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Effective LTB models

arXiv: 2308.10949 and 2308.10953

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12.12.2023, International Loop Quantum Gravity Seminar

Funded by
Deutsche Forschungsgemeinschaft
German Research Foundation

ERLANGEN CENTRE FOR ASTROPARTICLE PHYSICS

Main Question

How do quantum gravity effects influence black holes?

Extensive literature with contributions from many researchers:

[Ashtekar, Bojowald, Modesto, Cartin, Khanna, Boehmer, Vandersloot, Chiou, Campiglia, Gambini, Pullin, Sabharwal, Brannlund, Kloster, De Benedictis, Olmedo, Dadhich, Joe, Singh, Haggard, Rovelli, Vidotto, Corichi, Saini, Cortez, Cuervo, Morales-Técotl, Ruelas, Pawlowski, Bianchi, Giesel, Christodoulo, D'Ambrosio, Alesci, Bahrami, Pranzetti, Husain, Kelly, Santacruz, Wilson-Ewing, Lewandowski, Zhang, Ma, Song, Bodendorfer, Mele, Münch, Navascués, Mena Marugán, García-Quismondo, Perez, Speziale, Viollet, Han, Liu, Alonso-Bardaji, Brizuela, Vera,...]

- Eternal BH models: Based on results of LQC study quantum corrections Reviews: [Gambini, Olmedo, Pullin '22], [Ashtekar, Olmedo, Singh '23]
- Dynamical process of gravitational collapse?
- Typical playground: spherical symmetric model with dust (perfect fluid, no pressure), i.e. in classical GR Lemaître-Tolman-Bondi spacetimes
- Effective description: classical model with correction functions

- Most models describe Oppenheimer-Snyder scenario: homogeneous dust ball embeddded in vacuum
- Use classical junction conditions to glue interior Friedmann model to exterior
- Question: Is there a discontinuity in the gravitational field after the bounce? [Achour, Brahma, Uzan '20]
- Can we have decoupled equations of motion as in classical case?

We want to contribute to this discussion by

- Construction of effective LTB models from underlying effective spherical symmetric model as a 1+1 field theory under certain assumptions
- Start with general ansatz for effective model (not only motivated from $\overline{\mu}$-scheme and its reduced quantization [Chiou, Ni, Tang '12], [Gambini, Olmedo, Pullin '20])
- In this setup we can have arbitrary dust mass profiles
- Can consider different coordinates (aerial gauge) due to underlying spherical symmetric model to relate to other models

Plan of the talk

1. Classical LTB in the canonical framework with connection variables
2. Constructing effective LTB models
   - Analyze stability of LTB condition in effective dynamics
3. Concrete model: Adapt to improved LQC dynamics
   - Analyze solution in marginal bound case
   - Extended mimetic gravity is underlying Lagrangian
4. Polymerized vacuum solution and comparison to other models
5. Summary and Outlook

Classical LTB model

classical LTB sector in spherical sym. spacetimes

- Work with real Ashtekar-Barbero variables, Hamiltonian has standard form

$$H = \int \mathrm{d}x \, (NC + N^x C_x + \lambda G)$$

- Impose spherical symmetry on triad and connection (A, E) [Bojowald Kastrup '00], [Bojowald, Swiderski '06]
- Spherical symmetric metric has form (with $(E^\phi)^2 = (E^1)^2 + (E^2)^2$)

$$\mathrm{d}s^2 = -N(x,t)^2 dt^2 + \frac{(E^\phi)^2}{|E^x|}(dx + N^x dt)^2 + |E^x| d\Omega^2 \,.$$

- Can fix Gauß constraint to get rid off remaining gauge freedom
- Reduced phase space can be coordinatized by [Modesto '04]

$$\{K_x(x), E^x(y)\} = G\delta(x,y) \qquad \{K_\phi(x), E^\phi(y)\} = G\delta(x,y)$$

- Add dust to the gravitational system: (T, P_T)

classical LTB sector in spherical sym. spacetimes

Lemaitre-Tolman-Bondi sector

The LTB solution in these variables is given by [Lemaître '33], [Tolman '34], [Bondi '47]

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \frac{((E^x)')^2}{4|E^x|(1+\mathcal{E}(x))}\mathrm{d}x^2 + |E^x|\,\mathrm{d}\Omega^2\,,$$

with the dynamical equation for each shell

$$\partial_t E^x = \pm 2\sqrt{E^x}\sqrt{\mathcal{E}(x) + \frac{\mathcal{F}(x)}{(E^x)^2}}\,.$$

- Comparing LTB to general spherical symmetric metric, we need

$$N = 1 \qquad N^x = 0 \qquad G_x(x) = \tfrac{E^{x\prime}}{2E^\phi}(x) - \sqrt{1+\mathcal{E}(x)} = 0$$

Gauge Fixings

The LTB sector can be reached by the two gauge fixings

$$\Big(C \longrightarrow G_T = T(x) - t\Big)\,, \qquad \Big(C_x \longrightarrow G_x = \frac{E^{x\prime}}{2E^\phi}(x) - \sqrt{1+\mathcal{E}(x)}\Big)$$

Note: In marginally bound case ($\mathcal{E} = 0$), in dust time gauge $\{G_x(x), C_x^{\mathrm{tot}}(y)\} \approx 0$

Effective LTB models

Effective primary Hamiltonian

Start with partially gauge fixed effective system (dust time gauge):

Effective primary Hamiltonian

Consider effective model with temporal gauge fixed primary Hamiltonian

$$H_P^\Delta[N^x] = \int \mathrm{d}x \, (C^\Delta + N^x C_x)(x) \, , \qquad C_x = \tfrac{1}{G}(E^\phi K_\phi' - K_x (E^x)')$$

and the polymerized gravitational contribution of the scalar constraint

$$C^\Delta(x) = \frac{E^\phi}{2G\sqrt{E^x}} \left[-(1+f)E^x \left(\frac{4K_x K_\phi}{E^\phi} + \frac{K_\phi^2}{E^x} \right) + h_1 \left(\left(\frac{E^{x\prime}}{2E^\phi} \right)^2 - 1 \right) + 2\frac{E^x}{E^\phi} h_2 \left(\frac{E^{x\prime}}{2E^\phi} \right)' \right] .$$

The polymerization functions have classical limit

$$h_1(E^x) \to 1 \qquad h_2(E^x) \to 1 \qquad f(K_x/E^\phi, K_\phi, E^x) \to 0$$

Note: density weight unchanged since combination K_x/E^ϕ in f, thus

$$\{C^\Delta[N], C_x[N^x]\} = C^\Delta[N^x(\partial_x N)]$$

$\Rightarrow$ Investigate dynamically stable reductions to LTB sector

Conservation of C^{Δ}?

Conservation of C^{Δ} (see Lemma 1 in [Giesel, Liu, Rullit, Singh, SW '23])

We have [Tibrewala '12], [Alonso-Bardaji, Brizuela '21]

$$\left\{H_P^{\Delta}[N^x = 0], C^{\Delta}(y)\right\}\Big|_{C_x=0} = 0\,,$$

if there is **no polymerization** of K_x and

$$\frac{h_1 - 2E^x\partial_{E^x}h_2}{h_2} = \frac{-4E^x\partial_{E^x}f^{(2)} + \partial_{K_\phi}f^{(1)}}{2f^{(2)}}\,.$$

The polymerization functions are defined in this case as

$$C^{\Delta}(x) = -\frac{E^\phi}{2G\sqrt{E^x}}\left[\frac{4K_x f^{(2)}(K_\phi, E^x)}{E^\phi} + \frac{f^{(1)}(K_\phi, E^x)}{E^x} - h_1 \ldots\right](x)$$

$\Rightarrow$ **Note:** this is not equivalent with closure of constraint algebra since this should be analyzed in fully gauge unfixed system

Implementing polymerized LTB condition

- Introduce effective LTB condition (classical: $G_x^\Delta = \frac{E^{x\prime}}{2E^\phi} - \sqrt{1+\mathcal{E}}$)

$$G_x^\Delta = \frac{E^{x\prime}}{2E^\phi} - g_\Delta\big(K_x/E^\phi, K_\phi, E^x, [\partial_x^n(K,E)], \mathcal{E}\big)$$

- Investigate stability of effective LTB condition under effective dynamics [Bojowald, Harada, Tibrewala '08], [Bojowald, Reyes, Tibrewala '09]
  - We call such LTB conditions *compatible*
- Our strategy: work on the level of equations of motion

Question

For which G_x^Δ do the four EOM of $\dot{K}_x, \dot{K}_\phi, \dot{E}^x, \dot{E}^\phi$ reduce to only **two** in the sector

$$N^x = 0, \qquad C_x = 0, \qquad G_x^\Delta = 0 \quad ?$$

- First result: compatible LTB conditions are of the form

$$g_\Delta = g_\Delta^{(1)}(K_\phi, E^x, \mathcal{E}) + g_\Delta^{(2)}(\widetilde{K}_x = \frac{\partial_x K_\phi}{\partial_x E^x}, K_\phi, E^x)$$

- Contribution $g_\Delta^{(1)}$ represents non-marginally and $g_\Delta^{(2)}$ marginally bound case

Implementing polymerized LTB condition

Key results on the form of the polymerization functions and compatible LTB condition:

Key results

1. Additional condition for K_x polymerization
   $\Rightarrow$ Non-marginal: always no K_x polymerization allowed
   $\Rightarrow$ Marginal: if $g_\Delta^{(2)} = g_\Delta^{(2)}(K_\phi, E^x)$ no K_x polymerization allowed

2. Separating LTB function dependence $g_\Delta^{(1)} = \tilde{g}_\Delta(K_\phi, E^x)\sqrt{1+\mathcal{E}}$

$$g_\Delta^{(1)} = \tilde{g}_\Delta(E^x)\sqrt{1+\mathcal{E}} \qquad 1 - \frac{2E^x\partial_{E^x}\tilde{g}_\Delta}{\tilde{g}_\Delta} = \frac{-4E^x\partial_{E^x}f^{(2)} + \partial_{K_\phi}f^{(1)}}{2f^{(2)}}$$

   second condition allows the conservation of C^Δ when we further have

$$2E^x\partial_{E^x}\tilde{g}_\Delta = \left(1 - \frac{h_1 - 2E^x\partial_{E^x}h_2}{h_2}\right)\tilde{g}_\Delta\,.$$

3. Classical LTB condition restricts inverse triad and holonomy corrections to

$$\partial_{K_\phi}f^{(1)} = 2f^{(2)} + 4E^x\partial_{E^x}f^{(2)} \qquad h_1 = h_2 + 2E^x\partial_{E^x}h_2$$

   "compatibility" "conservation"

Consider system with compatible LTB condition and C^{Δ} conserved (Lemma 1):

Dynamical equations (see Corollary 4 in [Giesel, Liu, Rullit, Singh, SW '23])

The dynamics of such models decouple in radial x-direction

$$\partial_t E^x = 2\sqrt{E^x} f^{(2)}$$
$$\partial_t K_\phi = -\frac{1}{2\sqrt{E^x}} \left(f^{(1)} - \tilde{g}^2_{\Delta}(1+\mathcal{E})(2h_2 + 4E^x \partial_{E^x} h_2 - h_1) + h_1 \right)$$

- Equations applicable in marginally and non-marginally bound case
- Non-marginal: same result as implementing gauge fixing G^{Δ}_x and computing associated Dirac bracket
- Result supports assumption of decoupled shells in dust collapse models, e.g. [Kiefer, Schmitz '19], [Giesel, Li, Singh'21]
- Solution parametrized by energy $\mathcal{E}(x_0)$ and conserved quantity mass $M(x_0)$ of a shell at $x = x_0$

Concrete model from improved LQC dynamics

Matching LQC dynamics

General strategy

- Choose an effective LQC model as starting point
- Use Cor. 4 to identify effective spherically symmetric model and LTB condition

In this way we get

- Underlying spherical symmetric model, that has no areal gauge implemented yet
- Dynamically stable reduction to LTB sector through effective LTB condition
- Equations of motion are decoupled and coincide with chosen LQC model

Sometimes one can relate effective spherical symmetric model to an underlying covariant Lagrangian

- In our model this will be extended mimetic gravity in comoving gauge
- Redefinition of time dependence of mimetic field corresponds to coordinate transformations in the temporal coordinate

Matching LQC dynamics

Adapt decoupled effective LTB sector to improved LQC dynamics [Ashtekar, Pawlowski, Singh '06]

$$\partial_t v = 3v\frac{\sin(2\alpha b)}{2\alpha}, \quad \partial_t b = -\frac{1}{2}\left(\frac{\mathcal{E}(x)}{v^{\frac{2}{3}}} + \frac{3\sin^2(\alpha b)}{\alpha^2}\right),$$

where we defined $v = (E^x)^{3/2}$, $b = \frac{K_\phi}{\sqrt{E^x}}$, $\alpha = \beta\sqrt{\Delta}$.

Underlying spherical symmetric model

The corresponding gauge unfixed effective spherical symmetric model is [Tibrewala '12]

$$C^\Delta = -\frac{E^\phi\sqrt{E^x}}{2G}\left[\frac{3}{\alpha^2}\sin^2\left(\frac{\alpha K_\phi}{\sqrt{E^x}}\right) + \frac{(2E^x K_x - E^\phi K_\phi)}{\alpha\sqrt{E^x}E^\phi}\sin\left(\frac{2\alpha K_\phi}{\sqrt{E^x}}\right) + \right.$$
$$\left. + \frac{1-\left(\frac{E^{x\prime}}{2E^\phi}\right)^2}{E^x} - \frac{2}{E^\phi}\left(\frac{E^{x\prime}}{2E^\phi}\right)'\right].$$

No inverse triad corrections: compatible LTB condition is classical one

$$G_x^\Delta = G_x = \frac{E^{x\prime}}{2E^\phi} - \sqrt{1+\mathcal{E}}$$

and C^Δ is conserved quantity.

We can write dynamical equation as modified Friedmann equation

$$\frac{\dot{R}^2}{R^2}(x) = \left(\frac{\kappa\rho}{6} + \frac{\mathcal{E}}{R^2}\right)\left(1 - \alpha^2\left(\frac{\kappa\rho}{6} + \frac{\mathcal{E}}{R^2}\right)\right)(x)\,,$$

where the metric has the form (we work in LTB coordinates)

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \left(\partial_x R\right)^2 \mathrm{d}x^2 + R^2 \mathrm{d}\Omega^2\,.$$

Marginally bound case

In marginally bound case $\mathcal{E} = 0$ the solution is

$$R(x,t) = \sqrt{E^x} = \left(\mathcal{F}(x)\left(\tfrac{9}{4}(\tilde{\beta}(x) - t)^2 + \alpha^2\right)\right)^{\frac{1}{3}}$$

for homogeneous dust already in [Giesel, Han, Li, Liu, Singh '22], [Fazzini, Rovelli, Soltani '23]

- In vacuum: time symmetry, and metric is stationary
- No shell crossing singularities for vacuum and OS collapse, but in general inhomogeneous case not true [Fazzini, Husain, Wilson-Ewing '23]
- Horizons can form when $M(x) = \frac{\mathcal{F}}{2G} > M_c = \frac{8\alpha}{3\sqrt{3}G}$ [Kelly, Santacruz, Wilson-Ewing '20], [Giesel, Han, Li, Liu, Singh '22], [Lewandowski, Ma, Yang, Zhang '22]

Extended mimetic model

Underlying covariant Lagrangian

Primary Hamiltonian can be generated from 2d action [Achour, Lamy, Liu, Noui '18], [Han, Liu '22]

$$S_2 = \frac{1}{4G}\int_{\mathcal{M}_2} \mathrm{d}^2x \,\det(e) e^{2\psi}\Big\{\mathcal{R} + L_\phi(X,Y) + \tfrac{\lambda}{2}\left(\phi_{,j}\phi^{,j}+1\right)\Big\}\,,$$

in comoving gauge $\phi(t,x) = t$, note $\det(e) = E^\phi\sqrt{E^x}$ coupling

- (smooth) mimetic field naturally defines foliation into spacelike surfaces defined by $\phi = const.$, w.l.o.g. $\phi(x,t) = \phi(t)$
- Higher derivative coupling in X, Y relates to extrinsic curvature

$$X = -\Box_h\phi + Y = \frac{\partial_t E^\phi}{E^\phi}\,, \qquad Y = -h^{ij}\partial_i\psi\partial_j\phi = \frac{\partial_t E^x}{2E^x} = \frac{\sin(2\alpha b)}{2\alpha}\,,$$

- Pendant of the Einstein equations of this model are

$$G^{\Delta}_{\mu\nu} := G_{\mu\nu} - T^{\phi}_{\mu\nu} = -\lambda\partial_\mu\phi\partial_\nu\phi, \quad \partial_\mu\phi\partial^\mu\phi = -1\,.$$

Polymerized vacuum solution

Polymerized vacuum

Specialize to matter profile $\mathcal{F} = R_s = \mathrm{const}$ [Giesel, Han, Li, Liu, Singh '23], [Fazzini, Rovelli, Soltani '23]

$$R(x,t) = \left(R_s\left(\frac{9}{4}z^2 + \alpha^2\right)\right)^{\frac{1}{3}}, \qquad z := x - t$$

metric clearly stationary

- Corresponds to $\lambda = C^{\Delta} = 0$ but curvature non-vanishing

$$\mathcal{R} = -\frac{96\alpha^2}{\left(4\alpha^2 + 9z^2\right)^2}$$

  due to non trivial coupling of ϕ in $T^{\phi}_{\mu\nu} \Rightarrow$ QG effects
- Everything bounded, at bounce $z = 0$ no shell crossing singularity
- (smooth) signature change of E^{ϕ} at the bounce

$$E^{\phi}(t,x) = \frac{1}{2}(E^x)' = R(z)\partial_z R(z)$$

  allowed due to $\det(e)$ coupling in Lagrangian and consistent with degeneracy of metric

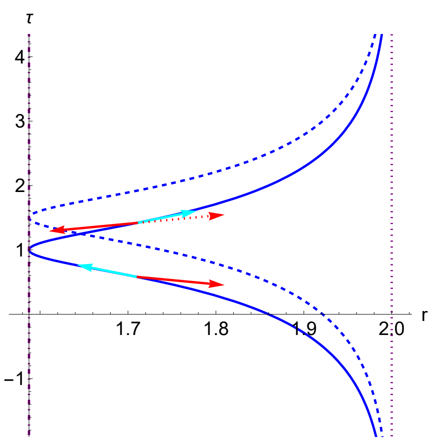
Comparison to other models

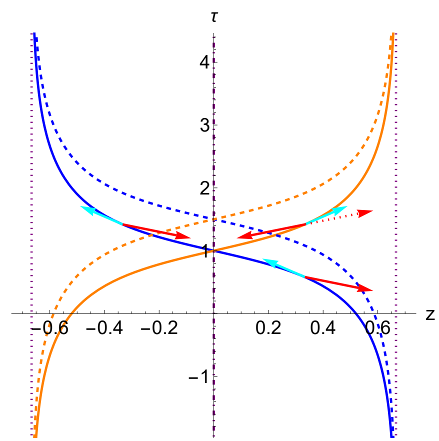
Underlying covariant model allows to perform coordinate transformations in t and x
$\Rightarrow$ Relate to other models in the literature and discuss implications (shocks?)

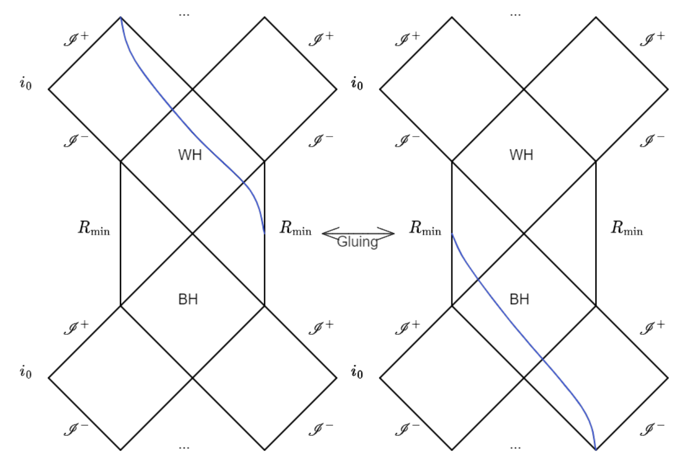
- Polymerized vacuum:
  - Transform to Schwarzschild-like coordinates $(t,x) \to (\tau, z)$
    $$\mathrm{d}s^2 = -A(r)\mathrm{d}\tau^2 + \frac{1}{A(r)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2 \qquad A(r) = 1 - \frac{2Gm_s}{r}\left(1 - \frac{\alpha^2}{r^2}\frac{2Gm_s}{r}\right)$$
    same solution as [Kelly,Santacruz,Wilson-Ewing '20],[Parvizi,Pawlowski,Tavakoli,Lewandowski '21],[Lewandowski,Ma,Yang,Zhang '22]
  - transformation only well defined for monotonic $R(z)$ (not at bounce)
- Areal gauge:
  - Gauge fix C_x with $G^{\mathrm{ar}} = E^x - r^2$, shift vector $N^r = -\frac{r}{2\alpha}\sin\left(\frac{2\alpha}{r}K_\phi\right)$
    $\Rightarrow$ We can exactly reproduce model in [Husain, Kelly, Santacruz, Wilson-Ewing '22]
  - We can transform our solution in LTB coordinates to Gullstrand-Painlevé
    $$N^r = -\partial_t R(t,x) = -\text{sign}(\partial_t R(t,x))\sqrt{\frac{2GM(x)}{r}(\dots)}$$
    smooth signature change at bounce
  - Not observed when working directly in radial coordinates in vacuum case
    $\Rightarrow$ gives rise to discontinuity/shocks in OS collapse

Comparison to other models

- Different global structure of spacetime can be seen in $x = \mathrm{const}$ geodesics
- These are also world lines of clock field ϕ, they intersect in (τ, r) after bounce
  $\Rightarrow$ discontinuity in clock field [Fazzini, Rovelli, Soltani '23] violates smoothness of mimetic field
- To have same global structure in r coordinates need to consider gluing of two patches with different orientation [Münch '21]







- OS collapse: Gluing with effective junction conditions does not allow shocks without violating smoothness of mimetic field

Inhomogeneous dust collapse

- Consider inhomogeneous dust profile on initial slice
- In OS collapse everything bounded
  $\Rightarrow$now shell crossing singularity[Fazzini,Husain,Wilson-Ewing '23]
- Prob. weak singularity: geodesics can pass through
- Separates spacetime into regions with different orientation

$\Rightarrow$ Needs further investigation, work in progress

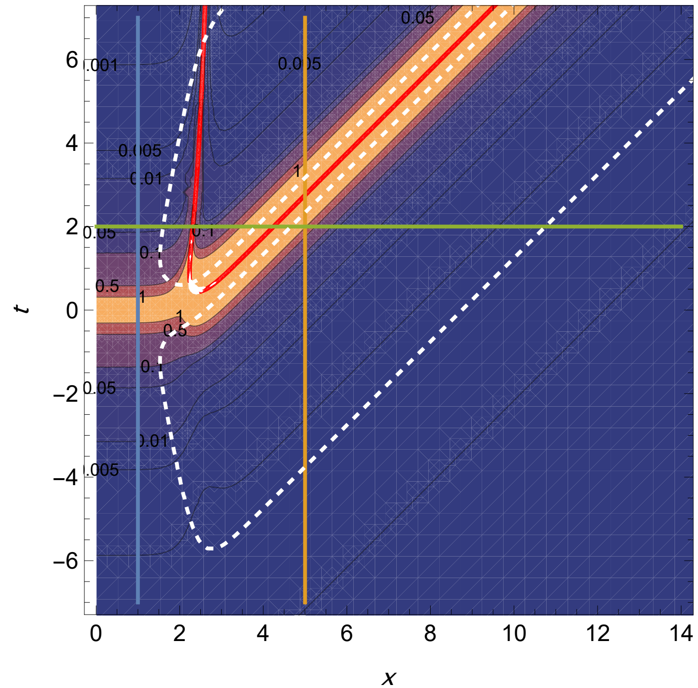


Figure: Kretschmann scalar

Conclusion

Summary

- Our framework allows construction of effective LTB models with holonomy and inverse triad corrections under certain assumptions (no polymerization of diffeo)
- Certain class of effective LTB models has decoupled dynamics
- LQC model as starting point: field theoretic model for inhomogeneous dust collapses
- Underlying mimetic model provides all coordinate transformations

Future work

- study further phenomenological properties like BH evaporation
- Extend analysis to LQC models with asymmetric bounce (work in progress)
- study polymerized vacua for more general polymerizations (work in progress)

Thank you for your attention!