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TOO MANY BARGAINERS SPOIL THE BROTH: THE IMPACT OF BARGAINING ON MARKETS WITH PRICE TAKERS

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Too Many Bargainers Spoil The Broth:

The Impact of Bargaining on Markets with Price Takers

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Abstract

In this paper we study how bargainers impact on markets in which firms set a list price to sell to those consumers who take prices as given. The list price acts as an outside option for the bargainers, so the higher the list price, the more the firms can extract from bargainers. We find that an increase in the proportion of consumers seeking to bargain can lower consumer surplus overall, even though new bargainers receive a lower price. The reason is that the list price for those who don't bargain and the bargained prices for those who were already bargaining rise: sellers have a greater incentive to make the bargainers' outside option less attractive, at a cost to profits from non-bargainers. Competition Authority exhortations to bargain can therefore be misplaced. We also consider the implications for optimal seller bargaining.

JEL Codes: L13, D43

Key words: Bargaining; Price takers; List Price; Consumer Surplus

1 Introduction

Consumers often ask retailers or service providers for discounts off their list prices. This is particularly prevalent for large ticket items such as expensive electronic and household goods, cars, package holidays, estate agency (realtor) fees, etc. The United Kingdom's market for automobiles is a striking example where discounts are common and large. As an example, discounts off the list price for Ford models sold in the UK in 1997/98 averaged 10.6% with reductions as large as 30% for some buyers. Similar discounts were available for other models (see Figure 1 from the UK's Competition Commission (2000) report on the car market.)

APPENDIX 7.1
(referred to in paragraphs 7.32 and 7.36)

Percentage discount given on make of car

Number of sales

Percentage discount	Vauxhall	Ford	Fiat	Peugeot	Toyota	Nissan	Citröen	Mitsubishi	Mazda	Renault	Rover	Honda	Audi	Volkswagen	Volvo	BMW	Mercedes-Benz
0	4	55	12	16	4	2	44	0	26	29	64	30	42	71	33	158	74
0–5	9	53	16	53	21	25	31	19	29	140	47	49	117	191	73	234	167
5–10	51	144	36	58	59	52	43	35	50	82	60	73	72	113	19	75	17
10–15	27	96	12	44	65	16	23	6	10	14	13	13	13	2	2	6	1
15–20	5	17	6	9	2	6	4		10	16	8	1	2				2
20–25	18	14	11	20		1	13		3	16	1						
25–30	7	23	5	8			6		1								
30–40	2	21	4	1													
Average	12.3	10.6	10.2	9.3	8.7	7.6	7.3	6.7	6.5	6.2	4.7	4.6	3.9	3.4	2.6	2.5	2.0
Standard deviation	7.5	8.7	8.6	7.5	3.8	4.4	7.7	2.9	5.9	6.2	4.9	3.6	3.3	2.9	2.7	2.5	2.4
Total number of sales	123	423	102	209	151	102	164	60	129	297	193	166	246	377	127	473	261
Percentage given discount	96.7	87.0	88.2	92.3	97.4	98.0	73.2	100.0	79.8	90.2	66.8	81.9	82.9	81.2	74.8	66.6	71.7

Source: Commission calculations based on OFT data.

Figure 1: Discounts on UK cars received by private purchasers from data sampled by the Office of Fair Trading between October 1997 and May 1998. The numbers in the table are the numbers of observations.

The estate agency market also illustrates the point. The UK's Office of Fair Trading (2004) assessed this market and the pricing behaviour of estate agents finding that

"almost half of those using an estate agent said that they had tried to negotiate fees with their chosen agent and of those who had done so around four out of five were successful in obtaining a reduction on the initial quote" (§4.48) and further that "those who did shop around and negotiate fees, paid on average 14 per cent lower fees than those who did neither" (§4.6).

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In this paper we model markets in which a proportion of the consumers bargain with sellers. To capture the effects of bargaining most succinctly, we introduce an original two-stage model of the price setting and subsequent bargaining process. In the first stage competing sellers establish a market price via simultaneous quantity choices. This market price acts as a publicly observed list price which will be the transaction price of the non-bargaining price takers. In the second stage the bargainers bargain with the sellers. An important innovation in this paper is in how bargaining is modelled: the bargainers are assumed to approach one or more of the sellers and request a price better than the posted list price. The number of sellers given the opportunity to lower price by any given buyer is private information. This model of bargaining is an adaptation of Burdett and Judd's (1983) work on non-sequential search except here the upper bound of prices is endogenously decided by the first stage competition for the non-bargainers. Paralleling Burdett and Judd (1983), the sellers offered the chance to re-quote will select a price below the upper bound (here the list price) according to a probability distribution. In choosing what price to offer bargainers, the sellers trade off the possibility that (i) the bargainer may not approach any other firm and so a high price would be profitable for the seller; and (ii) the bargainer may approach multiple sellers for second quotes and so a seller would need to offer a low price to be successful. Our bargainers are restricted to taking or leaving any second quote offered to them. One can clearly imagine more complicated bargaining scenarios with other sequences of quotes and perhaps counter offers. However, our simple model is both tractable and captures four key features of bargaining: (a) bargaining involves actively requesting a seller to lower her initial price offer; (b) not all consumers bargain; (c) not all bargainers bargain with the same number of sellers; and (d) the bargained price is random with some buyers doing better than others.

This bargaining model highlights a key trade-off faced by sellers. On the one hand sellers wish to compete for the price takers in the standard way by increasing their market share which pushes the list price down. However, the list price acts as an upper bound on the bargained prices: the lower the list price the lower the prices which have to be offered to bargainers. Thus sellers wish to see a high list price to allow rents to be extracted even after bargained reductions are offered to the bargainers. As the proportion of bargainers changes, the relative importance of competing for price takers' business changes. If the proportion of bargainers goes up, competition for price takers becomes less desirable allowing the market price to rise towards

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collusive levels. Therefore consider the following not unusual policy recommendation made in the Office of Fair Trading (2004) report on estate agency:

> "Greater shopping around and negotiation by consumers will increase competitive pressures on estate agents and result in better value for money in terms of both lower prices and higher service quality" (OFT 2004, §1.12)

We find that as the proportion of bargainers rises, consumers are split into three distinct camps: those who remain not bargaining, those who start bargaining, and those who were already bargaining. The new bargainers unambiguously gain as they swap a high list price for a lower bargained price. However, the list price and the bargained prices rise as the proportion of bargainers grows - thus the first and third camp of consumers experience higher prices. Overall this effect can dominate and at best an exhortation to bargain is a mixed bag.[1]

Our model of bargaining departs from the standard Nash and Rubinstein approaches as it offers a mixed strategy equilibrium and hence a probability distribution of prices. Evidence consistent with our predicted mixed strategy equilibrium can be found in Figure 1 displayed above, which shows that the discounts offered to customers in the UK car market vary widely. One might wonder whether this is an artefact of the aggregation of data to the manufacturer level, but Figure 2 from the same Competition Commission report shows that variance in discounts is similarly large at the model range level.

There is little available literature analysing the impact of bargaining on consumer markets and, to our knowledge, none that considers how the proportion of bargainers affects the level of list prices and consumer surplus. Bester (1988) looked at consumers bargaining with competing firms, but sellers did not post list prices and all consumers bargained so the effect of the presence of bargainers and non-bargainers couldn't be addressed. Bester (1993), in the presence of consumers imperfectly informed about quality, compared markets in which all sellers bargain against those where sellers post list prices, while Thanassoulis (2006) considers the effect of bargaining between buyers and sellers on the productivity dispersion which can be sustained amongst sellers. Neither paper, however, analyses changes in the proportion of consumers who

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[1] We note that the Internet may increase the proportion of bargainers by making bargaining easier. Zettelmeyer et al. (2006) find that customers who use the Internet reduce automobile dealers' gross margins by 22% and that much of this effect is driven by online referral services, such as Autobytel.com, which request quotes from dealers on behalf of buyers. Such services are used by about 20% of new car buyers in the sample.

Average discounts and standard deviation for some of the model ranges of the leading six suppliers, 1997 and 1998

Supplier	Model range*	Average discount %	Standard deviation
Vauxhall		13.5	8.3
		12.2	7.6
Ford		12.2	10.2
		14.5	11.2
		10.1	4.6
Renault		4.4	3.7
		8.9	8.6
		8.9	5.7
Peugeot		8.2	6.7
		10.0	7.6
		11.9	9.6
Rover		3.5	4.1
		7.0	4.8
		6.0	4.9
Volkswagen		3.2	2.5
		2.7	3.0
		4.0	3.4

Source: Commission calculations on OFT data.

*Includes model ranges with more than 20 observations.
†65 per cent of the observations were from the same dealer group.
‡88 per cent of the observations were from the same dealer group.

Figure 2: UK Car Discounts at the Model Range Level from Competition Commission (2000)

bargain.

There is a literature which looks at sellers' choice between committing to a fixed price and allowing consumers to haggle. See for instance Camera and Delacroix (2004), Desai and Purohit (2004) and the references therein. In general haggling allows better price discrimination, but compared to a credible commitment to a fixed price may allow strong bargainers to appropriate too much surplus (or put off buyers with weak bargaining power from entering the market). The usual comparison is between fixed posted pricing and bargaining with no posted price. More closely related to our analysis, Desai and Purohit (2004) consider two firms which post prices and choose whether to commit to their posted price or allow bargaining with the consumers who try to haggle to a price better than the one posted. When both firms allow haggling, Desai and Purohit only consider parameter values for which the posted prices do not act as effective outside options for the hagglers, so the posted prices are unaffected by the hagglers.

We have already noted that our model reinterprets and extends Burdett and Judd's (1983) analysis of price dispersion as the second part of a bargaining interface between sellers and consumers.[2] Janssen and Moraga-Gonzalez (2004) extend Burdett and Judd by endogenizing

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[2] There are other related papers with a mixed strategy dispersed price equilibrium arising from differential search, including seminal examples such as Varian (1980) and Stahl (1989).

the number of firms searched. They find that increasing rival numbers can lower welfare: the price distribution becomes more extreme as informed consumers see more prices, and fewer uninformed consumers search at all. Though we do not endogenize the decision of the number of sellers to seek second quotes from, we conjecture that the Janssen and Moraga-Gonzalez result would not extend to our setting: introducing more sellers increases the incentive to push down the market price to steal price takers, and so lowers prices to all types of buyers. Cason et al. (2003) experimentally study the difference between markets with take-it-or-leave-it posted prices and ones where the consumers can haggle, using the posted prices as a starting point. They find that where the consumers can haggle, prices tend to be higher and efficiency lower. Our model provides a possible theoretical justification for these findings.

The rest of this paper is organized as follows. Section 2 introduces the model. Section 3 derives the main results of the paper through comparative statics analyses involving the proportion of bargainers, the number of competitors and other variables. Section 4 analyses the sellers' optimal bargaining strategy. Section 5 extends the analysis using numerical methods. Section 6 concludes. All proofs are relegated to the Appendix.

2 The Model

Consider a market in which $N \geq 2$ firms compete to sell a homogeneous good. Each firm has the same constant marginal cost of production which we normalize to 0. There are M consumers who each purchase $\frac{1}{M}$ units of the product if and only if their valuation v_i exceeds the price. Each consumer's valuation is drawn from the uniform distribution: $v_i \sim U[0, 1]$. Taking the limit as $M \to \infty$, so that each consumer is small and atomistic, the proportion of consumers buying at price p becomes deterministic and tends to $\Pr[v_i > p] = 1 - p$. Thus the aggregate demand will be

$$Q(p) = \lim_{M \to \infty} \frac{1}{M} \times M \times (1 - p) = 1 - p \tag{1}$$

We also note that if a randomly drawn consumer is offered price p then their expected consumer surplus (CS) is given by

$$E[CS_i] \text{ at price } p = \int_p^1 (v_i - p) \, dv_i = \frac{1}{2}(1 - p)^2 \tag{2}$$

A proportion $\mu \in (0, 1]$ of the population are 'price takers'. The prevailing market (or list) price z is assumed to be the outcome of Cournot competition: each firm chooses a volume x_i to supply to the price takers, and the market price equates demand and supply. Each firm can adjust this market price by increasing or decreasing the volumes it supplies to the price takers, who do not seek to alter or bargain down from the market price z. We restrict $\mu > 0$ to ensure that there are always some price takers so the Cournot market price is well-defined.

Together with the downward-sloping demand, we can think of the Cournot competition for the price takers as a reduced form of an oligopolistic interaction which captures the following key features: (i) a more aggressive action (modelled as a higher volume) raises the aggressor's market share amongst the price takers without gaining the entire market, so competition is imperfect; (ii) by reducing the market price, the aggressive action increases the size of the total price taker market while having a cannibalizing effect on the aggressor's existing sales to price takers; and (iii) the oligopolistic competition for price takers determines a market (or list) price which acts as an upper bound on the prices that can be quoted to any consumers who attempt to bargain.

The other $1 - \mu \in [0, 1)$ consumers are 'bargainers'. After the Cournot market price is posted the bargainers each ask some firms for second quotes. A proportion q_k of the bargainers randomly approach k of the firms for second quotes, for $k \in \{1, 2, 3, \ldots N\}$. Of course $\sum_{k=1}^{N} q_k = 1$, and we assume that $q_1 \in (0, 1)$ so some bargainers ask for only a single second quote while the others ask for more. If a seller is asked to offer a better price it is ignorant of how many other (if any) firms are being similarly approached by this consumer, and the price it quotes is assumed to be a final and binding take-it-or-leave-it offer to the consumer. Each consumer then buys the good at the lowest price from the k quotes (randomizing in the event of a tie). The Cournot market price is assumed to be binding on the sellers - quotes cannot be offered above this level. The propensity to bargain with multiple firms is assumed independent of the consumers' valuations. That is the distribution of valuations is independent of the distribution of the number of quotes requested $\{q_1, q_2, ..., q_N\}$.[3]

The firms have two strategic decisions to make. The first is to decide how aggressively to

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[3] Note that empirically this may not always be the case. For example, the differences in bargained reductions in the UK car market between luxury cars (small reductions) and mass market cars (big reductions) highlighted in Figure 1 may, in part, be explained by the luxury brand buyers seeking multiple second quotes less often. This might be due to the greater value of time amongst luxury buyers and therefore a reduced propensity to bargain.

compete for price takers: this will dictate the market price z which the price takers receive in a Cournot type process. Should a firm be invited to offer a better price then it must decide what to bid. This is the second strategic choice. The firms seek to maximize their expected profits. The model's parameters $\{\mu, N, q_1, q_2, ..., q_N\}$ and payoff functions are common knowledge. We restrict attention to symmetric equilibria and, where possible, to pure strategies.

3   The Consumer Surplus Effects of Bargaining

In this section, we analyse how the surplus of consumers is affected by changes in the proportion of bargainers and by the number of firms in the market. We start by stating the main results in Theorem 1. We then solve the model by backwards induction, looking first at the bargaining subgame given the market price, and then at the market price setting game. Finally, we provide intuitions and a sketch proof for Theorem 1.

Theorem 1

1. *The price takers' consumer surplus is strictly decreasing as the proportion of price takers declines;*

2. *The bargainers' expected consumer surplus is strictly decreasing as the proportion of price takers declines;*

3. *If a small set of consumers swap from price taking to bargaining, they pay less*

4. *More bargainers is*

    (a) *bad for overall consumer surplus if the bargainers seek multiple second quotes with low enough probability;*

    (b) *good for overall consumer surplus if the bargainers seek multiple second quotes with high enough probability;*

5. *Increasing the number of competing sellers unambiguously raises overall consumer surplus.*

3.1 The Bargaining Subgame

We begin our analysis by considering the bargaining subgame. Suppose that the Cournot market clears at a price $z > 0$ at which the price takers purchase. There then remains a measure $(1 - \mu) M$ of bargaining consumers who each desire $\frac{1}{M}$ units which they each value at a random draw from $U[0, 1]$. We search for a symmetric equilibrium in which each firm decides to set its quoted prices from some distribution $F(p)$. Prices in excess of z cannot be sustained in equilibrium in this subgame as the Cournot market price forms a binding upper bound by assumption. Hence the support of F is contained within $[0, z]$, so $F(z) = 1$.

The monopoly price in this model is $\frac{1}{2}$: if a monopolist offers a price p it will get a demand of $1 - p$, so expected profits $p(1 - p)$ are maximized at $\frac{1}{2}$. Letting $\widehat{z} \equiv \min\left\{z, \frac{1}{2}\right\}$, we get the following lemma.

Lemma 1 *Given $z > 0$, any symmetric equilibrium must be mixed with (i) $F(p)$ continuous, i.e., with no mass points in the density function at any price strictly above zero; (ii) $F(p) < 1$ for $p < \widehat{z}$; and (iii) $F(\widehat{z}) = 1$.*

In the mixed strategy equilibrium the firms are trading off the incentive to price high to extract surplus from the bargainers who ask for few second quotes against the incentive to price low to sell to customers who ask for many quotes. No pure strategy equilibrium can exist: if the price were positive the firms would just undercut it, and if the price were zero the firms would want to raise price towards z to extract surplus from the bargainers who ask for just one second quote.

The expected profit from the $1 - \mu$ bargainers for a firm which sets a price of $p \leq \widehat{z}$ when asked for a second quote (that is a quote subsequent to the Cournot market price) is then given by

$$\frac{\pi(p)}{1 - \mu} = p \underbrace{(1 - p)}_{\substack{\text{probability} \\ \text{consumer} \\ \text{willing to} \\ \text{buy at price } p}} \cdot \left[\sum_{k=1}^{N} q_k \underbrace{\frac{k}{N}}_{\substack{\text{probability} \\ \text{selected as} \\ \text{one of the} \\ k \text{ firms}}} (1 - F(p))^{k-1} \right] \qquad (3)$$

For any p in the support of F we must have $\pi(p) = \pi$ a constant implying that

$$\frac{N\pi}{(1-\mu)\,p\,(1-p)} = \sum_{k=1}^{N} k q_k \left(1 - F(p)\right)^{k-1} \tag{4}$$

The right hand side is a smooth strictly increasing function of $1 - F(p)$ and so must have a strictly increasing inverse function $\Phi(\cdot)$ such that for any p in the support of F:[4]

$$F(p) = 1 - \Phi\left(\frac{N\pi}{(1-\mu)\,p\,(1-p)}\right) \tag{5}$$

Note that as $p(1-p)$ is strictly increasing in p for $p < \widehat{z}$ and Φ is a strictly increasing function, $F(p)$ must also be strictly increasing. Thus there can be no gaps in the density function.

From Lemma 1, the upper bound of the support of F is at $\widehat{z}$ so from (3) we can write profits from the bargainers as

$$\pi = \pi(\widehat{z}) = \frac{1-\mu}{N}\widehat{z}\,(1-\widehat{z})\,q_1 \tag{6}$$

As expected, profits are increasing in the proportion of bargainers who ask for just one quote. Profits are also increasing in the market price, but only up to $z = \frac{1}{2}$, after which profits remain constant as the firms do not wish to quote prices above the monopoly level.

Substituting (6) into (5), we get

$$F(p) = 1 - \Phi\left(\frac{\widehat{z}\,(1-\widehat{z})\,q_1}{p\,(1-p)}\right) \tag{7}$$

This tells us that given the Cournot market price and the bargaining strategy of consumers, the prices offered to bargainers are independent of the number of firms N.[5] Intuitively this is because a firm chooses a price to offer, when asked to lower its price below the market rate, optimally against the number of quotes it will be competing against in expectation *conditional on being chosen to give a second quote*. This depends on how consumers bargain and not on the total number of firms available.

In the subsequent analysis it will be useful to explicitly note the lower bound on prices, $\underline{p}$, in the bargaining phase. This can be deduced from the equal profit condition, so using (3) once

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[4] Burdett and Judd (1983) make an equivalent argument.

[5] Of course, for a given bargaining strategy, we are not permitting N to fall below the highest k for which $q_k > 0$ or the bargaining strategy would have to change.

more $\underline{p}$ is given implicitly by [6]

$$\widehat{z}\left(1-\widehat{z}\right)q_1 = \underline{p}\left(1-\underline{p}\right)\sum_{k=1}^{N}kq_k \tag{8}$$

3.2 The Market Price Setting Stage

By deciding how much volume to bring to the Cournot market for price takers each firm can influence the market price which bargainers see as their outside option. This has a knock on effect on expected profits from the bargaining stage, which the firms anticipate. The market price therefore depends on what proportion of consumers are bargainers $(1-\mu)$ and on how they bargain. The result of this analysis is:

Lemma 2 *The unique equilibrium in pure strategies of the market price setting stage is symmetric with a market price given by*

$$z = \frac{\mu + q_1\left(1-\mu\right)}{\mu\left(N+1\right) + 2q_1\left(1-\mu\right)} < \frac{1}{2}$$

Note first that as $z < \frac{1}{2}$, $z = \widehat{z}$ so from this point on we can discard the notation $\widehat{z}$. Note further that this result has the expected limiting properties. That is, as the proportion of price takers μ tends to one (no one bargains) or the proportion of bargainers asking for just one quote q_1 tends to zero (no profit can be made from bargainers), this market price tends to $\frac{1}{N+1}$, the "standard" Cournot price. As the proportion of price takers μ tends to zero, the market price tends to the monopoly level of $\frac{1}{2}$: making profits from bargainers becomes key. The market price is falling in the number of competing firms N; and as the number of firms tends to ∞, the market price goes to zero (firms' normalized marginal cost).

3.3 Intuitions For Theorem 1

The market price offered to price takers (non-bargainers) depends upon the proportion of consumers who will bargain and how effective they are at it (that is, how likely the buyers are to request second quotes from more than one seller). Lemma 2 and the distribution of prices in the bargaining stage established in (7) above are the foundations for proving Theorem 1. The

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[6] Note that $\underline{p}$ is uniquely defined on $(0, \widehat{z})$ as $p\left(1-p\right)$ is falling as p falls below $\widehat{z}$ and $\sum_{k=1}^{N} kq_k > q_1$. Also, $\underline{p}$ is increasing as $\widehat{z}$ rises and as $q_1 \to 1$, $\sum_{k=1}^{N} kq_k \to q_1$, so $\underline{p}\left(1-\underline{p}\right) \to \widehat{z}(1-\widehat{z})$ and hence $\underline{p} \to \widehat{z}$.

proof remains technical however and so we confine it to the appendix. We instead provide an intuition and a sketch proof for the 5 parts of the theorem below.

Part 1 of Theorem 1 states that consumer surplus for the price takers goes down if the proportion of price takers declines. Algebraically this result follows quickly from Lemma 2. The intuition behind this very general result is as follows. Consider the bargaining stage first: the firms do not want the consumers to have a very good outside option. That is the firms do not want the consumers to have the option to buy at a low prevailing market price as this pulls down the expected profits from the bargaining consumers. More specifically, note that when the seller is approached by a bargainer, the Cournot market price must be in the distribution from which the seller chooses her price (if not, a firm setting the highest price, which only sells to bargainers asking for just one quote, could profitably raise price towards the monopoly level). If the seller offers the Cournot market price in the bargaining phase, it will sell only if the bargainer does not approach any other firm for a second quote. As the Cournot market price declines the profit made from these consumers declines also, and hence the expected profits from bargainers falls as the seller is indifferent between all prices in her strategy. So to maximize profits from the bargainers, a high market price is required. However, a high market price implies that a seller could increase profits from price takers by unilaterally expanding output, lowering the market price, but taking market share from her rivals. As the proportion of bargainers grows, the lost profits from less active competition for the price takers become less significant and market prices are set with the bargainers more in mind. That is, for the price takers left behind, prices rise towards collusive levels and their consumer surplus falls.

Part 2 of Theorem 1 states that the incumbent bargainers' expected consumer surplus also declines as former price takers become bargaining consumers. To show this we prove that as the proportion of the consumers who bargain grows, the distribution of prices quoted in the bargaining stage is deformed in a first order stochastically dominant way. That is, the probability of a bargained price above any given level is higher if there are lots of bargainers than if there are few. It is clear from part 1 that as the proportion of bargainers grows the market price the price takers accept, and so the upper bound to bargained prices, also grows. Thus the expected profits from the bargaining consumers grow, and so the consumers must accept higher prices more frequently. This is captured formally in Lemma 3, stated in the

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appendix, which proves the first order stochastic dominance result. Given that bargained prices grow, in expectation, with more bargainers, it is then immediate that consumer surplus for the incumbent bargainers falls if the proportion of bargainers increases.

Part 3 of Theorem 1 follows from the fact that the bargained prices are strictly below the market clearing price. This is immediate from Lemma 1: sellers lower their prices in the bargaining phase as the buyer might solicit multiple second quotes $(q_1 < 1)$.

Therefore we've shown that if more people bargain then the people who haven't changed behaviour (stay bargaining or stay not bargaining) both lose out as prices for both groups rise relative to their positions before the change. This is of some policy interest as any exhortation to bargain - that is to actively give sellers the chance to revise their quoted prices downwards - creates losers amongst consumers who either ignore, or were already following, the advice. However, those consumers who do start to bargain swap a high market price for a lower bargained price. Is overall consumer surplus raised or lowered in this case? This question is taken up in Part 4 of Theorem 1. This part of Theorem 1 offers an answer in the two extremes of consumers being either very good or very bad at bargaining. In this model good bargaining is when *multiple* sellers are offered the chance to revise their quotes downwards $(q_1 \searrow 0)$: this forces sellers to price down aggressively and results in low prices for bargainers. Bad bargaining is when buyers tend to only ask one seller for a better quote: the seller has little incentive to re-price below the prevailing market price in this case. Technically the proof considers the sign of the derivative of total consumer surplus with respect to the proportion of price takers in the extremes of good $(q_1 \searrow 0)$ and bad $(q_1 \nearrow 1)$ bargaining.

As the bargainers become bad at bargaining $(q_1 \nearrow 1)$, that is they tend to ask only one seller to move below the market price, there is little incentive to lower the prices below the market price level. The benefits of having a high market price are therefore more fully extracted during the bargaining phase. Raising the proportion of bargainers then strengthens the incentive to force a high market price for the remaining price takers in order to extract more from the bargainers. So all prices rise further, while the consumers switching to becoming bargainers only get a small discount, and hence aggregate consumer surplus falls.

The flip side of this reasoning is that if bargainers become adept at bargaining $(q_1 \searrow 0)$ and so often seek multiple quotes, then bargainers are able to force large discounts. This diminishes

the value to the sellers of having a high market price which foregoes profit increases from unilateral moves to increase market share amongst the price takers as few rents are extracted from bargainers in any case. Therefore sellers become relatively keener to extract maximum gains at each others' expense from the price takers and so prices are pushed down towards regular Cournot levels. Thus raising the proportion of bargainers has little impact on prices, but the consumers who switch to bargaining get the much lower prices offered to bargainers and total consumer surplus therefore rises.

Part 5 of Theorem 1 notes that more sellers unambiguously raises consumer surplus whatever the proportion of bargainers. For price takers increasing the number of competitors lowers the Cournot market price in the standard way: any price reduction is internalized only for a smaller fraction of total demand and so sellers are keener to lower prices by increasing their volumes. For the bargainers, a lower Cournot market price provides a lower upper bound to prices in the bargaining phase. Further, the lower maximum price pushes the distribution function of prices offered to bargainers down in a first order stochastically dominated way. Thus more probability weight is put on lower prices and so bargainers receive lower expected prices also. Hence all consumers gain if the number of competing firms rises.

Theorem 1 therefore highlights that a change in the proportion of bargainers in the population alters the prices agreed by price takers and bargainers alike. This can, in theory, be detrimental to consumer surplus at extremes of poor bargaining. Section 5 shows that the result applies more widely still. Before that we first analyse the sellers' optimal bargaining strategy in the face of the mixed population of bargaining and non bargaining buyers.

4   The Sellers' Optimal Bargaining Strategy

This section aims to explore how the market price and sellers' choice of price distribution to offer to bargainers change in response to alterations in the proportion of bargainers (μ), the number of rivals (N) or the consumers' bargaining strategy $\{q_k\}$. We start with Theorem 2 which looks at the impact of μ and N, and then move on to Theorem 3 which analyses the impact of $\{q_k\}$. Finally, Section 4.1 provides an example of a possible distribution of discounts to bargainers based on data about the UK estate agency market from OFT (2004).

Theorem 2

*If the proportion of bargainers increases, **or** if the number of competing sellers falls then:*

1. *The market price set for price takers rises;*

2. *The lowest price offered to bargainers rises;*

3. *The distribution of prices offered to bargainers places more weight on higher prices in a first order stochastically dominant fashion;*

4. *The spread of possible bargained prices grows.*

Theorem 2 is closely related to parts 1, 2 and 5 of Theorem 1. The intuition for the first part follows closely from the discussion in Section 3.3. If the proportion of bargainers increases, or if the number of competing sellers falls then the market price for the price takers rises. With fewer competing sellers this happens for the standard Cournot reason - each seller internalizes more of the industry losses from over supply to the market and so acts to restrict demand and drive up the price. With more bargainers the firms rank reducing profit loss from bargainers more heavily than reducing profit loss from price takers by under supply (too high a list price). In either case the higher market price causes the upper bound of prices which can be quoted to bargainers to rise.

Parts 2 and 3 of Theorem 2 then follow as a seller is only willing to quote a price if the profits it makes in expectation are equal to the expected profits which the seller would make from not discounting and requoting the market price. With the latter behaviour only those buyers who solicit a second quote from one seller will purchase, and as the market price goes up so do profits from this strategy. Hence the lower bound of the offered prices and the price distribution in general migrate up to higher price quotes.

Part 4 of Theorem 2 is a useful mnemonic for the optimal seller bargaining strategy. The result can be understood by noting that if the number of competitors were to rise high enough then the market price would approach marginal cost (0 here) and bargainers would enjoy no reduction: that is the spread of bargained prices would shrink to nothing. As the market price for the price takers rises, the lowest bargained price will rise too - but not as fast as the market price as it always lags behind. Part 4 confirms this insight as a robust general result.

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Theorem 3 considers two bargaining strategies $\{q_i^{\text{poor}}\}$ and $\{q_i^{\text{good}}\}$. The good bargaining strategy has buyers strictly less likely to solicit a second quote from only one seller, and weakly more likely to solicit a second quote for any given multiple number of sellers. In particular:

$$
\begin{aligned}
q_1^{\text{good}} \quad &< \quad q_1^{\text{poor}} \text{ and} \\
q_k^{\text{good}} \quad &\geq \quad q_k^{\text{poor}} \ \forall k \in \{2, 3, \cdots, N\} \text{ with strict inequality for some } k
\end{aligned}
\tag{9}
$$

In this case the following holds:

Theorem 3

If the population of bargainers should move to using the good bargaining strategy $\{q_i^{good}\}$ as opposed to the poor strategy $\{q_i^{poor}\}$ then:

1. *The equilibrium market price falls;*

2. *The lowest price offered to bargainers falls;*

3. *The distribution of prices offered to bargainers places more weight on lower prices in a first order stochastically dominated fashion.*

Note that it immediately follows from Theorem 3 that total consumer surplus rises as the bargainers become better at bargaining as all prices are falling while the proportion of bargainers remains unchanged. To understand Theorem 3 note that in the bargaining stage the sellers will only make quotes which give them the same profit level in expectation: otherwise some price quote would be suboptimal. The expected profits which can be made from the bargaining stage can therefore be deduced as the profit level if the seller should refuse to offer a reduction against the market price, in which case only those buyers who solicit a second quote from only that seller will purchase. Thus the expected profits from the bargaining stage depend only on the probability of soliciting one second quote (q_1), and not on any of the other probabilities. Hence part 1 follows as $q_1^{\text{good}} < q_1^{\text{poor}}$ and so if buyers use the good bargaining strategy sellers expect to make less profit from bargainers and so the incentive to overprice to price takers is reduced.

The lowest second quoted price to bargainers also falls under the good buyer bargaining strategy (part 2). This is because bargainers are interacting with more sellers on average, and

so to maintain some probability of quoting the lowest second price amongst those sought, the seller will have to push her lowest quoted price down. Part 3 then confirms that this insight applies to all prices which might be quoted: with the good buyer bargaining strategy (in the sense of condition (9)) the seller has to push prices down across the board.

4.1 Selecting Second Quote Price Points

We now turn to the optimal probability with which given second quoted prices might be offered. The market price and the lower bound on prices are readily found using Lemma 2 and solving the quadratic given by (8). However deriving the distribution function from (6) and (4) involves solving up to an $(N-1)^{\text{th}}$ order polynomial. Though this can be done numerically, it cannot in general be done analytically. Nevertheless, in Theorems 2 and 3 we were able to prove general results about how $F(p)$ changes as μ, N and $\{q_k\}$ change.

Here we provide an example of how the optimal bargaining strategy can be calculated in practice. Consider the UK estate agency market analysed by the Office of Fair Trading in the UK (OFT (2004)). In Table 4.5 of OFT (2004) survey evidence of the number of estate agents visited by those consumers who claimed to 'shop around' is presented. We interpret shopping around as being a bargainer in this model. Limiting the maximum number of estate agents who are requested for a requote to 3 to preserve algebraic tractability the OFT data would imply that $q_1 = 0.51$, $q_2 = 0.22$ and $q_3 = 0.27$.[7] In paragraph 4.47 it is noted that 40% of the population looking for estate agent services 'do not shop around'. Thus we interpret $\mu = 0.4$. Assuming that 5 estate agents are in active competition for the business of a given house, the equilibrium bargaining strategy can be calculated and is depicted in Figure 3.

Recall that the marginal costs have been normalized to 0 in this work. Thus the market price z is actually to be interpreted as the profit margin made on sales at the market price to the non-bargainers. The first graph of Figure 3 then depicts what proportion of this margin a seller would offer back to bargainers in the hope of winning the business. This figure is perhaps

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[7] The source data actually had 21% visiting 3 estate agents with 4% visiting 4 and 2% visiting 5. To facilitate replication, note that if bargainers have strategy $\{q_1, q_2, q_3\}$ and $q_4 = 0 = q_5 = \cdots q_N$ then the distribution of offered bargained prices satisfies

$$1 - F(p) = \frac{1}{3q_3} \left[\sqrt{q_2^2 + 3q_1 q_3 \left[\frac{z(1-z)}{p(1-p)} - 1 \right]} - q_2 \right]$$

Graph 1: Percentage reduction _in margin_ offered to bargainers

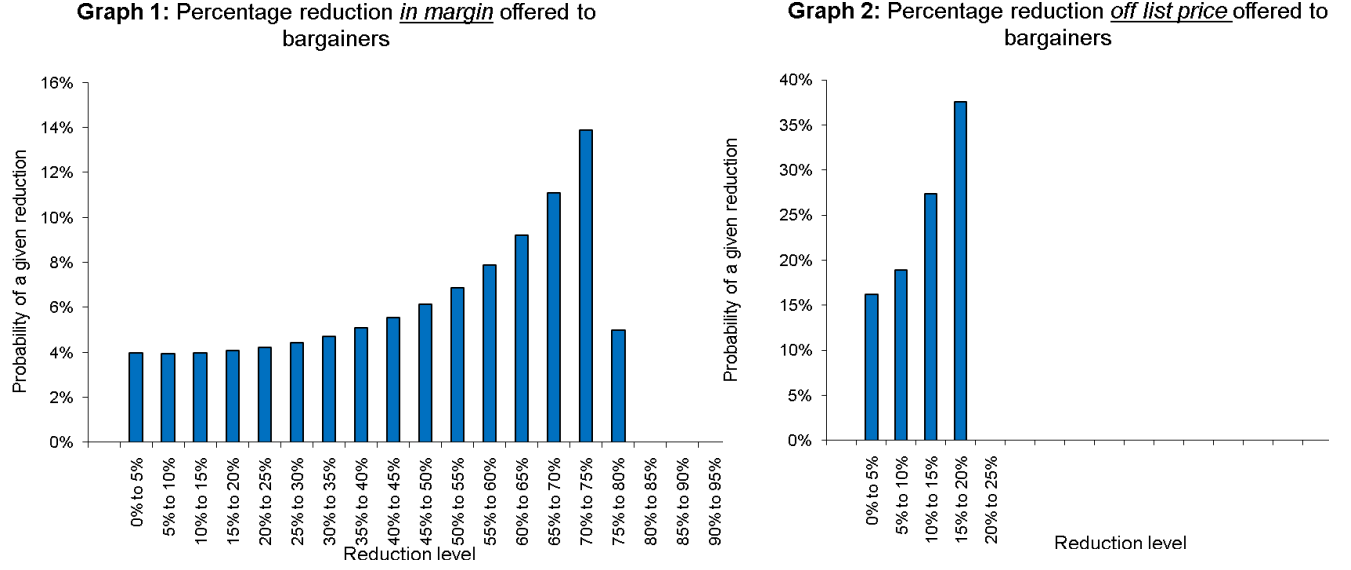**Graph 2:** Percentage reduction _off list price_ offered to bargainers

Figure 3: Equilibrium price reductions offered to bargainers. The calculations are based on the evidence on consumer bargaining with UK estate agents provided by OFT 2004. See the discussion in Section 4.1 of this paper.

useful as a benchmark: as the parameters of the market change, the optimal bargaining strategy will also change in a way captured by Theorems 2 and 3.

From paragraph 4.6 of OFT (2004), those who shopped around received an average discount of 14%. Together with the distribution of discounts off the profit margin calculated above, we can use this average discount to back out an estimate of the marginal cost c. Given this estimated marginal cost, we can then calculate the distribution of percentage equilibrium discounts off the list price of $z + c$.[8] The second graph of Figure 3 illustrates the equilibrium discounts offered.[9]

5 Numerical Analysis

We have established that more bargainers can be detrimental to consumer welfare at the extremes of bargaining proficiency (Theorem 1, part 4). What remains unclear however is whether more consumers bargaining can be detrimental at more realistic intermediate levels of consumer bargaining prowess. Here we consider a number of numerical examples to explore this and other questions.

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[8] The calculations are carried out assuming that the valuations are distributed uniformly on $[c, 1 + c]$.

[9] Note that a higher marginal cost might be a further explanation for the lower percentage discounts off list prices offered to buyers of luxury cars in Figure 1 (see also footnote 3).

More specifically, this section uses numerical simulations to explore the relationship between total consumer surplus and the proportion of the population bargaining $(1 - \mu)$, the probability that bargainers only request a better price from one seller (q_1), and the number of competing sellers (N). To facilitate the analysis we assume that buyers request better prices from one or two sellers only; not from three or more. In the notation of the model we are assuming $q_3 = q_4 = \cdots q_N = 0$. Calculating the total consumer surplus for this population is involved but essentially straightforward. We outline the main steps in Appendix C. Figure 4 graphs the results of this exercise.
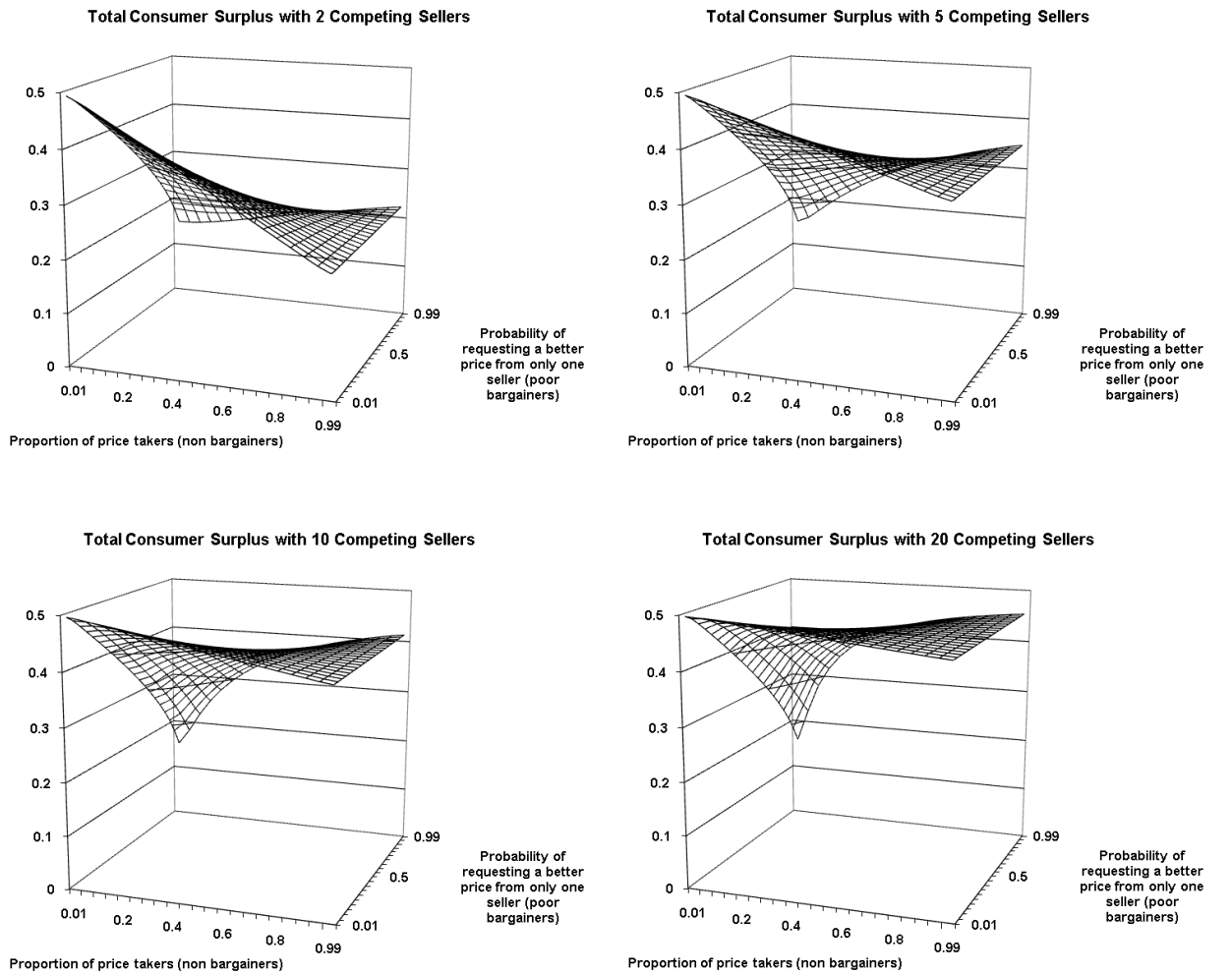

Figure 4: A numerical analysis: bargainers are assumed to either request better prices from one or two sellers only ($q_3 = q_4 = \cdots = q_N = 0$).

Figure 4 contains a panel of 4 graphs. Each graph represents a different number of competing

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sellers ($N \in \{2, 5, 10, 20\}$). In each graph the total consumer surplus is plotted as a function of both the proportion of non-bargainers (μ) and the probability that a bargainer will request a better price from one seller only (q_1). The graphs depict the following stylized facts:

- For any given proportion of price takers, the better bargainers are at bargaining (low q_1 and so high probability that second quotes are requested from multiple sellers), the greater is total consumer surplus. This is an immediate consequence of Theorem 3, which finds that all prices fall as the consumers become better bargainers.

- Theorem 1, part 4 is confirmed in all 4 graphs: If bargainers are very good at bargaining (q_1 low), then the consumer surplus falls monotonically as more buyers become price takers. If bargainers are poor at bargaining (q_1 high) then the consumer surplus rises monotonically as more buyers become price takers.

- No matter how many sellers compete, if almost all buyers bargain then consumer surplus is independent of the number of sellers and improves the better the buyers are at bargaining (the lower is q_1).[10] If almost all buyers bargain ($\mu \to 0$), the market price goes to the monopoly level of $\frac{1}{2}$ and the sellers post prices strategically based on how many sellers they'll be competing against in the bargaining phase, *conditional on being chosen*. The absolute number of sellers is not relevant.

- The result in part 5 of Theorem 1 that consumer surplus is increasing in the number of sellers is confirmed. Furthermore, the increases in consumer surplus available from bargaining (which occur for low q_1) become smaller as the number of rivals increases, while the decreases in consumer surplus from bargaining (which occur for high q_1) become larger. This is a natural result of the fact that when almost all buyers bargain, surplus is independent of the number of firms (see the previous bullet) but declining in q_1, while when almost nobody bargains, almost everybody buys at the "standard" Cournot price, so surplus is independent of q_1 but increasing in the number of sellers.

- Finally, changing the proportion of price takers while keeping bargaining ability fixed does not always alter the overall consumer surplus in a monotonic way. Consumer surplus

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[10] The downward sloping curve at $\mu = 0$ is the same in all four graphs.

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can, for some parameter values, be maximized at intermediate proportions of price takers versus bargainers under given bargaining prowess. For example, for $q_1 = \frac{1}{2}$ and $N = 5$ total consumer surplus is maximized at $\mu^* = 0.39$, i.e., where 39% of consumers take prices as given while the other 61% bargain. If q_1 rises to $\frac{3}{5}$, the optimal proportion of price takers rises to 0.78, so we only want a handful of bargainers.

6   Conclusion

In conclusion, using our relatively simple set-up we have been able to derive a rich picture of the impact of bargainers on markets with price takers. The model highlighted the strategic dependence of the market price (which the price takers see as final) on the rents which bargainers were able to extract. In the absence of bargainers, if market prices are high each firm has an incentive to try to take market share off its rivals by increasing output and accepting a small drop in the market price. However, when bargainers are present this price drop results in substantial extra profits being lost to those buyers able to bargain a reduction off the market price. Thus the presence of bargainers allows the market price to non-bargainers to be pushed closer to the collusive level. Of course bargainers do transact below the market price, but whether overall consumer surplus is benefited by bargaining depends on the parameters. Thus the best position for a consumer is to be one of few bargainers amongst a population consisting mainly of price takers. If more consumers start to bargain then the increase in prices to the remaining price takers and the incumbent bargainers can actually mean that total consumer surplus declines. Hence the title of this paper: too many bargainers can indeed spoil the broth.

The optimal proportion of bargainers for consumer surplus can be interior, or at an extreme where nobody bargains or almost everybody does. When bargainers approach few sellers, consumers tend to be best off in the absence of bargaining. When almost all bargainers approach more than one firm, prices to the bargainers go to cost so consumers overall are best off if almost everyone bargains. Finally, increasing the number of sellers always lowers prices to both price takers and bargainers and the more sellers the bargainers are likely to approach, the lower are all prices on average.

Perhaps the main route for further research is to allow the number of secondary quotes sourced to be endogenized. This would have bargainers getting more than one quote if they

don't receive a good enough result after bargaining with one firm. We conjecture that the main forces discussed here would however remain robust: profits are always higher from bargainers if the outside option (the prevailing list or market price) is high. If more are encouraged to bargain then those left behind are seen as a less key group by the sellers and so their price can be forced higher in a bid to secure larger profits from bargainers.

A   Proofs from Section 3

Proof of Lemma 1.   Suppose first that there is a mass point at price $p > 0$ in the density function. A firm could profitably deviate by lowering its quote price to $p - \varepsilon > 0$ just below the mass point whenever it would have proposed p. This increases total sales by a discrete amount (when the consumer receives a lowest quote from a rival firm of p and has $v_i \geq p$) in return for a vanishingly small loss and so is a profitable deviation. Suppose second that the support of F stops strictly below $\widehat{z}$. Any firm charging the highest price could profitably deviate by raising price towards $\widehat{z}$ as in either case it will make a sale only if the bargainer asks for just one second quote, and expected profits from such a consumer of $p(1-p)$ are increasing as p rises towards $\frac{1}{2}$.[11] For part (iii) we need to show that sellers would never quote prices above the monopoly price of $\frac{1}{2}$ to bargainers. To see this suppose for a contradiction that $F(\widehat{z}) < 1$. As $F(z) = 1$, we must have $z > \frac{1}{2}$ and $\widehat{z} = \frac{1}{2}$. Then $\exists p > \frac{1}{2}$ such that $f(p) > 0$. However the firm would do better to lower price towards $\frac{1}{2}$. Expected profits from a consumer asking for k quotes are $p(1-p)(1-F(p))^{k-1}$. These increase as p falls towards $\frac{1}{2}$ as $p(1-p)$ goes up while $F(p)$ falls. ∎

Proof of Lemma 2.   Suppose firm i elects to make a volume x_i available on the (first stage) Cournot market which is aimed at the price takers. Let $X = \sum x_i$ be the total volume supplied. At a market price $z \in [0, 1]$, from (1) the price takers demand a quantity $\mu(1-z)$, so given X the Cournot market price is given by

$$z(X) = \max\left\{1 - \frac{X}{\mu}, 0\right\} \tag{10}$$

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[11] This argument extends directly if the support of $F(\cdot)$ is open so that $\sup\{p : F(p) < 1\} = \bar{p} < \hat{z}$ as then a deviation from price p sufficiently close to $\bar{p}$ that $1 - F(p) < \varepsilon$ to a price of $\hat{z}$ is profitable if ε is sufficiently small.

Using (6), the seller's total profits are therefore given by

$$\Pi\left(x_i\right) = x_i z\left(X\right) + \text{expected profits from bargainers}$$

$$= x_i z\left(X\right) + \frac{1-\mu}{N}\widehat{z}\left(X\right)\left(1 - \widehat{z}\left(X\right)\right)q_1 \tag{11}$$

Recall that $\widehat{z} \equiv \min\left\{z, \frac{1}{2}\right\}$; and note that $\frac{\partial z(X)}{\partial x_i} = -\frac{1}{\mu}$ for $z\left(X\right) > 0$.

The next step is to determine firm $i's$ optimal volumes. For $z > \frac{1}{2}$,

$$\frac{\partial \Pi\left(x_i\right)}{\partial x_i} = z\left(X\right) - \frac{x_i}{\mu} \tag{12}$$

$$\frac{\partial^2 \Pi\left(x_i\right)}{\partial x_i^2} = -\frac{2}{\mu} < 0 \tag{13}$$

For $z \in \left(0, \frac{1}{2}\right)$,

$$\frac{\partial \Pi\left(x_i\right)}{\partial x_i} = z\left(X\right) - \frac{x_i}{\mu} + q_1\left(\frac{1-\mu}{N\mu}\right)\left(2z\left(X\right) - 1\right) \tag{14}$$

$$\frac{\partial^2 \Pi\left(x_i\right)}{\partial x_i^2} = -\frac{2}{\mu} + q_1\left(\frac{1-\mu}{N\mu}\right)\left(-\frac{2}{\mu}\right) < 0 \tag{15}$$

At $z = \frac{1}{2}$, the right-hand side derivative is (12) while the left-hand side derivative is (14). However, as $2z\left(X\right) - 1 = 0$ these both equal (12). The right-hand side second derivative is (13) while the left-hand side second derivative is (15).

In all these cases the optimal x_i^* is uniquely determined (possibly at the corner solution of $x_i^* = 0$) so the equilibrium must be symmetric. Note also that the objective function is everywhere strictly concave.

Suppose first that $z \geq \frac{1}{2}$ is an equilibrium. From (12) the first order condition gives $x_i^* = \mu z$. By symmetry, $X = Nx_i^* = N\mu z$. Thus (10) implies that $z = 1 - \frac{X}{\mu} = 1 - Nz$ and so $z = \frac{1}{N+1}$. As $N \geq 2$, we therefore get $z < \frac{1}{2}$, a contradiction.

Suppose instead that $0 < z < \frac{1}{2}$ is an equilibrium. From (14) the first order condition gives

$$Nx_i^* = \max\left\{N\mu z + q_1\left(1 - \mu\right)\left(2z - 1\right), 0\right\} \tag{16}$$

As $z = 1 - \frac{X}{\mu}$ from (10), and $X = Nx_i^*$ by symmetry, $Nx_i^* = \mu - z\mu > 0$. Equating this with

(16), we get the market price for price takers given by

$$z\left[N\mu + 2q_1(1-\mu) + \mu\right] = \mu + q_1(1-\mu) \Leftrightarrow$$
$$z = \frac{\mu + q_1(1-\mu)}{\mu(N+1) + 2q_1(1-\mu)} < \frac{1}{2}$$

as required.

Finally, suppose that $z = 0$ is an equilibrium, so $\sum x_i \geq \mu$. All firms make zero profit. Suppose first that there is a firm i for which $\sum_{j \neq i} x_j < \mu$. This firm can raise profits by lowering output and hence raising price above zero. If no such firm exists, we have an equilibrium with excess supply and zero price. However, we will disregard such equilibria as they are not robust to introducing any positive marginal cost, which would induce all the firms to deviate to zero output. ∎

A.1 Proofs of All Parts of Theorem 1

Proof of Part 1 of Theorem 1. If the Cournot market price is z, then the average consumer surplus per non-bargainer is given by (2) as $\frac{1}{2}(1-z)^2$. The consumer surplus of the price takers falls if the market price, z rises. The proof then follows by showing that $\frac{\partial z}{\partial \mu} < 0$ so that more bargainers (lower μ) implies a higher market price for the remaining price takers. In particular, using Lemma 2 we have

$$\frac{\partial z}{\partial \mu} = {}_{\text{sign}} \left[\mu + q_1(1-\mu) + N\mu + q_1(1-\mu)\right](1-q_1) - \left[\mu + q_1(1-\mu)\right](1-q_1 + N - q_1)$$
$$= -q_1(N-1) < 0 \text{ as } N \geq 2 \text{ by assumption} \tag{17}$$

∎

To make further progress we require the following technical result which states that if the proportion of bargainers rises, then the distribution of prices to the bargainers shifts upwards so that higher prices are always more likely.

Lemma 3 *If the proportion of price takers falls from μ_2 down to μ_1, then the distribution of prices to the bargainers under μ_1 (denoted $F_1(p)$) first order stochastically dominates the distribution of prices to bargainers under μ_2 (denoted $F_2(p)$).*

Proof. Define the Cournot market price with a proportion μ_i of price takers as z_i. By the proof of part 1 of Theorem 1 we have that $\mu_1 < \mu_2 \Rightarrow z_1 > z_2$. This just says that with fewer price takers the prices to the remaining price takers rises. Recalling from Lemma 2 that the Cournot market price must satisfy $z < \frac{1}{2}$ we must have that $z(1-z)$ is an increasing function of z. Hence

$$z_1(1-z_1) > z_2(1-z_2) \tag{18}$$

Now denote the distribution function of prices with μ_i price takers as F_i, and the lower bound of its support as $\underline{p}_i$. Using (7), this can be written

$$F_i(p) = 1 - \Phi\left(\frac{z_i(1-z_i)}{p(1-p)} q_1\right) \text{ for } p \in \left[\underline{p}_i, z_i\right] \tag{19}$$
$$F_i(p) = 0 \text{ for } p \leq \underline{p}_i; \quad F_i(p) = 1 \text{ for } p \geq z_i$$

where $\Phi(\cdot)$ is a strictly increasing function. From footnote 6 $z_1 > z_2 \Rightarrow \underline{p}_1 > \underline{p}_2$. Then for any $p \leq \underline{p}_2$, $F_1(p) = F_2(p) = 0$, for any $p \in \left(\underline{p}_2, \underline{p}_1\right]$ $F_1(p) = 0 < F_2(p)$, for any $p \in \left(\underline{p}_1, z_1\right)$ (18) implies that $0 < F_1(p) < F_2(p) \leq 1$ and for any $p \geq z_1$ $F_1(p) = F_2(p) = 1$. Thus $F_1(p)$ first order stochastically dominates $F_2(p)$. ∎

We can now analyse the consumer surplus of bargainers and prove part 2 of Theorem 1.

Proof of Part 2 of Theorem 1. Suppose that the proportion of price takers falls from μ_2 down to μ_1. The distribution of prices for the bargainers with μ_i price takers is given by F_i. By Lemma 3 we have $F_1 \succ_{\text{FOSD}} F_2$. If a bargainer gets k second quotes ($k \in \{2, 3, ..., N\}$), then the lowest price she receives has distribution function $H_i^k(p) = 1 - (1 - F_i(p))^k$ and so we have

$$F_1 \succ_{\text{FOSD}} F_2 \quad \Leftrightarrow \quad F_1(p) \leq F_2(p) \ \forall p \ \& \ \exists p \text{ s.t. } F_1(p) < F_2(p)$$
$$\Leftrightarrow \quad H_1^k(p) \leq H_2^k(p) \ \forall p \ \& \ \exists p \text{ s.t. } H_1^k(p) < H_2^k(p)$$
$$\Leftrightarrow \quad H_1^k \succ_{\text{FOSD}} H_2^k$$

That is if there are lots of bargainers (and so few price takers, state μ_1), then the bargained prices tend to be high, and if two or more prices are drawn the minimum of these tends to be higher also.

Now note that with μ_i price takers the consumer surplus per bargainer is given by

$$CS \text{ pp for bargainers}\big|_{\mu_i} = q_1 \underbrace{E_{F_i}\left[\frac{1}{2}(1-p)^2\right]}_{\substack{\text{Expected surplus} \\ \text{if ask for 1 quote}}} + \sum_{k=2}^N q_k \underbrace{E_{H_i^k}\left[\frac{1}{2}(1-p)^2\right]}_{\substack{\text{Expected surplus} \\ \text{if ask for } k \text{ quotes}}}$$

where pp stands for per person. Clearly $(1-p)^2$ is a decreasing function of p for prices in the feasible range of $\left[0, \frac{1}{2}\right]$ and so

$$F_1 \succ_{\text{FOSD}} F_2 \Rightarrow E_{F_1}\left[\frac{1}{2}(1-p)^2\right] < E_{F_2}\left[\frac{1}{2}(1-p)^2\right]$$

and similarly for $\left\{H_i^k\right\}$. Therefore we have

$$\mu_1 < \mu_2 \Rightarrow CS \text{ pp for bargainers}\big|_{\mu_1} < CS \text{ bargainers}\big|_{\mu_2}$$

which gives the result. ∎

Proof of Part 3 of Theorem 1. Immediate from Lemma 1 as bargainers receive prices strictly below the market price which price takers accept, and z is a continuous function of μ so as $\Delta\mu \to 0$, $\Delta z \to 0$. ∎

Proof of Parts 4a. and 4b. of Theorem 1. We define total consumer surplus (Tot CS) as

$$\text{Tot } CS \equiv \mu \cdot (CS \text{ pp for price takers}) + (1-\mu) \cdot (CS \text{ pp for bargainers})$$

The consumer surplus terms are continuous in μ. Therefore for part (a) it is sufficient to show that

$$\lim_{q_1 \to 1} \frac{d\text{Tot } CS}{d\mu} > 0 \tag{20}$$

as then the result will hold for some range of high q_1 as required. Now note that

$$\frac{d\text{Tot } CS}{d\mu} = \begin{array}{l} (CS \text{ pp for price takers}) - (CS \text{ pp for bargainers}) \\ +\mu\frac{\partial(CS \text{ pp for price takers})}{\partial\mu} + (1-\mu)\frac{\partial(CS \text{ pp for bargainers})}{\partial\mu} \end{array}$$

First note that as $q_1 \to 1$ the quoted price in the bargaining stage tends to z (see footnote

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6): if consumers don't get multiple second quotes then there is no incentive to lower prices to bargainers. But in this case the consumer surplus per person for both the bargainers and price takers tend to equality as the prices they face tend to the same limit. Next by parts 1 and 2 of Theorem 1, proved above, we have

$$\frac{\partial \left(CS \text{ pp for price takers}\right)}{\partial \mu} > 0 \; , \; \frac{\partial \left(CS \text{ pp for bargainers}\right)}{\partial \mu} > 0$$

Therefore if these don't both vanish as $q_1 \to 1$ we are done. Now note that $\frac{\partial(CS \text{ pp for price takers})}{\partial \mu} = -\left(1 - z\right)\frac{\partial z}{\partial \mu}$ and

$$\lim_{q_1 \to 1} z = \left[2 + \mu\left(N - 1\right)\right]^{-1} < 1$$

$$\text{And from (17)} \quad -\frac{\partial z}{\partial \mu} = \frac{q_1\left(N - 1\right)}{\left[\mu\left(N + 1\right) + 2q_1\left(1 - \mu\right)\right]^2} \to \frac{N - 1}{\left[2 + \mu\left(N - 1\right)\right]^2} > 0 \text{ as } q_1 \to 1$$

Hence $\lim_{q_1 \to 1} \frac{\partial(CS \text{ pp for price takers})}{\partial \mu} > 0$ and so (20) holds. This gives part 4a. of Theorem 1

Part 4b. of Theorem 1 is more straightforward. If all bargainers request more than one quote then the standard Bertrand pricing outcome follows for bargainers; prices fall to marginal cost and so all bargainers receive a price of 0 and have expected consumer surplus of $\frac{1}{2}$. Formally, as $q_1 \to 0$ profits from the bargainers fall to zero from (6) which means that $F(p) \to 1$ for all p. As the bargained price is independent of the Cournot market price this drops to its standard value $\left(z = \frac{1}{N+1} > 0\right)$. The total consumer surplus is therefore

$$\lim_{q_1 \to 0} \text{Tot } CS = \left(1 - \mu\right)\frac{1}{2} + \mu\frac{1}{2}\left(1 - \frac{1}{N + 1}\right)^2$$

which rises as μ falls.[12] ■

Proof of Part 5 of Theorem 1.   Suppose we compare two markets, one with N_1 competing sellers, the other with $N_2 > N_1$. Let z_i denote the Cournot market price with N_i competitors. Then by Lemma 2 we have $z_2 < z_1$. Thus the price takers have greater consumer

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[12] Note that Tot CS is smooth in q_1 as $\Phi\left(\cdot\right)$ is and so

$$\frac{\partial}{\partial \mu}\left(\lim_{q_1 \to 0} \text{Tot } CS\right) = \lim_{q_1 \to 0}\left(\frac{\partial \text{Tot } CS}{\partial \mu}\right)$$

by a Taylor expansion around $q_1 = 0$.

surplus with the larger number of competitors. Turning to the bargainers, we denote the distribution of prices they receive with N_i competitors as F_i. Following the proof of Lemma 3, F_1 first order stochastically dominates F_2. That the bargainers have higher consumer surplus with the larger N_2 competitors then follows identically to the proof of part 2 of Theorem 1. Combining we have the desired result. ■

B Proofs from Section 4

Proof of Theorem 2. Part 1 requires us to confirm that $\frac{\partial z}{\partial \mu} < 0$ which follows from the proof of Theorem 1, part 1. We also need to confirm that $\frac{\partial z}{\partial N} < 0$ which is clear from Lemma 2. For part 2 revisit (8) to give

$$q_1 z (1 - z) = \underline{p} (1 - \underline{p}) \sum_{k=1}^{N} k q_k \qquad (21)$$

As the market price (z) is less than $\frac{1}{2}$, $z (1 - z)$ is monotonically increasing in z. As the lowest price offered to bargainers $(\underline{p})$ is smaller than z, then $\underline{p} (1 - \underline{p})$ is monotonically increasing in $\underline{p}$ also. Hence z and $\underline{p}$ move in the same direction giving part 2. Part 3 is a direct application of Lemma 3 for the effect of μ and of the proof of part 5 of Theorem 1 for the effect of N. For part 4 differentiate (21) with respect to z to give

$$\frac{d\underline{p}}{dz} = \underbrace{\left[\frac{q_1}{\sum_{k=1}^{N} k q_k} \right]}_{<1} \underbrace{\left[\frac{1 - 2z}{1 - 2\underline{p}} \right]}_{<1} < 1$$

which gives the result. ■

Proof of Theorem 3. Consider the price takers first. The market price is determined in Lemma 2 and only depends on q_1, not on the value of q_k for $k > 1$. Further, by (9) we have $q_1^{\text{good}} < q_1^{\text{poor}}$. It is immediate from Lemma 2 that $\frac{\partial z}{\partial q_1} =_{\text{sign}} \mu (1 - \mu) (N - 1) > 0$. Therefore the market price for the price takers if bargainers use the good strategy is lower.

Turning to the bargainers, as the market price for the price takers is below the monopoly level of $\frac{1}{2}$ by Lemma 2, $z (1 - z)$ is an increasing function of z and so

$$z^{\text{good}} \left(1 - z^{\text{good}} \right) < z^{\text{poor}} (1 - z^{\text{poor}})$$

Substituting (6) into (4), we can show that for any p in the support of F^i (the distribution of prices given $\{q_1^i, q_2^i, ..., q_N^i\}$) we have

$$q_1^i \left[\frac{z^i(1-z^i)}{p(1-p)} - 1 \right] = \sum_{k=2}^{N} kq_k^i \left(1 - F^i(p)\right)^{k-1} \tag{22}$$

From (8) $\underline{p}^i$, the lower bound on the support of F^i, is defined by $\underline{p}^i(1 - \underline{p}^i) = \frac{z_i(1-z_i)q_1^i}{\sum_{k=1}^{N} kq_k^i}$. Thus $\underline{p}^{\text{good}}(1 - \underline{p}^{\text{good}}) < \underline{p}^{\text{poor}}(1 - \underline{p}^{\text{poor}})$ as (i) $z^{\text{good}}\left(1 - z^{\text{good}}\right) < z^{\text{poor}}\left(1 - z^{\text{poor}}\right)$, (ii) $q_1^{\text{good}} < q_1^{\text{poor}}$ and (iii) $\sum_{k=1}^{N} kq_k^{\text{good}} > \sum_{k=1}^{N} kq_k^{\text{poor}}$ by (9). Hence as $\underline{p} < z^i < \frac{1}{2}$ then $\underline{p}\left(1 - \underline{p}\right)$ is an increasing function of $\underline{p}$ and so $\underline{p}^{\text{good}} < \underline{p}^{\text{poor}}$. That is the lowest price a population of good bargainers can get is lower then the lowest price a population of poor bargainers can get.

We now aim to show that the prices a population of good bargainers would get are consistently lower than the prices a population of poor bargainers would get, in the sense of first order stochastic dominance. To see this note that for any $p \leq \underline{p}^{\text{good}}$, $F^{\text{poor}}(p) = F^{\text{good}}(p) = 0$, for any $p \in \left(\underline{p}^{\text{good}}, \underline{p}^{\text{poor}}\right]$, $F^{\text{poor}}(p) = 0 < F^{\text{good}}(p)$, for any $p \geq z^{\text{poor}}$, $F^{\text{poor}}(p) = F^{\text{good}}(p) = 1$. In the range $p \in \left(\underline{p}^{\text{poor}}, z^{\text{poor}}\right)$ we must have $F^{\text{poor}}(p) < F^{\text{good}}(p)$. Suppose not. Then p is in the support of both F^{good} and F^{poor} so (22) must hold for both. The left hand side of (22) is smaller under good bargaining than under poor bargaining. However, by (9) $kq_k^{\text{good}} \geq kq_k^{\text{poor}}$ for $k \geq 2$. Thus for (22) to hold we need $1 - F^{\text{good}}(p) < 1 - F^{\text{poor}}(p)$, a contradiction to the initial assumption. But this argument works for any p in $\left(\underline{p}^{\text{poor}}, z^{\text{poor}}\right)$ and so $F^{\text{poor}}(p)$ first order stochastically dominates $F^{\text{good}}(p)$. Hence we have part 3. ■

C The Proofs Underlying the Numerical Analysis of Section 5

Consider a population of buyers with a proportion of price takers μ, N firms competing, and bargainers requesting better prices from one seller with probability q_1 or two sellers with probability $1 - q_1$. The market price, z, offered to the price takers is given by Lemma 2 and the consumer surplus per person for the price takers is explicitly given as $\frac{1}{2}(1 - z)^2$ in equation (2). Once the following lemma is established, the total consumer surplus is found by noting that there are μ price takers and $1 - \mu$ bargainers and so the weighted average of the consumer surplus per person expressions can be found.

Lemma 4 *The consumer surplus per person for the bargainers is given by*

$$\frac{1}{4}\frac{q_1^2}{1-q_1}z^2\left(1-z\right)^2\left[-\frac{1}{2p^2}+\frac{1}{p}-\ln p+\ln\left(1-p\right)\right]_{p=\underline{p}}^{z}$$

with

$$\underline{p}=\frac{1}{2}\left(1-\sqrt{1-\frac{4z\left(1-z\right)q_1}{2-q_1}}\right)$$

Proof. Setting $q_3 = q_4 = \cdots = 0$ in (4) and substituting in (6) where the market price is established as $z < \frac{1}{2}$ we have

$$1-F\left(p\right)=\frac{1}{2}\frac{q_1}{1-q_1}\left[\frac{z\left(1-z\right)}{p\left(1-p\right)}-1\right]\text{ and }\frac{d}{dp}\left[1-F\left(p\right)\right]=\frac{1}{2}\frac{q_1}{1-q_1}z\left(1-z\right)\frac{2p-1}{p^2\left(1-p\right)^2}\quad(23)$$

Now let $H\left(p\right)$ denote the distribution function of the lowest price from two quotes. We can therefore write the expected consumer surplus of a bargainer by

$$q_1E_F\left[\frac{1}{2}\left(1-p\right)^2\right]+\left(1-q_1\right)E_H\left[\frac{1}{2}\left(1-p\right)^2\right]$$

Using the fact that $H\left(p\right)=1-\left(1-F\left(p\right)\right)^2$ we can produce the composite distribution function

$$\begin{aligned}G\left(p\right)&=&q_1F\left(p\right)+\left(1-q_1\right)H\left(p\right)\\&=&1-q_1\left[1-F\left(p\right)\right]-\left(1-q_1\right)\left[1-F\left(p\right)\right]^2\end{aligned}$$

Using (23) we can calculate the composite density function as

$$g\left(p\right)=\frac{1}{2}\frac{q_1^2}{1-q_1}z^2\left(1-z\right)^2\frac{1-2p}{p^3\left(1-p\right)^3}$$

We therefore have the consumer surplus per bargainer as

$$E_G\left[\frac{1}{2}\left(1-p\right)^2\right]=\int_{p=\underline{p}}^{z}\frac{1}{4}\frac{q_1^2}{1-q_1}z^2\left(1-z\right)^2\frac{1-2p}{p^3\left(1-p\right)}dp$$

Now noting that

$$\frac{1-2p}{p^3\left(1-p\right)}=\frac{1}{p^3}-\frac{1}{p^2}-\frac{1}{p}-\frac{1}{1-p}$$

allows the integral to be solved and the result to be derived.

Finally the lower bound on bargained prices $(\underline{p})$ is found by setting $F(\underline{p}) = 0$ in (23) and solving the resultant quadratic equation. ∎

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