Carnap versus Gödel:

On Syntax and Tolerance

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One thing we have found out about logical empiricism, now that people are examining it more closely again, is that it was more a framework for a number of related views than a single doctrine. The pluralism of different approaches among various adherents to the Vienna and Berlin groups has been much emphasized. Some have gone so far as to suggest that the kind of speculative philosophy now often called "continental" (including, say, phenomenology) can be seen as falling within the framework of views shared by the Vienna Circle. Much is made of Felix Kaufmann's membership in the Vienna Circle, for instance, or of its "Austrian" roots in the phenomenology of Brentano.

This paper will argue that this goes too far, and that although logical empiricism is not a single doctrine, it does have a core position that is actually *not* compatible with most of what counts as continental philosophy today. (Carnap's views were diametrically opposed, for instance, to those of Heidegger, as Michael Friedman has discussed.) The Vienna Circle, though not doctrinaire, did see itself and its allies as having very definite values; they have been appropriately called a late form of the European Enlightenment. Their critique of metaphysics, though later qualified and revised in a number of respects, was not revoked. It was, supposedly, refuted, by Quine, Kuhn, and others, and this supposed refutation seems to have made metaphysics respectable again. But apart perhaps from Hempel, none of the original logical empiricists accepted this refutation, and there is no reason to believe they would even now.

The supposed refutation of logical empiricism has recently, as you know, received new reinforcement from the heart of the Vienna Circle itself — from Goedel, who without making it very apparent at the time, had apparently been an adherent of continental metaphysics all along. Gödel's critique has provoked considerable discussion since its publication a few years ago. More directly than the previous refutations, it seeks to undermine the principle of tolerance, a central feature of Carnap's late philosophy, which has, until quite recently, been less discussed and understood than some of Carnap's other doctrines. It was only Gödel's critique that has aroused interest in Carnap's later view again, especially in the writings of Goldfarb, Ricketts, Creath, Parsons, Potter, and Friedman.

Despite this revival of interest, the impression persists that the essential doctrines of the logical empiricists, insofar as they are still distinguishable from almost any doctrine (including even "continental philosophy"), have been decisively refuted. We will argue that this is in fact not so, at least for the case of Gödels supposed refutation (we also think that related defenses can be mustered against Quine and company, but that is another matter). Not only is the framework of logical empiricism quite distinct from, and incompatible with, that of traditional and of continental philosophy; we believe that in its most developed form, in Carnap's later philosophy, it remains a viable platform for a continuation of the Vienna Circle's mission of enlightenment.

Gödel's posthumously published critique of Carnap is given in a paper entitled "Is mathematics syntax of language", which was written for the Schilpp volume on Carnap, but not included in it. The reason it was not included is apparently that Gödel was never quite satisfied with it; indeed he produced at least 6 different versions over that many years. When he finally withdrew from the project, he wrote to Schilpp:

> The fact is that I have completed several different versions, but none of them satisfies me. It is easy to allege very weighty and striking arguments in favor of my views, but a complete elucidation of the situation turned out to be more difficult than I had anticipated, doubtless in consequence of the fact that the subject matter is closely related to, and in part identical with, one of the basic problems of philosophy, namely the question of the objectivity of concepts and their relations. (quoted by Goldfarb 1995, p. 324)

Gödel's paper addresses what he calls the "syntactic program", which he summarizes in the two assertions that *mathematics can be interpreted to be syntax of language*, and *mathematical sentences have no content.* About these claims, he thinks, Carnap's investigations in *Logical Syntax*, and those of the Hilbert school, have shown the following:

> (1) *Mathematics can be interpreted to be syntax of language* only if the terms "language" or "syntax" or "interpreting" are taken in a very generalized or attenuated sense, or if only a small part of what is commonly regarded as "mathematics" is acknowledged as such. . . .

> (2) *Mathematical sentences have no content* only if the term "content" is taken from the beginning in a sense acceptable only to empiricists and not well founded even from the empirical standpoint. (Gödel *1953/9-III, p. 337)

Regarding assertion (2), Gödel maintains that the examination of the syntactical viewpoint "leads to the conclusion that there *do* exist mathematical objects and facts which are exactly as objective (i.e., independent of our conventions or constructions) as physical or psychological objects and facts". (ibid.) It is clear how Carnap would have responded to this point; he generally regarded questions (asked outside the context of a particular linguistic framework) about the existence or non-existence of any objects, whether physical or mathematical, as empty of cognitive significance. To *that* extent, Gödel and Carnap are in complete agreement that there is no difference between the two kinds of objects.

Regarding the assertion (1), that mathematics can be interpreted to be syntax of language, Gödel maintains that it can be *proved false* (ibid.). This argument is the one that has attracted most of the recent philosophical interest. Most commentators, while acknowledging that Carnap would not have accepted the terms in which Gödel's criticism is framed, see it as having some force, and as showing how Carnap's principle of tolerance (which he continued to maintain all his life, long after the 1934 syntax doctrine had been left behind) is not as neutral as he intended. The responses to this diagnosis vary; some (e.g. Goldfarb and Ricketts 1992) think it possible to maintain a weakened or diluted (though rather empty) principle of tolerance despite Gödel's criticism; others (e.g. Friedman 1999, Potter 2000) are more pessimistic about such prospects, and see Gödel's criticism as more or less conclusive, and the principle of tolerance as self-undermining (Friedman 1999, p. 229).

In what follows, we reject the *premise* of all such responses, and we hold that Carnap need not make any concessions to Gödel at all. Indeed, we diagnose in Gödel's argument a rather subtle fallacy that has so far escaped the commentators.

I.

Here is one of Gödel's formulations of the particular argument at issue:

> ... a rule about the truth of sentences can be called *syntactical* only if it is clear from its formulation, or if it somehow can be known beforehand, that it does not imply the truth or

falsehood of any "factual" sentence (i.e., one whose truth, owing to the semantical rules of the language, depends on extralinguistic facts). This requirement not only follows from the concept of a convention about the use of symbols, but also from the fact that it is the lack of content of mathematics upon which its apriori admissability in spite of strict empiricism is to be based. The requirement under discussion implies that the rules of syntaxt must be demonstrably consistent, since from an inconsistency *every* proposition follows, all factual propositions included. (Gödel *1953/9-III, p. 339)

And similarly:

To eliminate mathematical intuition or empirical induction by positing the mathematical axioms to be true by convention is not possible. For, before any such convention can be made, mathematical axioms of the same power or empirical findings with a similar content are necessary already in order to prove the consistency of the envisaged convention. A consistency proof, however, is indispensable because it belongs to the concept of a convention that one knows it does not imply any propositions which can be falsified by observation (which, in the case of mathematical "conventions", is equivalent with consistency ...).

This argument can be paraphrased in something like the following four steps:

(i) For mathematics to be interpreted as syntax of language — and thus empty of empirical content — it must be proved that no syntactic (i.e. purely linguistic) stipulation can possibly have empirical consequences; otherwise that supposed convention is in danger of making claims about the empirical world on purely arbitrary, definitional (however convenient or practical) grounds.

(ii) But even a decision for a quite weak language framework (as restricted as primitive recursive arithmetic) has the consequence, by Gödel's own second incompleteness theorem, that the consistency of the chosen language cannot be proved without further resources.

(iii) Any proof that our chosen language is consistent, then, presupposes the consistency of the stronger metalanguage required for the proof; so the attempt to prove consistency -- at any level -- incurs an infinite regress. We therefore cannot with certainty exclude the possibility that the chosen language is inconsistent and thus has empirical consequences (as it would then imply not only every mathematical sentence, but every empirical sentence).

(iv) Conclusion: The requirement of step (i) is not met, so mathematics cannot be syntax of language.

Now our *main point* is this: Gödel's argument — or one close to it — does indeed establish that it cannot be *proved* that mathematics *is* syntax. However it does not, as Gödel claims, *prove* that mathematics *is not* syntax.

A rather subtle fallacy appears to have occurred right at the outset, in step (i). Gödel there requires that the language in question (i.e. whatever is taken as the mathematical system) be *provably* consistent. He says e.g. that "the rules of syntax must be demonstrably consistent, since from an inconsistency *every* proposition follows" (*ibid.*). But why "demonstrably consistent"? Why not just "consistent"? Consider for instance the following analogous argument:

(i´) For spacetime to be flat, it must be shown that such and such conditions (indicating curvature) do not obtain in any region of the universe.

(ii´) But the required observations may be affected by the presence of curvature — e.g. measurements may be distorted or instruments become unreliable. Moreover, the universe may be infinite, or there may be regions that are in principle inaccessible to us.

(iii´) We can therefore never be certain that our observations are conclusive, and that the said conditions do not obtain.

(iv´) Conclusion: The requirement of step (i´) cannot be met, so spacetime cannot be flat.

Of course, this is an argument about empirical matters, not mathematical ones; but it has the same form, and it is no less sound than Gödel's. The conclusion is unwarranted; what follows instead is just that we cannot *know* that spacetime is flat. The physical conditions that must be met for spacetime to be curved might obtain and yet be undetectable. The key phrase in the argument is "it must be shown that ..." in (i´). For spacetime to be flat, one requires instead only "it must be the case that...".

Analogously, in step (i) of Gödel's argument the key phrase is "it must be proved that ..."; and of course, this condition cannot be met. But here, too, the requirement is too strong; for mathematics to be syntax of language one requires only that the language *is* consistent, not that its consistency be provable. While Gödel has therefore *not* proved that mathematics cannot be interpreted as syntax of language, he has shown the weaker fact that the syntactic interpretation itself cannot be proved. That is, if mathematics *is* indeed syntax of language, then thiscannot be proved correct. But *this* result, far from undermining Carnap's position, seems to be in complete harmony with it. For the syntactic view implies the vacuity of mathematics, which would surely be violated if that viewpoint could itself be *proved* mathematically, since that result would itself be a non-trivial mathematical proposition.

II.

In the very first passage quoted above — and elsewhere in his critical essay — Gödel speaks somewhat loosely of an "interpretation" of mathematics. Let us follow him in this terminology, meaning a particular conception of mathematics, for example as is provided by a formal system for the statement and proof of propositions of mathematics. Different systems like set theory, type theory, or first-order arithmetic may be regarded as descriptions of different philosophical or conceptual standpoints like platonism, logicism, formalism, and so on. This is one way of making the notions of an "interpretation" or "viewpoint" more precise, at least for the purpose at hand.

But what then is the "syntactic interpretation" that Gödel refers to? For Carnap, it is not to be identified with any particular formal system, but is rather a conception of how mathematics relates to the various different systems that can be used to interpret it. Specifically, it is a way of expressing *neutrality* among the various other interpretations. None of them, Carnap thought, can claim to be *true* or correct in any absolute or objective sense; mathematical truth is language-relative, just as is mathematical existence. The choice of an interpretation then becomes a matter, not of truth, but of the most useful language for some purpose, e.g. for natural science.

This pragmatic neutrality, for Carnap, is fundamental. It does not rest on antecedent assumptions about the nature of reality or of language, such as, for instance, the existence or necessity of an analytic-synthetic distinction. On the contrary, Carnap regards the availability of that distinction as a *constraint* on a good (i.e. useful) interpretation; an interpretation must, he thought, account for the central role of mathematics in empirical science, while also doing justice to the importance, stressed by so many scientists,[1] of separating statements internal to the mathematical framework from those that involve a physical interpretation of this framework.

Its ability to account for this dual role of mathematics (as analytic and purely logical on the one hand, and yet central to empirical science on the other) gave credence, Carnap thought, to the syntactic interpretation of mathematics. But he never, of course, claimed this interpretation to be right; in fact he went to some lengths to deny any such claim. As is well-known, he argued that *all* interpretations of mathematics should be regarded rather as *proposals* for the construction of our scientific language.

This would of course seem most unsatisfactory to Gödel, as it makes the selection among different possible interpretations at least partly a practical matter; the interpretation of mathematics is not itself a mathematical problem, nor even one that can be regarded as having a single "correct" answer. He might have objected (like Beth in his contribution to the Schilpp volume) that the various proposed interpretations of mathematics cannot, after all, be treated on an equal basis (as the principle of tolerance urged), since the standpoint adopted in the investigation of any one of them presupposes linguistic resources that those who favor simpler languages (or other interpretations) would rule out. Certainly there is a sense in which advocates or speakers of simpler languages fail to understand richer languages. They can manipulate the purely formal calculus of a richer language, of course, but are unable initially to give it any meaning (Carnap 1963, pp. 872-873).

At this stage in a pragmatic comparison of languages, the principle of tolerance is admittedly not perfectly neutral. This is the case in any conversation where one side does not know the other's language. But to join in the pragmatic discussion about the relative merits of different languages, one must *learn* the other's language. This is always the case where two different cultures or intellectual frameworks confront each other; knowing the languages of both sides is the price of entering the discussion. To Carnap, it would have seemed that the *refusal* to learn

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[1] Carnap often quoted Einstein, "Insofar as the sentences of mathematics refer to reality, they are not certain, and insofar as they are certain, they do not refer to reality. . . I place such a high value on this conception of geometry because without it, the discovery of the theory of relativity would have been impossible for me." (1921, pp. 3-6)

another's language could only be motivated by a prejudice, most likely a conviction that one's own favored proposal (e.g. for the interpretation of mathematics) is *the correct one.*

Carnap's own skepticism about the objective correctness of any particular interpretation of mathematics was not motivated by any such considerations, but rather by the recognition that there is no fact of the matter about such proposal-statements. At best, they can be taken as *pragmatic* statements regarding the utility or desirability of interpreting mathematics in some specified way.

In *Logical Syntax*, where this view was put forward, Carnap took the additional step of proposing explicitly a principle of tolerance, according to which such pragmatic, interpretive views about the character of mathematics cannot serve as grounds for rejecting or restricting mathematics itself, or any part of it. It was of central importance to Carnap that the realm of logic and mathematics be independent of any moral or optative considerations (Carnap 1963, pp. 1001-1003); as he put it, "in logic there are no morals". This proposal remained of great importance to Carnap even after other elements of the syntax program were abandoned; if it is followed, all prohibitions of linguistic forms, and rules against learning certain languages lose their force.

III.

Gödel could still have objected that despite this apparent "neutrality", his own position is not accommodated; the so-called "principle of tolerance" is not really very tolerant after all. That it is not *completely* neutral, Carnap would readily have acknowledged, as we have just seen. However, before going on he would also have pointed out that there is a sense in which this issue, like so many other traditional philosophical issues, cannot be argued about. In his terms, it is not a *cognitive* issue; it is not an *internal* question (to use the terminology of "Empiricism, Semantics, and Ontology"). It is an *external* question, and thus necessarily pragmatic, not cognitive. This distinction between internal and external questions is tied to the Principle of Tolerance; as pointed out above, it does not depend on the analytic/synthetic distinction. On the contrary, the availability of such a distinction is taken as a "pragmatic" constraint on scientific languages, because experience has shown it to be useful for the purposes of science.

The question of neutrality is then an external question. Perhaps it belongs among those external questions that Carnap called "optative" questions, i.e. questions of values. This is not to say that it is off-limits for discussion, but by Carnap's standards, our language for dealing with such questions is extremely vague and unsatisfactory[2]. In the spirit of clarification, however, he might (we speculate) have said something like the following about the "neutrality" of the view suggested by the principle of tolerance. The sense in which this view really is *more neutral* than others (despite the fact that it disqualifies certain other views) is its adoption of neutrality *itself* as the governing value to be optimized. Of course some people may reject that value, and to them, obviously, the principle of tolerance is *not* neutral.

But this is a common problem, in many practical and scientific situations. Democratic and open societies, for instance, cannot be *completely* democratic and open; they have to limit the rights of those who reject democracy and openness itself. Science itself cannot be perfectly neutral or accommodating, as it must exclude or ignore those who refuse to recognize the evidence accepted by the scientific community.[3] In practical terms, then, *absolute* or *complete* neutrality is impossible. But Carnap's view that makes it an explicit goal to *maximize* neutrality (in the sense here suggested) may have a claim to be "*more* neutral" than others.

One way in which such a response is *uncharacteristic* of Carnap is that it leaves open the question how to explicate the vague term "neutrality". But it is not inconceivable that he might have advanced a further proposal, e.g. to explicate "neutrality" in terms of formal (or semantic) "explicitness", or some similar concept, as follows: neutrality would be regarded as maximized in a framework of discussion that allowed various views (proposed languages) to state their assumptions and their terms (including categorial terms) as clearly and unambiguously as possible, in a way that their own users regarded as adequate — but excluding all normative or optative components of those views. (Not that Carnap would suggest the latter should not be discussed; on the contrary, he regarded them as very important. But they cannot be adequately addressed until the views themselves are clearly and completely, i.e. explicitly, identified so that their consequences are known.)[4]

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[2] See his reflections on this issue in section 32 of Carnap 1963.
[3] As Carnap had noted long before Kuhn, e.g. in Carnap 1932.
[4] Michael Friedman (1999) argues, based on arguments of Beth and Gödel, that the principle of tolerance undermines itself because it fails to be neutral among all philosophical positions (pp. 227-233). But as we

IV.

Returning to the specious argument about spacetime in Section I, assume the truth of (ii´) for the sake of argument; that is, suppose we cannot determine with certainty whether curvature exists. For Carnap, "spacetime is flat" would then be either meaningless (because empirically untestable) or analytic (i.e. a definition). In the latter case, we *propose* to *call* spacetime "flat", as a matter of convention or stipulation — just as one might define the notion of being "at rest" relative to the position of the earth for the purpose of describing the motions of the heavenly bodies. In the syntactic interpretation of mathematics, such a stipulation amounts to identifying mathematical truth with truth in a particular, given language.

As a realist, Gödel would presumably have rejected the conclusion that the flatness of spacetime is a pseudo-question. For him, there was a fact of the matter about the true curvature of spacetime, however unknowable it may be, now or in principle. In mathematics, too, he would have insisted that the notion of truth is absolute, and not ultimately dependent on a particular choice of language-framework. His well-known mathematical platonism no more permits mathematical truth to be dependent on our linguistic conventions than his realism permits spacetime curvature to be dependent on our physical conventions.

But Gödel did not *refute* the syntactic interpretation, as he claimed. Instead he showed that Carnap could not prove the syntactic view. Far from claiming to have proved it, however, Carnap never even claimed it to be *true*. The syntactic interpretation was understood as a *proposal* for adopting certain ways of speaking and proceeding with respect to the relation between mathematics and its applications. The merits of such a proposal can, of course, be discussed. But the question whether we should adopt this proposal is, for Carnap, a *practical* one; the *proposal itself* is not to be taken as the kind of sentence to which the concepts "proof" or "disproof" apply. It is true that the syntactic interpretation presumes the consistency of mathematics, but so, for that matter, does Gödel's platonist one; neither position requires this consistency itself to be *provable*. This is not to say that no questions *relevant* to the merits of such a proposal can be decided by proof; Gödel's incompleteness results are an obvious example. But these results do not, by themselves, determine a single, correct interpretation of mathematics. Nor do they suffice to *refute* Carnap's syntactic interpretation.

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suggest here, complete neutrality is not only not what Carnap aspired to, but is in fact impossible. The point, rather, is to make something like the "highest feasible degree of neutrality" a goal.

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