac{\mathcal{E} - \Omega L - q\Phi}{\|\chi\|}$$

$$\mathcal{E}_{loc} \quad = \quad \frac{\kappa}{8\pi\|\chi\|}\delta A$$

$$\mathcal{E}_{loc} \quad = \quad \frac{\overline{\kappa}}{8\pi}\delta A$$

$$\overline{\kappa} \equiv \frac{\kappa}{\|\chi\|}$$

Local first law

Local BH energy

$$\mathcal{E}_{\ell oc} \equiv -w^a u_a \equiv \text{local energy of the absorbed particle}$$

The appropriate local energy notion must be the one such that:

$$\delta E = \mathcal{E}_{\ell oc}$$

$$\delta E = \frac{\bar{\kappa}}{8\pi}\delta A$$

Local first law

Local BH energy

$\ell^2 << A$

$\mathcal{E}_{\ell oc} \equiv -w^a u_a \equiv$ local energy of the absorbed particle

The appropriate local energy notion must be the one such that:

$$\delta E = \frac{\overline{\kappa}}{8\pi}\delta A$$

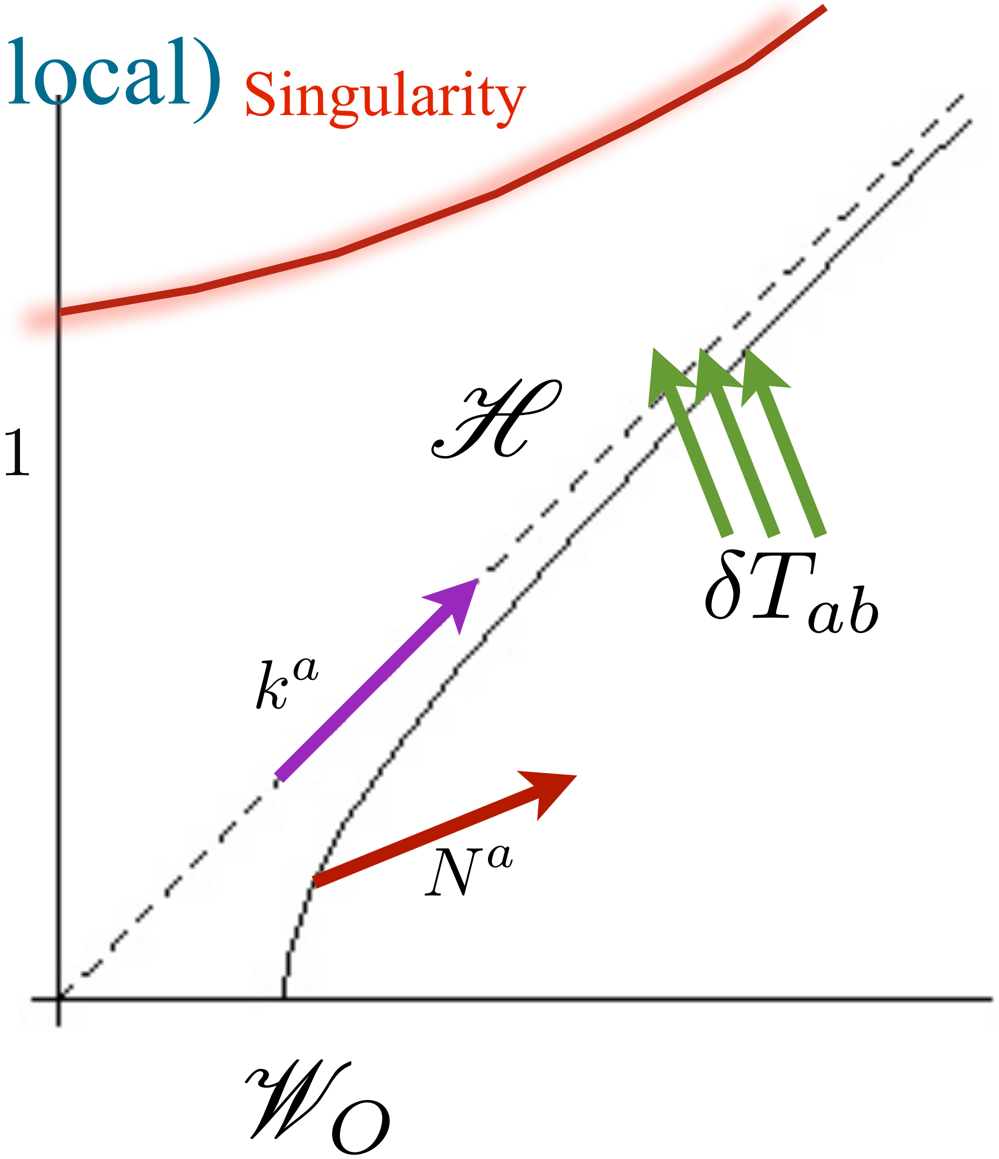
$$\overline{\kappa} \equiv \frac{\kappa}{||\chi||} = \frac{1}{\ell} + o(\ell)$$

$$E = \frac{A}{8\pi\ell}$$

Local first law

$\chi = \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial t} \quad \xi = \frac{\partial}{\partial t} \quad \psi = \frac{\partial}{\partial \phi}$

A refined argument
(dynamical and more local)



$J^a = \delta T^a{}_b \chi^b$ is conserved thus

(1) $$\int_{\mathscr{H}} dV dS \; J_b k^b = \int_{W_{\mathscr{O}}} J_b N^b \qquad \text{with} \qquad k(V) = 1$$

(2) $$\int_{\mathscr{H}} dV dS \; \delta T_{ab} \overbrace{\kappa V k^a}^{\chi^a} k^b = \| \chi \| \underbrace{\int_{W_{\mathscr{O}}} \delta T_{ab} u^a N^b}_{\delta E}$$

The Raychaudhuri equation

(3) $$\frac{d\theta}{dV} = -8\pi \delta T_{ab} k^a k^b$$

(4) $$-\frac{\kappa}{8\pi \| \chi \|} \underbrace{\int_{\mathscr{H}} dV dS \; V \frac{d\theta}{dV}}_{-\delta A} = \delta E,$$

$$\delta E = \frac{\kappa}{8\pi} \delta A$$

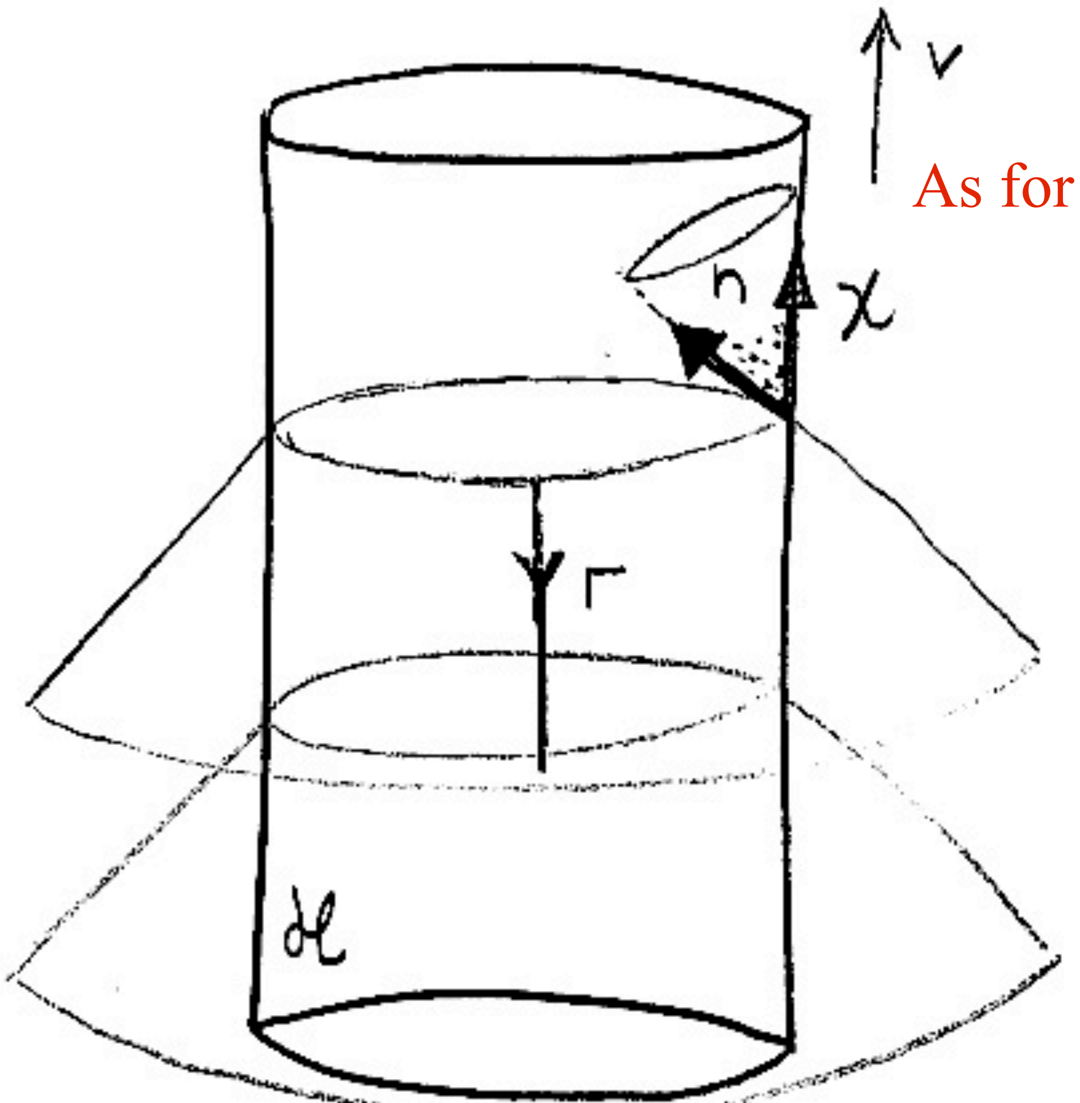
Local first law for IHs

(a completely local argument)

$$ds^2 = 2dv_{(a}dr_{b)} - 2(r-r_0)[2dv_{(a}\omega_{b)} - \kappa_{IH}dv_a dv_b] + q_{ab} + o[(r-r_0)^2]$$

Ashtekar-Beetle-Dreyer-Fairhurst-Krishnan-Lewandowski-Wisniewski PRL 85 (2000)

Stationary observers exist $\mathscr{L}_\chi g_{ab}|_{\mathscr{H}} = 0$



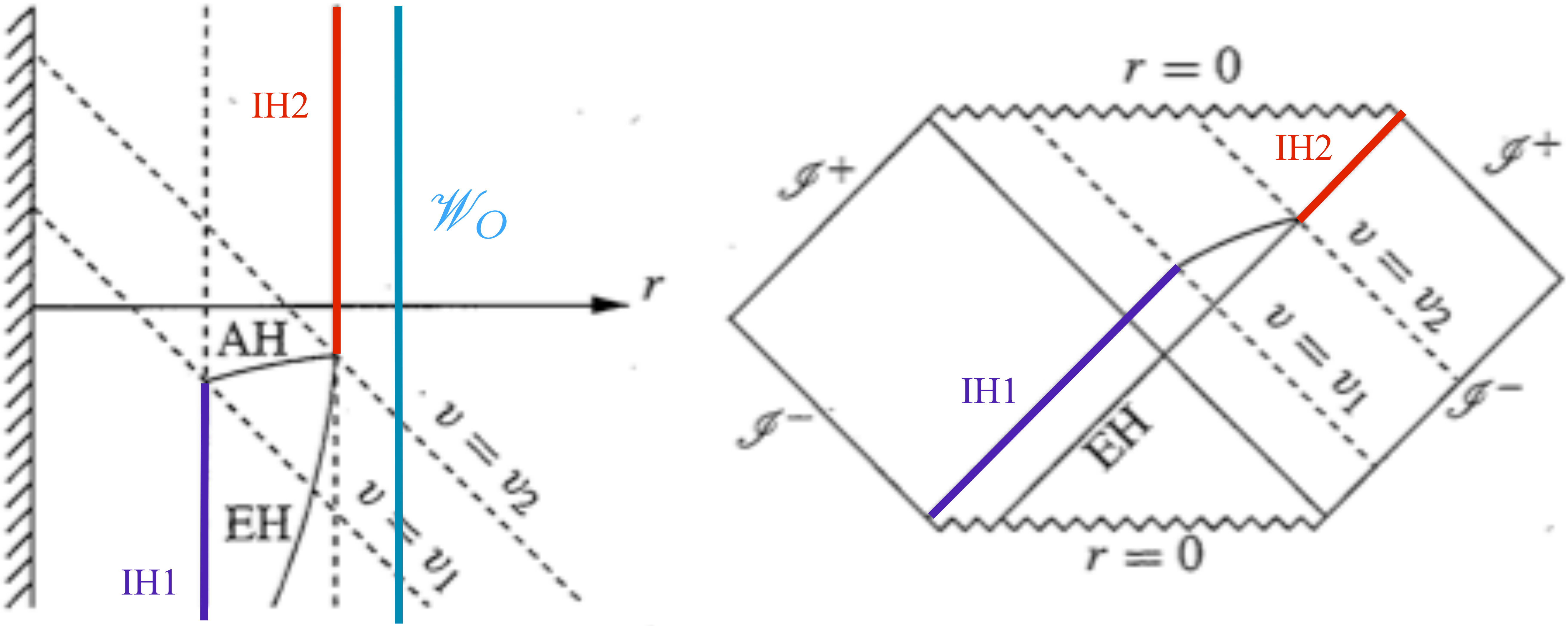
As for stationary BHs they are Rindler like $\overline{\kappa} = \frac{\kappa_{IH}}{||\chi||} = \frac{1}{\ell}$

Similar argument using Gauss and Raychaudhuri equation

$$\delta E = \frac{\overline{\kappa}}{8\pi}\delta A$$

The Local first law is dynamical

Simple example: Vaidya spacetime



$$\delta E = \frac{\overline{\kappa}}{8\pi}\delta A$$

A dynamical first law for IHs

Main classical results

$$\delta E = \frac{\overline{\kappa}}{8\pi}\delta A$$

$$\overline{\kappa} \equiv \frac{\kappa}{||\chi||} = \frac{1}{\ell} + o(\ell)$$

$$E = \frac{A}{8\pi\ell}$$

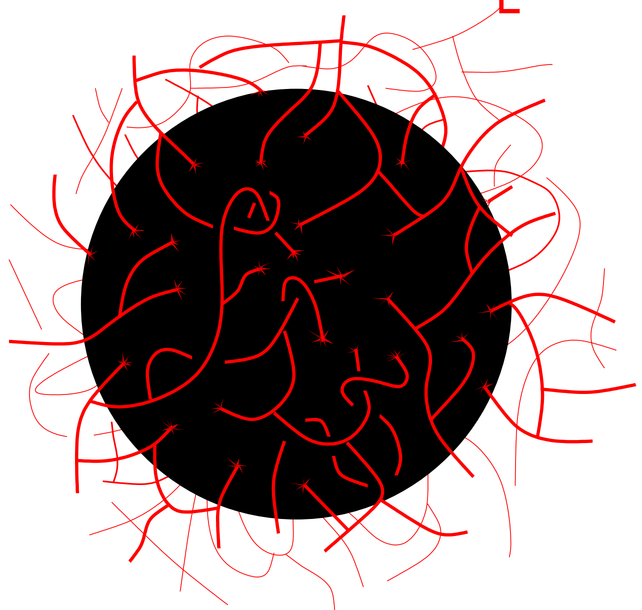
Quantization

Chern-Simons theory with spin-network punctures

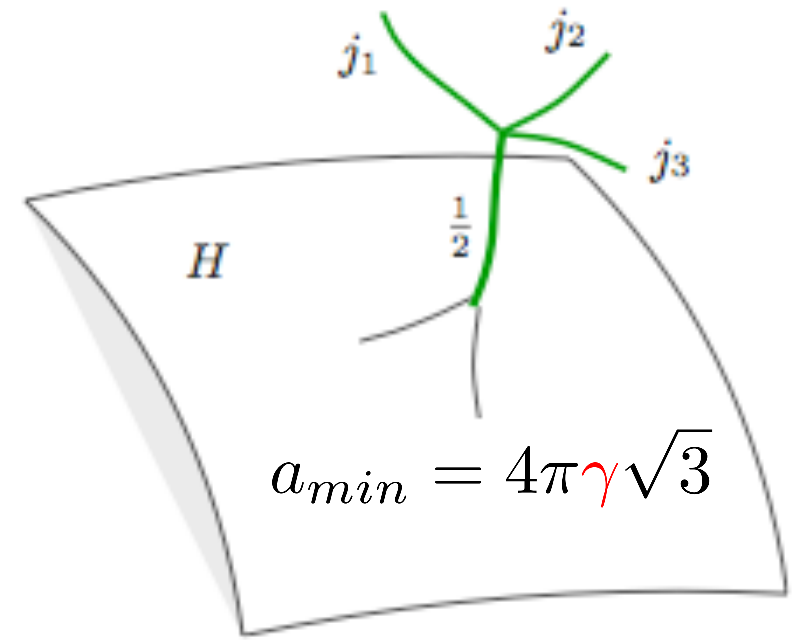
Loop quantum gravity

The area gap is an **energy gap**

$$\widehat{A}_S\,|j_1, j_2\cdots\rangle = \left[8\pi\gamma\ell_p^2 \sum_p \sqrt{j_p(j_p+1)}\right] |j_1, j_2\cdots\rangle$$



The AREA gap and BH quantum transitions



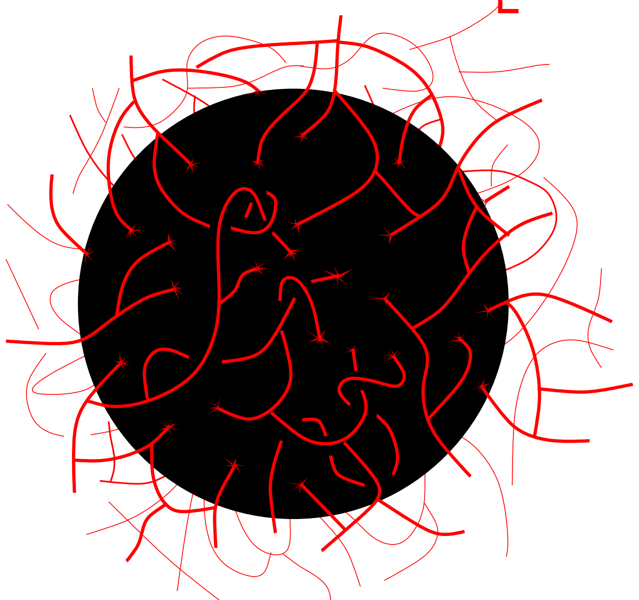
a) By a rearrangement of the spin quantum numbers labelling spin network links ending at punctures on the horizon without changing the number of punctures N (in the large area regime this kind of transitions allows for area jumps as small as one would like as the area spectrum becomes exponentially dense in $\mathbb{R}^+$ [Rovelli 96]

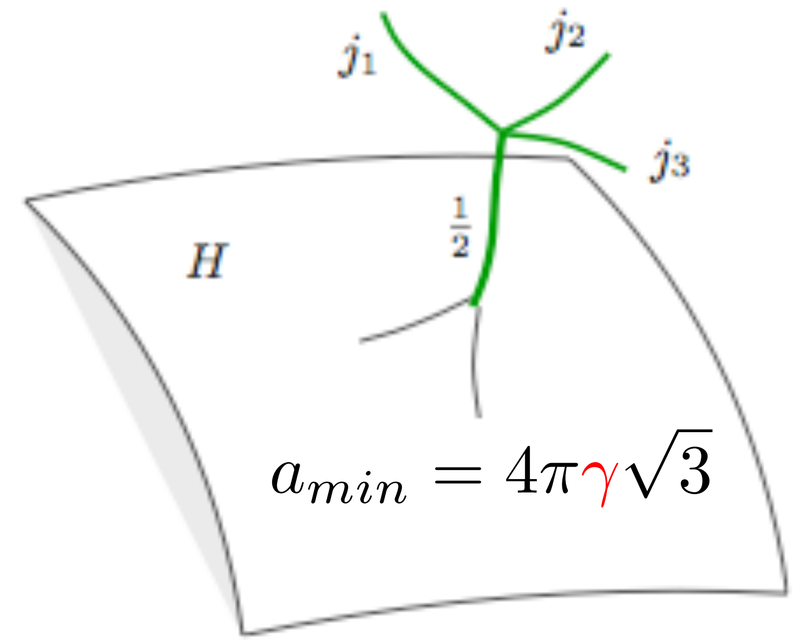
b) By the emission or absorption of punctures with arbitrary spin (such transitions remain discrete at all scales and are responsible for a modification of the first law: a chemical potential arises and encodes the mean value of the area change in the thermal mixture of possible values of spins j).

Loop quantum gravity

The area gap is an **energy gap**

$$\widehat{H}|j_1, j_2 \cdots\rangle = \left[\frac{\gamma \ell_p^2}{\ell} \sum_p \sqrt{j_p(j_p+1)}\right] |j_1, j_2 \cdots\rangle$$





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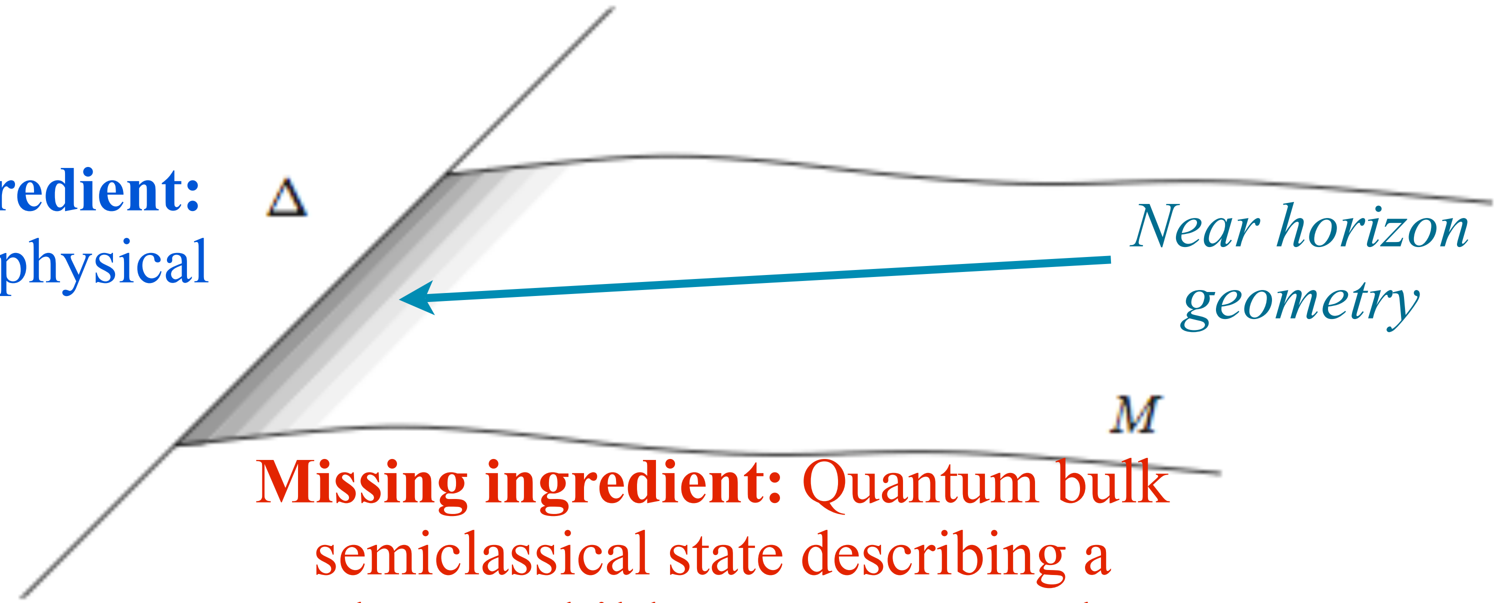
A missing ingredient

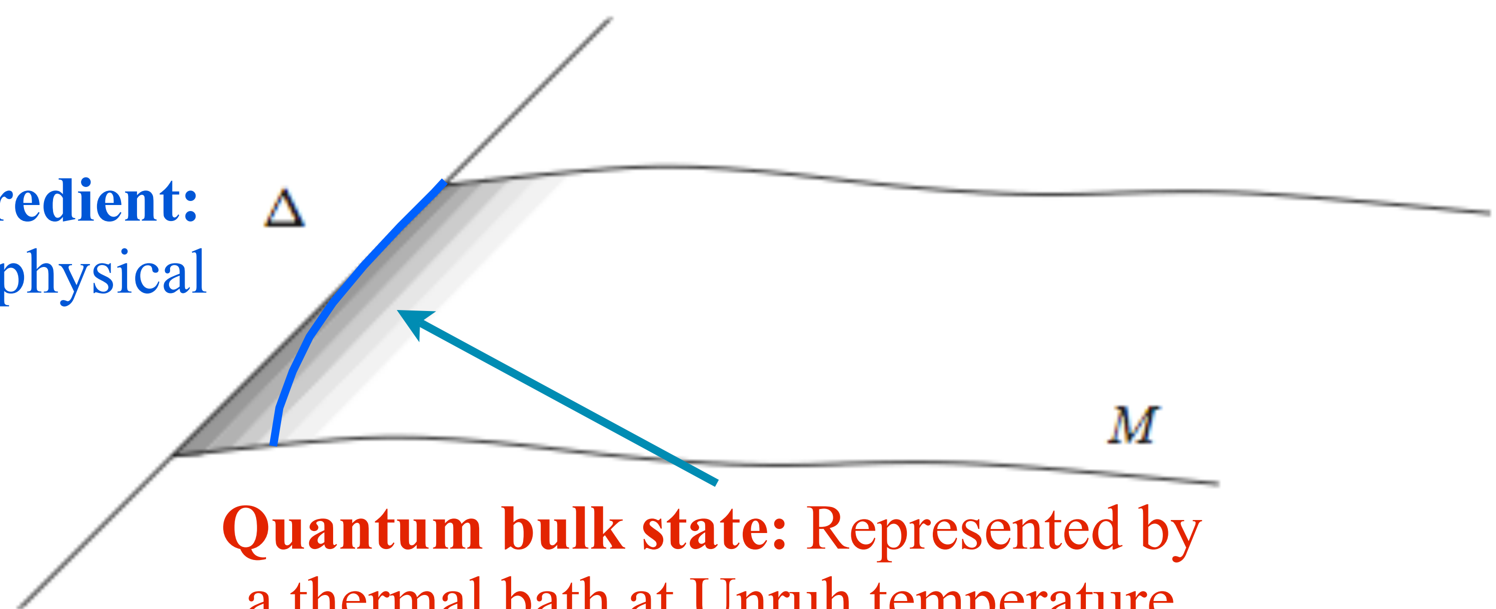
Solution: a physical input

Present ingredient: Quantum IH physical state



Missing ingredient: Quantum bulk semiclassical state describing a Schwarzschild geometry near the horizon

Present ingredient: Quantum IH physical state



Quantum bulk state: Represented by a thermal bath at Unruh temperature as seen by stationary observers.

$\ell \equiv$ arbitrary fixed proper distance to the horizon

Entropy calculation in the Canonical Ensemble

System is like a gas of non-interacting particles

Ghosh-AP available at http://inspirehep.net/record/917420

The canonical partition function is given by

K. Krasnov (1999), S. Major (2001), F. Barbero E. Villasenor (2011)

$$Z(N,\beta) = \sum_{\{s_j\}} \prod_j \frac{N!}{s_j!} (2j+1)^{s_j} e^{-\beta s_j E_j} \tag{1}$$

where $E_j = \ell_g^2 \sqrt{j(j+1)}/\ell$. A simple calculation gives

$$\log Z = N \log[\sum_j (2j+1) e^{-\beta E_j}] \tag{2}$$

For the entropy we get

$$S = -\beta^2 \frac{\partial}{\partial \beta}(\frac{1}{\beta}\log Z) = \log Z + \beta \frac{A}{8\pi\ell}. \tag{3}$$

finally in thermal equilibrium at $T_U = \ell_p^2 \frac{\overline{\kappa}}{2\pi} = \frac{\ell_p^2}{2\pi\ell}$

$$S = \sigma(\gamma) N + \frac{A}{4\ell_p^2}$$

where

$$\sigma(\gamma) = \log[\sum_{j=1/2}^{\infty} (2j+1) \exp -2\pi\gamma\sqrt{j(j+1)}]$$

Entropy calculation in the Canonical Ensemble

System is like a gas of non-interacting particles

At thermal equilibrium the average energy $\langle E \rangle = -\frac{\partial}{\partial \beta} \log Z$ at $T = T_U$ is a function of N only; this relates the number of punctures to the area

$$N = -\frac{A}{4\ell_g^2\, \sigma'(\gamma)}. \tag{1}$$

Note that for all values of γ the number of punctures $0 \leq N \leq \frac{A}{4\sqrt{3}\pi\ell_g^2}$. Moreover, for a fixed macroscopic area A, the number of punctures grows without limit as $\gamma \to 0$ while it goes to zero as $\gamma \to \infty$.

There are two equivalent expressions for the BH entropy

$$\boxed{S(A,N) = \sigma(\gamma)N + \frac{A}{4\ell_p^2}} \quad \text{or} \quad \boxed{S(A) = \frac{A}{4\ell_p^2}\left[1 - \frac{\sigma}{\sigma'}\right]}$$

Semiclassical consistency

back to the first law

(1)
$$S(A,N) = \sigma(\gamma)N + \frac{A}{4\ell_p^2} \quad \text{or} \quad S(A) = \frac{A}{4\ell_p^2}\left[1 - \frac{\sigma}{\sigma'}\right]$$

The (thermodynamical) local first law versus the (geometric) local first law

(2)
$$\delta E = \frac{\overline{\kappa}}{2\pi}\,\delta S + \overline{\mu}\,\delta N \quad \Longleftrightarrow \quad \delta E = \frac{\overline{\kappa}}{2\pi}\,\delta A$$

Recall

(3)
$$\overline{\mu} = T_U \frac{\partial S}{\partial N}|_E = \frac{\overline{\kappa}}{2\pi}\sigma(\gamma)$$

Finally going from local to global (when one can, i.e., in a stationary background BH spacetime)

(4)
$$\delta M = \frac{\kappa}{2\pi}\,\delta S + \Omega\,\delta J + \Phi\,\delta Q + \mu\,\delta N \quad \Longleftrightarrow \quad \delta M = \frac{\kappa}{2\pi}\,\delta A + \Omega\,\delta J + \Phi\,\delta Q$$

where

(5)
$$\mu = \frac{\kappa}{2\pi}\sigma(\gamma)$$

Entropy calculation

The old view

The usual LQG calculation was performed in the microcanonical ensemble (with an implicit assumption of a vanishing chemical potential) and gives

$$S = \frac{\gamma_0}{\gamma}\frac{A}{4\ell_p^2} = \frac{\gamma_0}{\gamma}\frac{2\pi\ell}{\ell_p^2}E \tag{1}$$

while semiclassical considerations (Hawking radiation) imply that

$$T_U^{-1} = \frac{\partial S}{\partial E} = \frac{2\pi}{\overline{\kappa}\ell_p^2} \tag{2}$$

Thermal equilibrium at Unruh temperature is achieved only if the *Immirzi parameter* is fine tuned according to

$$\gamma = \gamma_0 = 0.274067... \tag{3}$$

$$\sigma(\gamma) = \log[\sum_{j=1/2}^{\infty}(2j+1)\exp -2\pi\gamma\sqrt{j(j+1)}] \tag{4}$$

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where

(5) $$\mu = \frac{\kappa}{2\pi}\sigma(\gamma)$$

Conclusions

- A local definition is needed which corresponds to large semiclassical BHs: Isolated horizons [Ashtekar et al.] provides a suitable boundary condition.

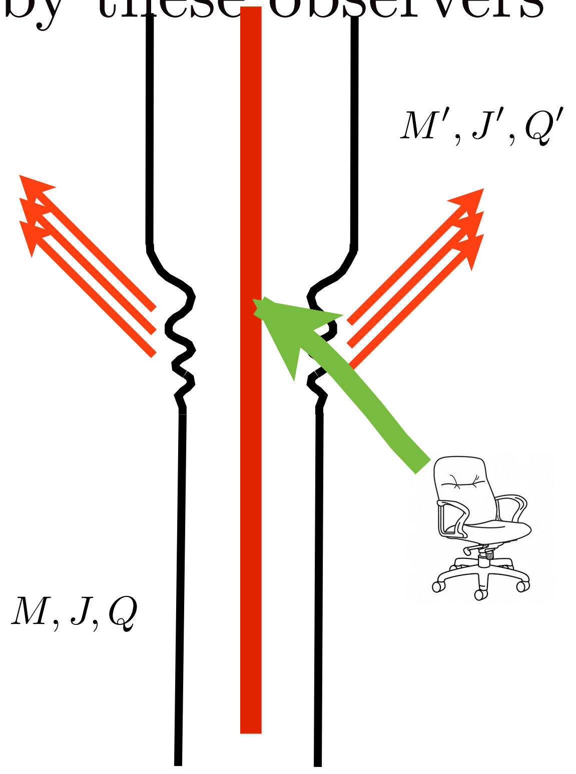
- Yet a little bit more (near horizon geometry) is necessary for dealing with BH thermodynamics.

- New [E. Frodden, A. Ghosh and AP (arXiv:1110.4055)]: A preferred notion of stationary observers can be introduced. These are the suitable observers for local thermodynamical considerations. There is:

  1. a unique notion of energy of the system described by these observers $E = A/(8\pi\ell)$.
  2. a universal surface gravity $\overline{\kappa} = 1/\ell$.
  3. and they are related by a local fist law

$$\delta E = \frac{\overline{\kappa}}{8\pi\ell}\delta A.$$

  4. The first law is of a dynamical nature.



- In progress [E. Wilson-Ewin, AP, D. Forni]: a first law for Rindler Horizons holds

At the quantum level

- The area gap of LQG=an energy gap in the local formulation.
- New [A. Ghosh and AP (PRL **107** 2011)]: The entropy computation yields and entropy formula that is consistent with Hawking semiclassical calculations for all values of the Immirzi parameter γ. One has
  1. $S = \frac{A}{4\ell_p^2} + \mu N$.
  2. The chemical potential $\mu = \frac{\kappa}{2\pi}\sigma(\gamma)$ where
  $$\sigma(\gamma) = \log[\sum_j (2j+1)\exp\left(-2\pi\gamma\sqrt{j(j+1)}\right)].$$
  3. If one fixes $\gamma = \gamma_0$ then $\mu = 0$!
  4. The usual law gets a LQG correction
  $$\delta M = \frac{\kappa}{2\pi}\delta S + \Omega\delta J + \Phi\delta Q + \mu\delta N \quad \Longleftrightarrow \quad \delta M = \frac{\kappa}{2\pi}\,\delta A + \Omega\,\delta J + \Phi\,\delta Q$$
- There are possible observational effects [Ghosh, AP]. Also in progress [Diaz-Polo, Borja] and [Pranzetti].

A potentially measurable effect

$$\langle \mathcal{N} \rangle \quad = \quad \frac{\Gamma}{\exp\left(\frac{2\pi}{\kappa}(\omega - \Omega m - q\Phi)\right) \pm 1} \approx \Gamma \exp\left(-\frac{2\pi}{\kappa}(\omega - \Omega m - q\Phi)\right)$$

$$\langle \mathcal{N}_{LQG} \rangle = \frac{\Gamma^{\infty}}{\exp\left(\frac{2\pi}{\kappa}(\omega - \Omega_H m - q\Phi) + \sigma(\gamma) n\right) \pm 1} \approx \Gamma e^{-n\sigma(\gamma)} \exp\left(-\frac{2\pi}{\kappa}(\omega - \Omega m - q\Phi)\right)$$

$$\frac{\langle \mathcal{N}_n \rangle}{\langle \mathcal{N}_0 \rangle} \approx e^{-n\sigma} = \frac{1}{\left[\sum_j (2j+1) \exp -2\pi\gamma\sqrt{j(j+1)}\right]^n}$$

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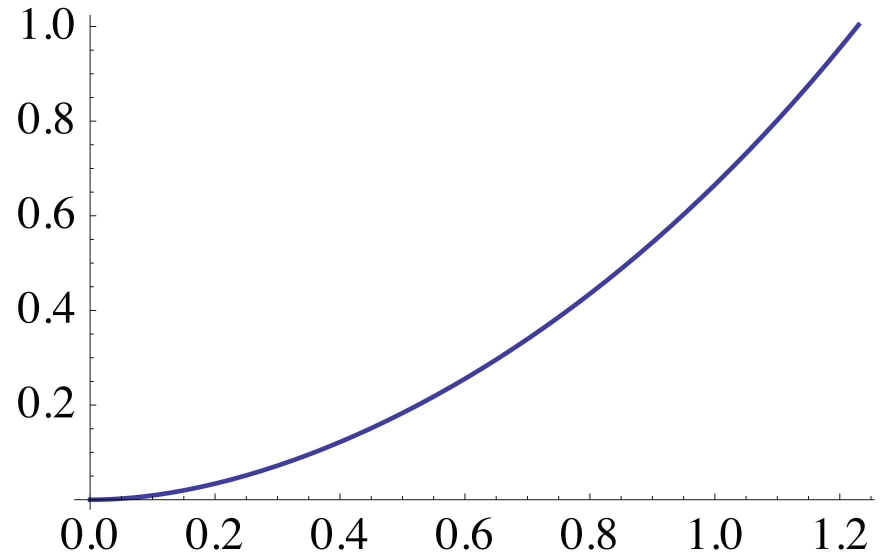
$$\frac{\langle \mathcal{N}_{neutrinos} \rangle}{\langle \mathcal{N}_{photons} \rangle} \approx e^{-\sigma} = \frac{1}{\sum_j (2j+1) \exp -2\pi\gamma\sqrt{j(j+1)}}$$



About 80% of the energy of a primordial black hole is lost in the neutrino chanel [Page 1976]

Thank you very much